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Juraj Hromkovič

Algorithmics for Hard Problems

Introduction to Combinatorial Optimization, Randomization, Approximation, and Heuristics

Second Edition

With 71 Figures

Springer

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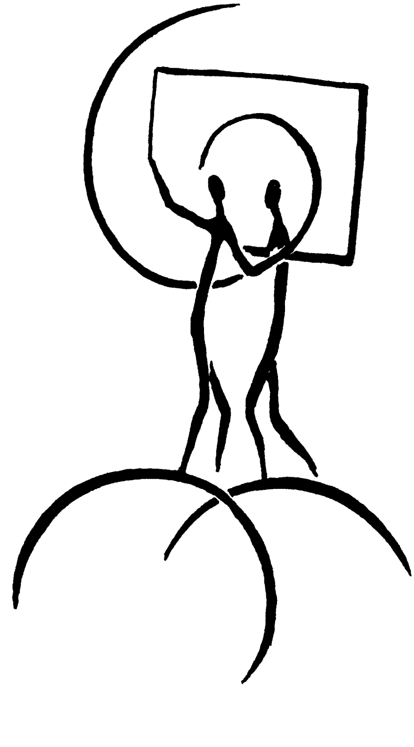
To Petra and Paula



You have been told also that life is darkness,
and in your weariness you echo what was said by the weary.

And I say that life is indeed darkness
save when there is urge,

And all urge is blind save when there is knowledge,

And all knowledge is vain save where there is work,

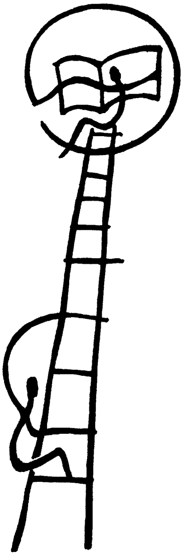
And all work is empty save when there is love;

And when you work with love you bind yourself
to yourself, and to one another, and to God . . .

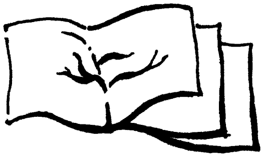
Work is love made visible.

And if you cannot work with love but only with distances,
it is better that you should leave your work
and sit at the gate of the temple and take alms of those
who work with joy.

Kahlil Gibran
The Prophet



Preface to the Second, Enlarged Edition



The term algorithm is the central notion of computer science and algorithmics is one of the few fundamental kernels of theoretical computer science. Recent developments confirm this claim. Hardly any other area of theoretical computer science has been more lively and has achieved comparably deep progress and breakthroughs so fascinating (such as the PCP-theorem and efficient algorithms for primality testing) in recent years. The most exciting development happened exactly in the field of algorithmics for hard problems, which is the topic of this book.

The goal of this textbook is to give a transparent, systematic introduction to the concepts and to the methods for designing algorithms for hard problems. Simplicity is the main educational characteristic of this textbook. All ideas, concepts, algorithms, analyses, and proofs are first explained in an informal way in order to develop the right intuition, and then carefully specified in detail. Following this strategy we preferred to illustrate the algorithm design methods using the most transparent examples rather than to present the best, but too technical, results. The consequence is that there are sections where the first edition of this book does not go deep enough for advanced courses.

To smooth this drawback in the second edition, we extended the materials for some of the topics of central interest – randomized algorithms for primality testing and applications of linear programming in the design of approximation algorithms. This second edition contains both the Solovay-Strassen algorithm and the Miller-Rabin algorithm for primality testing with a selfcontained analysis of their behaviour (error probability). In order to give all related details, we extended the section about algebra and number theory in an appropriate way. To explain the power of the method of relaxation to linear programming, we added the concept of LP-duality and the presentation of the primal-dual method. As an introduction to this topic we used the Ford-Fulkerson pseudo-polynomial-time algorithm for the maximum flow problem, which is in the section about pseudo-polynomial-time algorithms.

