An Accelerated Two-Step Iteration Hybrid-norm Algorithm for Image Restoration

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Abstract. Linear inverse problem is an important solution frame to solve image restoration. This paper develops an accelerated two-step iteration hybrid-norm reconstruction algorithm, exhibiting much faster convergence rate and better image than iteration shrinkage/thresholding based L1 norm algorithm. In the proposed method, hybrid norm model is built for image restoration objective function. Two-step iteration accelerates objective minimization optimization. Two-step iteration hybrid-norm algorithm converges to a minimizer of hybrid-norm objective function, for a given range of values of its parameters. Numerical examples are presented to validate that the effectiveness of the proposed algorithm is experimentally confirmed on problems of restoration with missing samples.

Keywords: Hybrid-norm · Image restoration · Compressive sensing · Two-step iteration

1 Introduction

Image restoration is still an important image processing research field and has played an important role in medical and astronomical imaging, image and video coding, remote sensing, radar imaging and many other applications [1-3].

Image restoration is to recover an image from distortions to its original image. These distortions usually are twisting, noising, blurring and so on. Image restoration can be described as an inverse problem [2,4]. Let $x \in \mathbb{R}^{n^2}$ be an original $n \times n$ image, $A \in \mathbb{R}^{m \times n}$ be an operator, and $y \in \mathbb{R}^m$ be an observation which satisfies this relationship:

$$y = \mathbb{N}\left(Ax\right) \in \mathbb{R}^m \tag{1}$$

Where $\mathbb{N}(\cdot)$ is an operation process that represents a noise contamination or corruption procedure. In this inverse problem, the goal is to estimate an unknown image

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for observation. In detail, given A, image restoration is a procedure that extract x from y, which is either under-determined or ill-conditioned problem. When A is a linear operator, it is called a linear inverse problem (LIP). Approaches to LIP define a solution $\hat{x}$ (a restored image) as a minimizer of an objective function f. Given by

$$f = \min_x \left\{ \Re_{reg}(x) + \mu \Re_{fid}(Ax - y) \right\} \tag{2}$$

Where $\Re_{reg}(\cdot)$ promotes solution regularity such as sparseness, $\Re_{fid}(\cdot)$ fits the observed data by penalizing the difference between Ax and y. μ balances the two terms to minimization.

For regularization term $\Re_{reg}(\cdot)$, sparseness is an important measurement which used in image restoration. According to sparseness definition, $\Re_{reg}(\cdot)$ should be L0 norm minimization problem which is NP-hard problem. L0 norm minimization is only the ideally accurate solutions. But it is hard to obtain by solve the L0 concave solutions. Conventional L1-norm minimization is to solve convex optimization that is able to guarantee stable solutions to acquire reconstructions. From this view point of accurate and stable reconstruction, the motivation of the proposed hybrid-norm is to balance both aspects.

The purpose of this paper is to develop a new fast two-step iteration hybrid-norm algorithm (TIH) for restoring x from observation y, where A is a general linear operator. In the section II, hybrid-norm model and two-step iteration solver is introduced which also contains the central theorem of the paper. Finally, experimental results are reported in section III. Conclusions are drawn in the final section.

2 Method

According to our design, restoration procedure consists of several parts shown as Fig. 1. When an original image transmits in transmission channel, it is usually corrupted by some noises or disturbances. Then we use hybrid-norm to build restoration model. In leading to the restoration objective function, two-step iteration solver is employed to solve the hybrid-norm restoration model. When iteration conditions have meet, the final results will be obtained. Hybrid-norm model and two-step iteration method are focused on following sections.

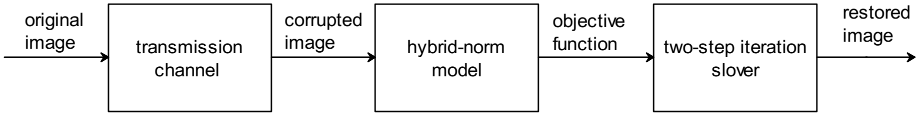

Fig. 1. Flowchart of restoring image based on two-step iteration hybrid-norm (TIH)

According to restoration objective function, a framework for image restoration is hybrid-norm based minimization which is a special case of image restoration where the linear operator is an identity matrix. Denote f as the image restoration objective function:

$$f = \arg\min_{x} \frac{1}{2}\|Ax - y\|_F^2 + \lambda \cdot H(x) \tag{3}$$

where the $H(\cdot)$ regularizer can be homotopic L0 norm which balances between L0 norm and L1 norm describing in hybrid-norm model.

2.1 Hybrid-norm Model

Given x is a sparse and measurement matrix is A, and then the restoration problem can be given by

$$\min_{x} H(x) \qquad s.t. \quad Ax = y \tag{4}$$

where $H(\cdot)$ is hybrid-norm that is transformed to unstrained equation as formula (3).

