

# A Graph Matching and Energy Minimization Based Algorithm for Lunar Surface Image Mosaic

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**Abstract.** This paper concerns the problem of lunar surface image mosaic, including both image registration and image stitching procedures. A wide viewing composite obtained by mosaic technique plays an important role in many lunar rover's operations. Considering particular characters in lunar surface images, such as large geometrical deformations, significant illumination differences and repeated patterns, previous image mosaic techniques often fail to create a qualified composite. In this paper, a novel algorithm is introduced to tackle the lunar surface image mosaic problem. Specifically, in the image registration procedure, to deal with the misregistration problem caused by large geometrical deformation and repeated patterns, structural information is introduced to solve the feature correspondence by formulating it as a graph matching problem. In the image stitching procedure, an energy minimization method is proposed based on the graduated nonconvexity and concavity procedure (GNCCP), to handle the visible seams caused by illumination differences and ghosting problem caused by large parallax in the overlapped area. Comparative experiments on real lunar surface images acquired by Yutu rover and Apollo image gallery validate the effectiveness of the proposed method.

**Keywords:** Lunar image mosaic · Graph matching · Energy Minimization · GNCCP

## 1 Introduction

Image mosaic is the technique to combine two or more images into a high resolution and wide viewing composite. It lays the foundation for many lunar rover operations. For instance, a wide viewing composite can benefit the lunar rover's self location, which further helps its navigation operations. The special environment of lunar surface leads to certain particular image characters. For instance, the lunar surface images taken by the moving lunar rover often have large geometrical deformations, and suffer from large illumination changes caused by different sunlight angles. Besides, the barren lunar surface, mainly composed of

rocks and dust, makes the acquired images contain lots of repeated patterns, and often lack salient features. Thus performing image mosaic on lunar surface images remains a challenging task, and there are few techniques dedicated to the lunar surface image mosaic problem.

Image mosaic mainly includes two procedures, i.e., image registration and image stitching. Image registration is the process that transforms images from different views into one coordinate system. A comprehensive survey on image registration methods is in [1], in which the image registration techniques are classified into area-based methods and feature-based methods. In the last two decades, together with the emergence of a bunch of splendid local feature descriptors, e.g. SIFT [2] and SURF [3], the feature-based methods become more popular in image registration. The success of feature-based methods can be attributed to the rotation and scale invariance of the features, thus they can be used to register images with significant deformations, while the area-based methods are only applicable on images with translational and rotational transformations. However, the feature-based methods still fail in dealing with lunar surface image registration, mainly due to less distinctive local features and repeated patterns in the lunar surface images. That is because they only utilized the appearance similarity, without considering other useful information, such as the structure information.

Image stitching takes the registered images as input to create a wide viewing composite. It can be very difficult for lunar surface images because perfect image registration can hardly be obtained and real lunar surface images are rarely under constant light exposure. Therefore, blurring caused by misregistration, visible seams caused by light exposure difference and ghosting caused by possible moving objects often occur in lunar surface images. Traditional mosaic algorithms also aimed at the above problems or part of them. For instance, early in the image mosaic research, researchers [4,5] tried to eliminate the blurring and visible seams by a weighted average method, which is called feathering. However, when the misregistration is significant and moving objects exist, the feathering usually results in visual artefacts and ghosting in the composite. Some other researchers used the optimal seam method [6,7] to deal with the moving objects. By introducing the Dijkstra's algorithm [9], the method can find a path avoiding cutting through the moving objects, which, therefore, makes the moving objects all in or all out of the composite. Unfortunately, this method may fail when the light exposure difference is significant, because it may treat the areas with different light exposures as areas with moving objects. In [8] they used the region of difference (ROD) to find the regions where the moving objects lie and then choose the right region to keep, which to some extent improved the the optimal seam method. But it is not robust against significant light exposure in lunar surface images. Generally, rare approaches can well tackle the image stitching problem under all the above specific difficulties in lunar surface images.

In this paper, we propose an novel algorithm for lunar surface image mosaic. Specifically, in the image registration procedure, to deal with the repeated patterns in the lunar surface images, the structural information of the feature points

is introduced to solve the feature correspondence problem by formulating it as a graph matching problem, which is approximately solved by a probabilistic spectral graph matching algorithm. In the image stitching procedure, exposure compensation is firstly conducted to correct the illumination difference between lunar surface images. Then an energy minimization method based on the recently proposed graduated nonconvexity and concavity procedure (GNCCP) [11] is used to handle the ghosting problem caused by large parallax in the overlapped area.