In addition to extending some parts of the book, numerous small improvements and corrections were performed. I am indebted to all those who sent me their comments and suggestions. Especially, I would like to thank Dirk Bongartz, Hans-Joachim Böckenhauer, David Buttgereit, Thomas Deselaers, Bernd Hentschel, Frank Kehren, Thorsten Uthke, Jan van Leeuven, Sebastian Seibert, Koichi Wada, Manuel Wahle, Dieter Weckauf, and Frank Wessel who carefully read and commented on large parts of this book. Special thanks go to Dirk Bongartz and Hans-Joachim Böckenhauer for fruitful discussions on new parts of the book and for their valuable suggestions. The expertise of our LaTeX experts Markus Mohr and Manuel Wahle was very helpful and is much appreciated. The excellent cooperation with Ingeborg Mayer and Alfred Hofmann from Springer-Verlag is gratefully acknowledged.

Last but not least I would like to express my deepest thanks to Peter Widmayer for encouraging me to make the work on this book a never-ending story.

Aachen, October 2002 *Juraj Hromkovič*

Preface

Algorithmic design, especially for hard problems, is more essential for success in solving them than any standard improvement of current computer technologies. Because of this, the design of algorithms for solving hard problems is the core of current algorithmic research from the theoretical point of view as well as from the practical point of view. There are many general textbooks on algorithmics, and several specialized books devoted to particular approaches such as local search, randomization, approximation algorithms, or heuristics. But there is no textbook that focuses on the design of algorithms for hard computing tasks, and that systematically explains, combines, and compares the main possibilities for attacking hard algorithmic problems. As this topic is fundamental for computer science, this book tries to close this gap.

Another motivation, and probably the main reason for writing this book, is connected to education. The considered area has developed very dynamically in recent years and the research on this topic discovered several profound results, new concepts, and new methods. Some of the achieved contributions are so fundamental that one can speak about paradigms which should be included in the education of every computer science student. Unfortunately, this is very far from reality. This is because these paradigms are not sufficiently known in the computer science community, and so they are insufficiently communicated to students and practitioners. The main reason for this unpleasant situation is that simple explanations and transparent presentations of the new contributions of algorithmics and complexity theory, especially in the area of randomized and approximation algorithms, are missing on the level of textbooks for introductory courses. This is the typical situation when principal contributions, whose seeping into the folklore of the particular scientific discipline is only a question of time, are still not recognized as paradigms in the broad community, and even considered to be too hard and too special for basic courses by non-specialists in this area. Our aim is to try to speed up this transformation of paradigmatic research results into educational folklore.

This book should provide a "cheap ticket" to algorithmics for hard problems. Cheap does not mean that the matter presented in this introductory material is not precisely explained in detail and in its context, but that it is presented as transparently as possible, and formalized by using mathematics that is as simple as possible for this purpose. Thus, the main goal of this book can be formulated as the following optimization problem.

Input: A computer science student or a practitioner

Constraints:

- To teach the input the main ideas, concepts, and algorithm design techniques (such as pseudo-polynomial-time algorithms, parameterized complexity, local search, branch-and-bound, relaxation to linear programming, randomized algorithms, approximation algorithms, simulated annealing, genetic algorithms, etc.) for solving hard problems in a transparent and well-understandable way.
- To explain the topic on the level of clear, informal ideas as well as on the precise formal level, and to be self-contained with respect to all mathematics used.
- To discuss the possibilities to combine different methods in order to attack specific hard problems as well as a possible speedup by parallelization.
- To explain methods for theoretical and experimental comparisons of different approaches to solving particular problems.

Costs: The expected time that an input needs to learn the topic of the book (particularly, the level of abstractions of mathematics used and the hardness of mathematical proofs).

Objective: Minimization.

I hope that this book provides a feasible solution to this hard optimization problem. To judge the quality (approximation ratio) of the solution provided in this book is left to the reader.

I would like to express my deepest thanks to Hans-Joachim Böckenhauer, Erich Valkema, and Koichi Wada for carefully reading the whole manuscript and for their numerous comments and suggestions. I am indebted to Ivana Černá, Vladimír Černý, Alexander Ferrein, Ralf Klasing, Dana Pardubská, Hartmut Schmeck, Georg Schnitger, Karol Tauber, Ingo Wegener, and Peter Widmayer for interesting discussions or their comments on earlier drafts of this book. Special thanks go to Hans Wössner and the team of Springer-Verlag for their excellent assistance during the whole process of the manuscript preparation. The expertise and helpfulness of our LaTeX expert Alexander Ferrein was very useful and is much appreciated.

Last but not least I would like to thank Tanja for her patience with me during the work on this book.

Aachen, March 2001 *Juraj Hromkovič*

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