Hybrid-norm model is defined by

$$H(x) = \sum_{\Omega} g(x) \tag{5}$$

$$g(u) = \begin{cases} a|u|/\tau, & |u| < \tau \\ \dfrac{\left||u| - b\right|}{\left||u| - b\right| + \varepsilon}, & |u| \geq \tau \end{cases} \tag{6}$$

where $H(\cdot)$ means hybrid-norm operator of the proposed method. Constants $a = \dfrac{\sqrt{\varepsilon^2 + 4\tau\varepsilon} - \varepsilon}{\sqrt{\varepsilon^2 + 4\tau\varepsilon} + \varepsilon}$ and $b = \tau - \dfrac{\sqrt{\varepsilon^2 + 4\tau\varepsilon} - \varepsilon}{2}$ are chosen to make the function continuous and differentiable at $|u| = \tau$. Parameter τ is a threshold and $0 < \tau < 1$ is introduced to provide stability. Functional g is related to parameter τ. $\varepsilon > 0$ is to avoid problems due to non-differentiability of hybrid-norm function around intersection point. Profile of hybrid-norm function is shown in Fig. 2. Meanwhile, to be convenient for comparison and understanding of profile functions, profiles of L1, L0, Lp(0<p<1) are added to Fig. 2.

It can be shown from Fig. 2, for any fixed value of ε and τ, hybrid-norm function curve consists of two sections. The first section in the small absolute u is straight with bigger slope than L1. The second section is a conic that is close to L0 under the control of ε. Intersection between two sections keeps smooth and differential when building variables a and b. In the section of L1, this function is strictly convex over $\mathbb{R}^+$. A unique and exact solution to the sparse reconstruction can be acquired. In the section of approaching to L0, solution is the sparsest reconstruction. Hybrid-norm metric combines the merits of L0 and L1 and keeps stable and accurate in reconstruction. In addition, the curve of hybrid-norm is smooth and continuous and its difference is existent and convergent.

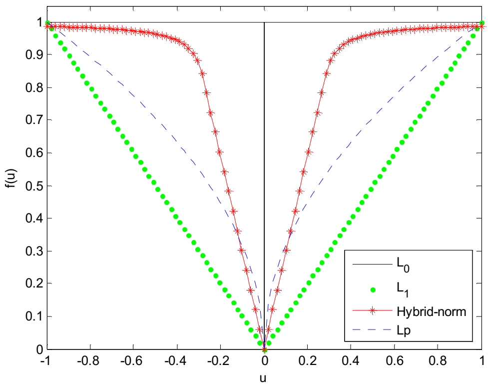

Fig. 2. Hybrid-norm function curve profile comparison with L0, L1 and Lp (0<p<1)

Furthermore, it is seen that the proposed hybrid-norm function includes L1 norm function as a special case when $\tau = 1$. As τ approaches zero, hybrid-norm becomes the L0 of signal. For any $0 < \tau < 1$, hybrid-norm mixes characteristic of both L1 and L0. The value of τ controls the contributions from L1 or L0 respectively. For large τ, hybrid-norm function is closer to a convex function and thus has better convergence to the global minimum. For small τ, it can acquire more accurately solutions because of the profile approaching to L0 norm. Therefore, an optimal τ would best compromise between these two cases.

2.2 Two-Step Iteration Solver

Two-step iteration solver is a fast and effective solver to solve linear inverse problem which developed in fundamental of iterative shrinkage/thresholding (IST)[5,6]. It has been recently used to handle high-dimensional convex optimization problems arising in image inverse problem. In the (k+1)-th iteration, the Two-step iterative solver is as follows.

$$\begin{cases} x_1 = \Gamma_\lambda(x_0) \\ x_{k+1} = (1-\alpha)x_{k-1} + (\alpha-\beta)x_k + \beta\Gamma_\lambda(x_k) \\ \Gamma_\lambda(x) = \Psi_\lambda\{x - A^*(A(x)-y)\} \end{cases} \tag{7}$$

where Ψ_λ is a denoising operator such as wavelet transformation. A* is an adjoint operator of A. α and β are two parameters. The convergence of the two-step iteration algorithm has been well established in [7,8]. Some details also can be found in ref [7,8]. From formula (7), hybrid-norm based restoration problem for equation (3) should be solved for each iteration of two iteration method. In real applications, this subproblem can be solved only approximately, resulting in non-monotonic decrease of the objective function value.

3 Experimental Results

This section reports some experiments to validate image restoration quality and the convergence speeds of the proposed two-step iteration hybrid-norm algorithm (TIH).