## 2 The Proposed Lunar Surface Image Mosaic Algorithm

There are mainly two procedures in the proposed algorithm: image registration and image stitching. The detailed discussion is given below.

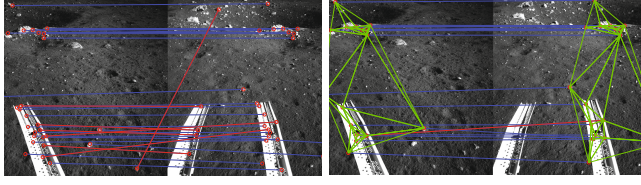
### 2.1 Graph Matching Based Image Registration

To automatically utilize the information from the input images to create a high resolution and wide viewing composite, the very beginning step is geometrically aligning the images, i.e. to overlapping the input images of the same scene.

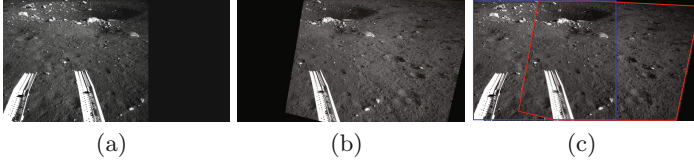
Given two images, the reference image  $I_1$  and the sensed image  $I_2$ , taken from different viewpoints with overlapping area, the image registration procedure is to find a geometric transformation matrix  $T$ . Matrix  $T$  can project the sensed image  $I_2$  to the reference image  $I_1$ , so that two images can share the same coordinate system and the overlapping scene of two images should have the same coordinates. The projective matrix  $T$  is estimated by a modified feature based algorithm. This is done in the following steps.

**Feature Extraction.** Salient and distinctive objects of the input images  $I_1, I_2$  are automatically detected by the Speeded Up Robust Features (SURF). For each feature point, it has a 64-dimensional descriptor gathering the information of the surrounding area. In this paper, we denote the descriptors of image  $I_1$  and  $I_2$  as  $D_1, D_2$ , where  $D_1 \in \mathbb{R}^{M \times 64}, D_2 \in \mathbb{R}^{N \times 64}$ .  $M, N$  is the number of feature points of  $I_1, I_2$  respectively.

**Feature Correspondence.** In this step, the correspondence between the features detected in the sensed image and reference image is established. Different from some conventional methods [19] which only use the descriptor similarity, in the proposed algorithm we take the structural information into account and apply a graph matching method to tackle the feature correspondence problem. We use our newly published probabilistic spectral graph matching algorithm [23] to tackle this combinational optimization problem. This algorithm approximately solve the optimization problem (4) by spectral decomposition. After the spectral matching procedure, a probabilistic assignment is obtained. We can also set an probabilistic threshold to control the numbers of the final correspondence feature points.



**Fig. 1.** Feature correspondence results. The correct correspondence points are connected by blue lines and The wrong correspondence points are connected by red lines. **a** Without combining the structural information. **b** Combining the structural information



**Fig. 2.** Image Registration results. **a** The reference image after registration; **b** The sensed image after registration; **c** The two input images in the same coordinate system

The example feature correspondence experiment is conducted on real lunar surface images acquired by Yutu lunar rover. The results are illustrated in Fig.1. As we can see, the proposed algorithm which combines the structural information gets a more accurate correspondence sets.

**Image Transformation and Interpolation.** After the feature correspondence has been established, we use RANSAC [13] to robustly get an estimation of the projective matrix  $T$  by utilizing the correspondence points. Next we apply the matrix  $T$ . The registration result of two lunar surface images is illustrated in Fig.2. As we can see, after the registration procedure, two input images have the same size and the overlapping area has the same coordinates in two images.

## 2.2 Energy Minimization Based Image Stitching

In the image stitching stage, we still have to decide how to blend the images to create a clean composite. In this paper, we use an energy minimization method to choose the right image for the composite at every pixel. The pre-processing and post-processing are also given to enhance composite quality.

**Exposure Compensation.** To adjust the exposure, we assume the reflective properties of the scene remain unchanged. This allows us to use a linear approximation to make the adjustment in intensity. Given two images  $I_1, I_2$ , the intensity of the images is denoted as  $e_1, e_2$ . Then the exposure compensation is done by a linear approximation as in

$$e_2 = \alpha e_1 + \beta. \quad (1)$$

The gain  $\alpha$  and bias  $\beta$  are found by utilizing the intensity of the correspondence feature points. We apply the linear regression to acquire the optimal  $\alpha$  and  $\beta$ , which has the least mean square error. Next  $\alpha$  and  $\beta$  are used to adjust the intensity of registered image  $I_2$  at every pixel.

