Automated Benchmarking of Incremental SAT and QBF Solvers*

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Abstract. Incremental SAT and QBF solving potentially yields improvements when sequences of related formulas are solved. An incremental application is usually tailored towards some specific solver and decomposes a problem into incremental solver calls. This hinders the independent comparison of different solvers, particularly when the application program is not available. As a remedy, we present an approach to automated benchmarking of incremental SAT and QBF solvers. Given a collection of formulas in (Q)DIMACS format generated incrementally by an application program, our approach automatically translates the formulas into instructions to import and solve a formula by an incremental SAT/QBF solver. The result of the translation is a program which replays the incremental solver calls and thus allows to evaluate incremental solvers independently from the application program. We illustrate our approach by different hardware verification problems for SAT and QBF solvers.

1 Introduction

Incremental solving has contributed to the success of SAT technology and potentially yields considerable improvements in applications where sequences of related formulas are solved. The logic of quantified Boolean formulas (QBF) extends propositional logic (SAT) by explicit existential and universal quantification of variables and lends itself for problems within PSPACE. Also for QBFs, incremental solving has been successfully applied in different domains [4,7,11,12].

The development of SAT and QBF solvers has been driven by competitive events like the SAT Competitions, QBF Evaluations (QBFEVAL), or the QBF Galleries. These events regularly result in publicly available benchmarks submitted by the participants which help to push the state of the art in SAT and QBF solving. In the past, the focus was on *non-incremental* SAT solving, and the evaluation of *incremental* solvers does not readily benefit from competitions and available benchmark collections.

Benchmarking incremental solvers requires to solve a sequence of related formulas. To this end, the formulas must be incrementally imported to the solver and solved by

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means of API calls. The API calls are typically generated by an application program, like a model checker or a formal verification or planning tool, for example, which tackles a problem by encoding it incrementally to a sequence of formulas. In order to compare different incremental solvers on that sequence of formulas, the solvers must be tightly coupled with the application program by linking them as a library. Hence benchmarking of incremental solvers relies on the application program used to generate the sequence of formulas which, however, often is not available. Even if the application program is available, it has to be adapted to support different solvers, where each solver might come with its own API. Further, the same sequence of formulas must be generated multiple times by the application program to compare different solvers.

To remedy this situation, we present an approach to automated benchmarking of incremental SAT and QBF solvers which decouples incremental SAT/QBF solving from incremental generation of formulas using an application program. This is achieved by translating a sequence of related CNFs and QBFs in prenex CNF (PCNF) into API calls of incremental solvers. Such a sequence might be the output of an application program or it was taken from existing benchmark collections. The formulas are then syntactically analyzed and instructions to incrementally import and solve them are generated. For CNFs, the instructions are function calls in the IPASIR API, which has been proposed for the Incremental Library Track of the SAT Race 2015.¹ For PCNFs, the instructions correspond to calls of the API of the QBF solver DepQBF,² which generalizes IPASIR and allows to update quantifier prefixes. The result of translating a sequence of formulas to solver API calls is a *standalone benchmarking program* which replays the incremental solver calls. Any incremental SAT/QBF solver supporting the IPASIR API or its QBF extension as implemented in DepQBF can be integrated by simply linking it to the program. This allows to compare different solvers independently from an application.

In some applications, the sequence of formulas depends on the used solver, e.g., if truth assignments are used to guide the process. Even then, our approach allows to compare different incremental solvers on the fixed sequences generated with one particular solver. However, then it is important to note that this comparison is limited to this particular fixed sequence, it would be unfair to conclude something about the performance of the solvers would they have been genuinely used within the application. This problem occurs also in sequences of formulas which are already present in benchmark collections. For experiments in this paper, we only considered applications where the sequences of generated formulas do not depend on intermediate truth assignments.

As our approach is also applicable to already generated formulas that are part of existing benchmark collections, such collections become available to developers of incremental solvers. Furthermore, comparisons between solvers in incremental and non-incremental mode are made possible. In addition, since the input for the benchmarking program describes only the differences between consecutive formulas, we obtain a quite succinct representation of incremental benchmarks. Our approach to automated benchmarking of incremental SAT and QBF solvers underpins the goal of the Incremental Library Track of the SAT Race 2015. We have generated benchmarks and submitted them to this competition.