We conduct extensive experiments in some examples. Due to the limitation of writing space, we only show two groups of tests in this paper. The goals of these experiments are to present restoration effectiveness from missing samples. The observed images are obtained by convolving the well-known “phantom”and“cameraman” images with a 9*9 uniform blur and then adding noise with variance 40dB below that of the blurred image. The evolution of the objective function and convergence performance are shown using iterative shrinkage/thresholding (IST) and the proposed TIH method in the results.

Example 1. In this group, test object is phantom that comes from typical medical test image. Table 1 lists its results in mean square error (MSE)and CPU time. Figure 1 shows the observed image and the restored image produced by IST and TIH. Figure 2 shows convergence processing of IST and TIH.

Table 1. Experimental results for phantom

	IST	Proposed TIH
MSE	0.17306	0.026431
CPU time	35.537028	33.633816

Quantitative index MSE and CPU time are shown in Tab1. In this table, IST and TIH take 0.17306 and 0.026431 of MSE. TIH improves image quality approximate one power of magnitude from 0.17306 to 0.026431. The proposed method is super to IST. In consuming time, TIH consumes 33.633816s and IST has 35.537028s. The proposed method improves little faster than IST. The reason is that the phantom is an ideal sparse image which we can not dig more sparse information. These factors decide iteration times.

In Fig. 3, subimage (a) is original image. (b) is corrupted image by noisy and blurred factor. (c) stands for the restored image using the proposed algorithm. (d) is the restored image using IST method. Restored image using TIH reduces noisy and blurred factor, which gets sharp boundaries and clear contents in several important part such as gray circle, two black ellipses. The white circle boundary is sharp and clear. In subfigure (d), IST method restores image which has many pseudo artifacts like dummy circle. White circle boundary of subfigure (d) is little blurred. In visual effectiveness of restored images, TIH is clearer and neater than IST algorithm.

Fig. 4 has two subfigures. The above is a curve representing the relationship between objective function and CPU time. The blow curve represents relationship between restored error MSE and CPU time. TIH method converges very faster than IST and obtains lower restored error MSE. IST spends 35.537028 seconds to gets 0.17306 error and TIH needs 33.633816 seconds to have 0.026431 restored errors. Fig. 4 shows that TIH converges considerably faster and more excellent than IST.

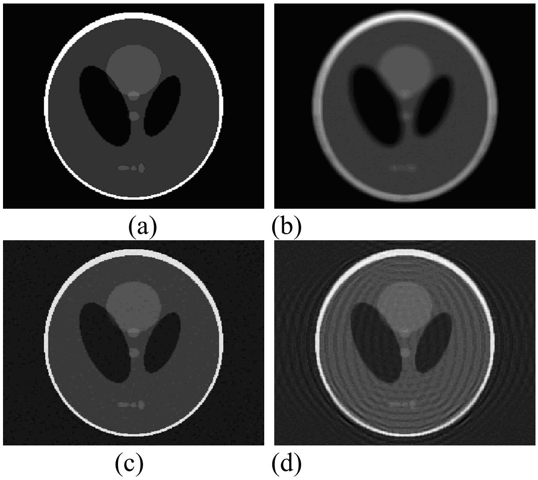

Fig. 3. Image restoration results for phantom. (a)Original image ;(b)Noisy and blurred image; (c)TIH restored image; (d)IST restored image

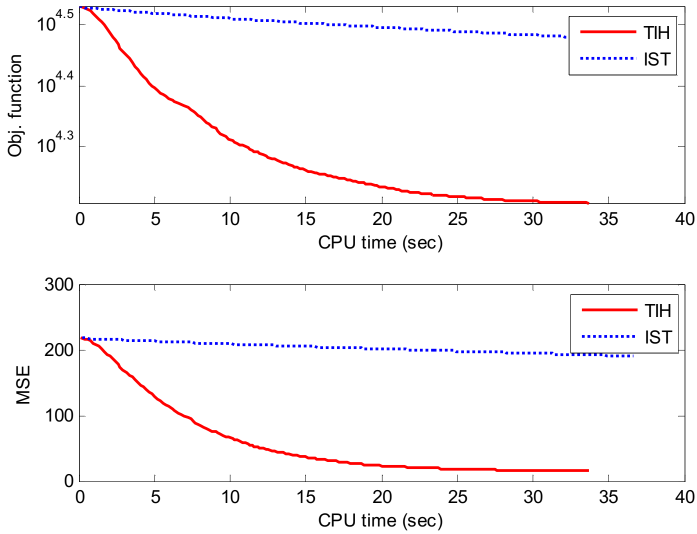

Fig. 4. Convergence behavior for phantom restoration. Above: objective; below: relative error MSE. In both plots, the horizontal axes denote CPU time in seconds.