**Pixel Labelling.** In this paper, we use the pixel labelling method to choose the right image for the composite to achieve a smooth transition between the images.

Given a registered source image sets  $I = \{I_1, I_2, \dots, I_k\}$ , indexed by a label sets  $L = \{l_1, l_2, \dots, l_k\}$ , where  $k$  is the number of source images. The image stitching problem is assigning every pixel  $p$  in composite a label in the label set  $P = \{p_1, p_2, \dots, p_n\}$ , where  $n$  is the number of pixels in the composite. Then the image stitching problem is converted to a pixel labelling task, i.e. finding a mapping function  $F$  between sets  $P$  and  $L$ .

$$F : P \rightarrow L; \quad F = \{f_1, f_2, \dots, f_n\}, \quad (2)$$

where  $f_i$  denotes the label of pixel  $p_i$ .

In the proposed algorithm, we use an energy minimization method to solve the pixel labelling problem. We build the energy function based on the assumption that the natural images can be formulated as Markov Random Field(MRF) [10], i.e. the images are local smoothing. The smooth prior makes the label of each pixel affect by both the information of current pixel and the neighbouring pixels. Therefore when moving objects and misregistration area exist in the overlapping scene, the label tend to totally keep or remove the moving objects and put a seam at the misregistration area.

In this paper, the energy function  $E(F)$  is defined by

$$E(F) = \sum_p S(p, f_p) + \sum_p \sum_{\{p,q\} \in \mathcal{N}} V_{pq}(f_p, f_q), \quad (3)$$

where  $S(p, f_p)$  is called data energy, it penalizes assigning the label  $f_p$  to pixel  $p$ .  $V_{pq}(f_p, f_q)$  denotes the smooth energy built based on the smooth prior. It penalizes the inconsistency between neighbouring pixels.  $\mathcal{N}$  is the neighbouring system of the image. Then to find an optimal label set is equivalent to find a mapping function  $f$ , which makes the energy function has the global minimum as  $F^* = \arg \min_F E(F)$ . Particularly in this paper, the data energy  $S(p, f_p)$  for selecting image  $f_p$  as the label at pixel  $p$  is given as

$$S(p, f_p) = \begin{cases} 0, & \text{if } I_{f_p}(p) \text{ is valid} \\ +\infty, & \text{otherwise} \end{cases}, \quad (4)$$

where a valid pixel is a pixel in the original image but not a padding from the image transformation. If all the registered images is valid at pixel  $p$ , the data energy is all set to zero, which is reasonable when no prior is given about which image is better.

The smooth energy penalizes the inconsistency between neighbouring pixels, i.e. assigning different labels to adjacent pixels. The smooth energy in this paper is a modified function derived from [12]. The energy is defined by the inconsistency in color and gradient space between neighbours as follow.

$$V_{pq}(f_p, f_q) = \begin{cases} Y + \lambda Z, & \text{if } \{p, q\} \in \mathcal{N} \\ 0, & \text{otherwise} \end{cases}, \quad (5)$$

where  $Y$  evaluates the inconsistency in color space. It is the euclidean distance in RGB space defined by  $Y = \|I_{f_p}(p) - I_{f_q}(p)\| + \|I_{f_p}(q) - I_{f_q}(q)\|$ .  $Z$  evaluates the inconsistency in gradient space, it can protect the edges in images. It is given by  $Z = \|\nabla_{pq} I_{f_p}\| + \|\nabla_{pq} I_{f_q}\|$ , where  $\nabla_{pq} I$  is the gradient of image  $I$  in RGB space.  $\lambda$  is used to balance the color and gradient inconsistency.

Minimizing the energy function(9) is a typical combinational optimization problem, which is an NP-hard problem with factorial complexity. In this paper we use our recently published GNCCP algorithm to tackle (9). GNCCP was originally proposed to approximately solve the assignment problem under one-to-one constraint [11]. Recently GNCCP was generalized to solve the Maximum A Posteriori(MAP) estimation in MRF, called GNCCP MAP algorithm [22]. Here in this paper we modify GNCCP to solve the energy minimization problem in MRF. The GNCCP energy minimization algorithm is given in Algorithm 1.

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**Algorithm 1** Energy minimization algorithm

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**Input:** Energy matrix  $Q$  corresponding to the pairwise smooth energy and  $A$  corresponding to the data energy

**Initialization:**  $x = \frac{1}{nk}$ ,  $\varsigma = 1$

**repeat**

**repeat**

$y = \arg \min_y \nabla E_\varsigma(x)^T y, s.t. y \in \Omega$

$\alpha = \arg \min_\alpha E_\varsigma(x + \alpha(y - x))^T, s.t. 0 \leq \alpha \leq 1$

$x \leftarrow x + \alpha(y - x)$

**until** converged

$\varsigma = \varsigma - d\varsigma$

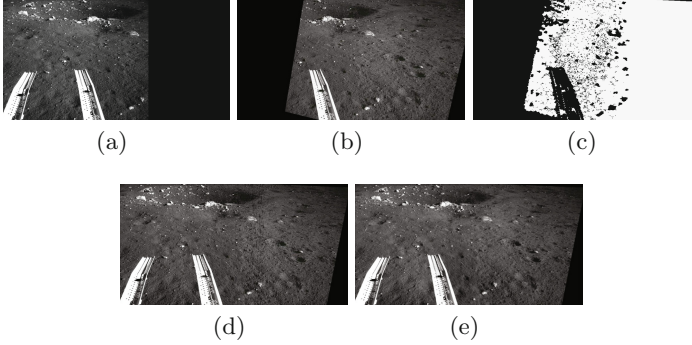
**until**  $\varsigma < -1 \vee x \in \Pi$

**Output:** An assignment vector  $x$

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In the algorithm, by relaxing the integer constrain  $\Pi$  to its convex hull  $\Omega$ , GNCCP is proved to realize the Convex-Concave Relaxation Procedure(CCRP) [14], which has an ideal convergence to the global minimum, detailed proof of GNCCP is given in [11]. The final label set  $F$  is finally obtained by utilizing assignment vector  $x$ . The labelling result conducted on two registered lunar surface images is given in Fig.3(c).

**Gradient Reconstruction.** Once the label of each pixel has been computed, we can directly copy the information of the input images according to the label



**Fig. 3.** An illustration of the image stitching procedure. **a b** are the registered input images. **c** is the labelling result of the proposed method. **d** is composite directly copying the values of the input images according to the labelling result. **e** is the composite reconstructed from the gradient domain

to form the final composite. But the pixels will still exist some degree of inconsistency. We use the gradient domain reconstruction technique [15] as the post-process to minimize the inconsistency. Rather than copying pixel values, the gradient domain reconstruction copies the gradients of the registered images according to the labelling result. The actual pixel values of the composite image  $C$  are then computed by solving a Poisson equation that best matches the gradients and satisfies the boundary condition  $\Omega$  given by the labelling result above.

$$\min_{C(p)} \|\nabla C(p) - \nabla I_{f_p}(p)\| \text{ s.t. } C(p) = I_{f_p}(p), \text{ for } p \in \Omega \quad (6)$$

In Eq(19),  $\nabla X$  denotes the gradient of  $X$  and  $\Omega$  is the boundary of the composite.

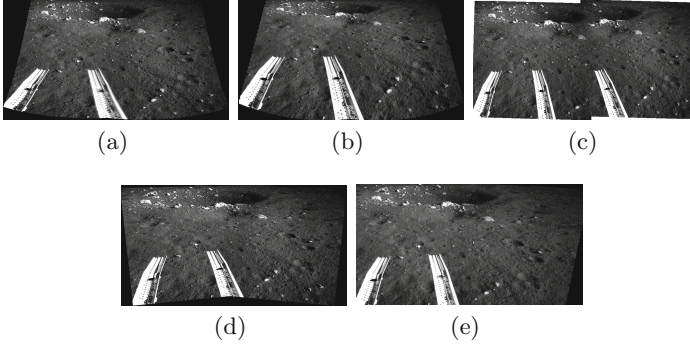
After the stitching procedure above, we can get the final composite. The image stitching result of the two example lunar surface images are shown in Fig.3. As we can see after an energy minimization procedure, the proposed algorithm make the composite smoothly transit from one image to another as shown in Fig.3(d), which is the result of directly copying the color information according to the labelling result. The gradient domain reconstruction result is given in Fig.3(e), which further eliminating the inconsistency in Fig.3(d).