Example 2. In the second experiment, we apply "cameraman" image to test effectiveness of the algorithm. In this group experiment, noisy and blurred image is obtained using the same method in example 1. "cameraman" image is a real natural image which is not completely different from phantom image in 1st experiment.

Table 2. Data results for cameraman

	IST	TIH
MSE	0.067087	0.027247
CPU time	39.078251	16.224104

According to Table 2, TIH obtains 0.027247 in MSE and 16.224104 seconds in CPU time. IST has 0.067087 MSE and 39.078251 seconds in CPU time. In restoration quality, TIH has 0.027247 errors of original image and restored image. IST only has 0.067087 differences between the original image and restored image. Restored image of TIH considerably is better than that of IST. Also, TIH is largely faster than IST.

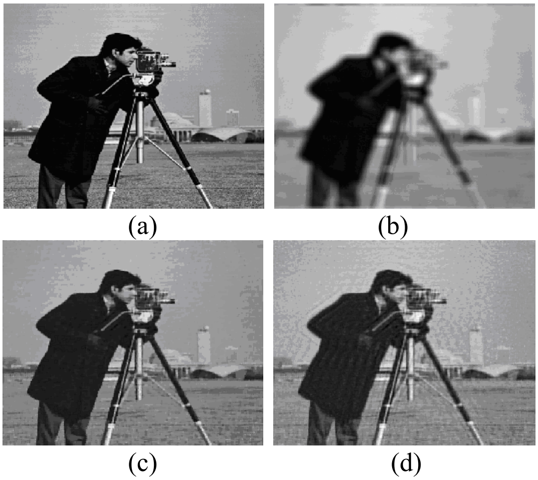

Fig. 5. Image restoration results for phantom. (a) Original image; (b) Noisy and blurred image; (c) TIH restored image; (d) IST restored image

Some results of "cameraman" restored image are shown in the Fig. 5. In the same statements as in first group experiment. Subfigures (a) to (d) are original image, noisy and blurred image, TIH restored image and IST restored image separately. Differences of restored images in subfigure (c) and subfigure (d) are distinguished apparently. From the view of vision, subfigure (c) is clearer and neater than subfigure (d). Image in subfigure (d) has large amount of artifacts and alias. Restored image in subfigure (c) has little drawbacks. But it can be seen clearly. Why we can not restore an image as same as original image. Restoration is an anti-process that can not completely restore image as original image in the condition of loss of some information. From data of experiments, MSE in subfigure (c) is 0.027247 and subfigure (d) only takes 0.067087. The proposed algorithm improves apparently both in vision and experimental data.

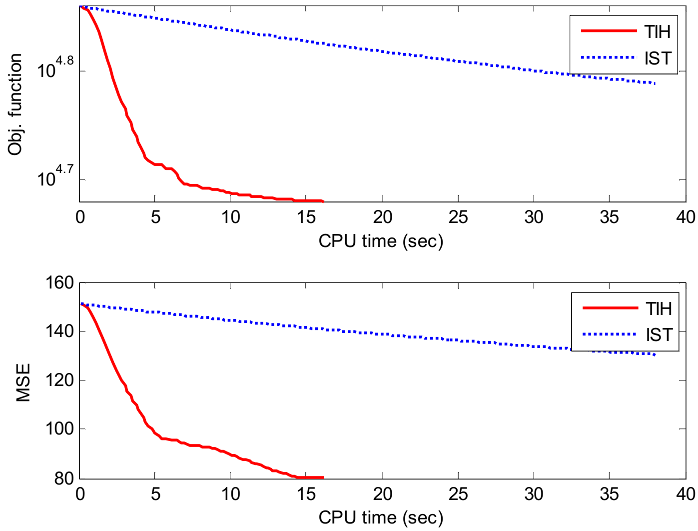

Fig. 6. Convergence curves for “cameraman” image restoration

In the figure 6, convergence curves of “cameraman” image restoration are shown. Relationship between objective function and CPU time is shown in the above subplot in Fig. 6 and restored image error is shown in the below subplot in Fig. 6. IST needs 39.078251 seconds to up to MSE value 0.067087 and TIH only requires 16.224104 seconds to obtain 0.027247 restored images. The two curves using TIH decrease sharper than that of IST. In other words, TIH converges rapidly and consumes little times.

Though different object images are employed in the two groups of experiments, nearly same conclusions are drawn that the proposed TIH method is superior to IST in both image quality and restoring speed.

4 Conclusion

This paper proposed a fast two step iteration hybrid-norm image restoration method to solve fast and high image restoration. The proposed method combined fastness of two-step iteration and effectiveness of hybrid-norm model which is a homotopical L0 norm method. Two groups of experiments in phantom and natural images give evidences of high image quality and fast speed in restoring image.

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