### 3 Experiment

In this paper, the proposed algorithm is implemented in Matlab 2013(a) and tested on real lunar images acquired by Yutu rover and Apollo image gallery. The size of the lunar surface images used in the experiments is  $460 \times 460$ . To sufficiently verify the effectiveness of our proposed algorithm, we compare it with four popular mosaic softwares, including Autostitch<sup>1</sup>, panorama maker<sup>2</sup>,

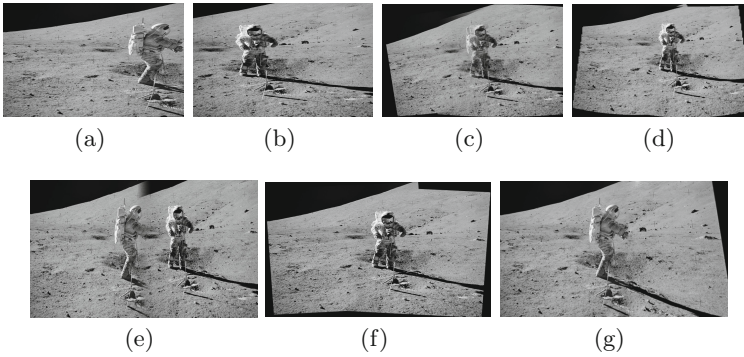
<sup>1</sup> [www.cs.bath.ac.uk/brown/autostitch/autostitch.html](http://www.cs.bath.ac.uk/brown/autostitch/autostitch.html)

<sup>2</sup> [www.arcsoft.com/panorama-maker/](http://www.arcsoft.com/panorama-maker/)



**Fig. 4.** Mosaic results on lunar images acquired by Yutu rover 1. **a** Autostitch. **b** Panorama maker. **c** Panorama factory. **d** Microsoft ICE. **e** Proposed algorithm

panorama factory<sup>3</sup> and Microsoft ICE<sup>4</sup>. All these mosaic softwares apply the traditional feature based method which utilizes the feature descriptor similarity only to automatically register the images. After registration, Autostitch uses a multi-band blending technique [21] to stitch the registered images together directly, while panorama maker, panorama factory and Microsoft ICE have an optimal seam searching procedure to deal with the moving objects and misregistration area and then use a blending technique to improve the quality of the composite. The mosaic results of these softwares and our algorithm tested on real lunar images acquired by Yutu rover and Apollo image gallery are given in Fig.4 and Fig.5 respectively.



**Fig. 5.** Mosaic results on Apollo image gallery 1. **a, b** Images for mosaic. **c** Autostitch. **d** Panorama maker. **e** Panorama factory. **f** Microsoft ICE. **g** Proposed algorithm

<sup>3</sup> [www.panoramafactory.com/](http://www.panoramafactory.com/)

<sup>4</sup> [research.microsoft.com/en-us/projects/ice/](http://research.microsoft.com/en-us/projects/ice/)



From the mosaic experiments above, we can see that the special environment of lunar surface adds particular character to the images. The acquired lunar images always lack significant features due to the barren lunar surface and usually suffer from different illumination conditions. In some special cases, there also exists possible moving objects. These factors makes the lunar surface images mosaic a challenging task. Firstly, the features extracted from lunar surface images are not distinctive, which increases the difficulty of feature matching and further increases the probability of misregistration. As in the Fig.4 and Fig.5, some mosaic softwares, like panorama maker and panorama factory, can hardly get a satisfactory registration of the input images. Secondly, since the perfect registration of the lunar images are hard to obtain and some possible moving objects exist in the overlapping area, simple blending of the registered images, as Autostitch does, may lead to ghost and visual artefacts. Thirdly, the exposure difference between the input images, if not properly handled, will cause visible seam in the final composite.

As shown in the experiments, traditional mosaic softwares are not capable of lunar surface images mosaic. Overall our algorithm are robust against various situations in the lunar images and all get pleasing composite.

## 4 Conclusion

In this paper, a general framework of lunar surface image mosaic is proposed. Considering the speciality of lunar surface images, like large exposure difference, structural deformation and repeating patterns, we improve the traditional feature based image registration method by incorporating the structural information of features. The experiments shows the robustness of the proposed method against environmental changes. In image stitching procedure, an energy minimization method is applied to choose the right image at every pixel of the composite. Our method get pleasing results when dealing with different situations as illustrated in experiments.

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