¹ http://baldur.iti.kit.edu/sat-race-2015/

² http://lonsing.github.io/depqbf/

2 Background

We consider propositional formulas in CNF and identify a CNF with the set of its clauses. A sequence $\sigma = (F_1, \ldots, F_n)$ of formulas represents the formulas that are incrementally generated and solved by an application program. A QBF $\psi = P.F$ in prenex CNF (PCNF) extends a CNF F by a quantifier prefix P. The prefix $P = Q_1, \ldots, Q_n$ of a QBF is a sequence of pairwise disjoint quantified sets Q_i . A quantified set Q is a set of variables with an associated quantifier $quant(Q) \in \{\exists, \forall\}$. We consider only closed PCNFs. For adjacent quantified sets Q_i and Q_{i+1} , $quant(Q_i) \neq quant(Q_{i+1})$. Given a prefix $P = Q_1, \ldots, Q_n$, index i is the *nesting level* of Q_i in P.

Our automated benchmarking approach is based on *solving under assumptions* [5,6] as implemented in modern SAT [1,9,13] and QBF solvers [10,11,12]. When solving a CNF under assumptions, the clauses are augmented with *selector variables*. Selector variables allow for temporary variable assignments made by the user via the solver API. If the value assigned to a selector variable satisfies the clauses where it occurs, then these clauses are effectively removed from the CNF. This way, the user controls which clauses appear in the CNF in the forthcoming incremental solver run. The IPASIR API proposed for the Incremental Library Track of the SAT Race 2015 consists of a set of functions for adding clauses to a CNF and handling assumptions. A disadvantage of this approach is that the user has to keep track of the used selector variables and assumptions manually.

For incremental QBF solving, additional API functions are needed to remove quantified sets and variables from and add them to a prefix. For QBF solvers, we generate calls in the API of DepQBF which generalizes IPASIR by functions to manipulate quantifier prefixes. Additionally, it allows to remove and add clauses in a stack-based way by push/pop operations where selector variables and assumptions are handled internal to the solver and hence are invisible to the user [10]. For details on the IPASIR and DepQBF interfaces, we refer to the respective webpages mentioned in the introduction.

3 Translating Related Formulas into Incremental Solver Calls

We present the workflow to translate a given sequence $\sigma = (\psi_1, \dots, \psi_n)$ of related (P)CNFs into a standalone benchmarking program which calls an integrated solver via its API to incrementally solve the formulas from ψ_1 up to ψ_n :

- 1. First, the formulas in σ are analyzed and the syntactic differences between each ψ_i and ψ_{i+1} are identified. This includes clauses and quantified sets that have to be added or removed to obtain ψ_{i+1} from ψ_i . Also, variables may be added to or removed from quantified sets. For CNFs, the prefix analysis is omitted.
- 2. The differences between the formulas identified in the first step are expressed by generic update instructions and are written to a file. A clause set is represented as a stack which can be updated via push and pop operations. The update instructions for quantifier prefixes are adding a quantified set at a nesting level and adding new variables to quantified sets already present in the prefix. Unused variables are deleted from the prefix be the solver.
- Files that contain generic update instructions are then interpreted by a *benchmarking* program which translates them into calls of the IPASIR API (for CNFs) or QBF solver calls (for PCNFs). For the latter, calls of DepQBF's API are generated.

The benchmarking program is standalone and independent from the application program used to generate σ . It takes the files containing the generic update instructions as the only input. Multiple solvers may be integrated in the benchmarking program by linking them as libraries. Files containing the update instructions can serve as standardised benchmarks for incremental SAT and QBF solvers.

Analyzing CNFs. The algorithm to analyze sequences $\sigma = (F_1, \ldots, F_n)$ of clause sets relies on a stack-based representation of F_i which allows for simple deletion of clauses that have been added most recently. A clause c which appears in some F_i and is removed later at some point to obtain F_j with $i < j \le n$ is called *volatile in* F_i . A clause which appears in some F_i for the first time and also appears in every F_j with $i < j \le n$ and hence is never deleted is called *cumulative in* F_i .

The algorithm to analyze sequence σ identifies volatile and cumulative clauses in all clause sets in σ . Cumulative clauses are pushed first on the stack representing the current clause set because they are not removed anymore after they have been added. Volatile clauses are pushed last because they are removed at some point by a pop operation when constructing a later formula in σ . For illustration, consider the following sequence $\sigma = (F_1, \ldots, F_4)$ of clause sets F_i along with their respective sets C_i of cumulative clauses and sets V_i of volatile clauses:

$F_1 = \{c_1, c_2, v_1\}$	$C_1 = \{c_1, c_2\}$	$V_1 = \{v_1\}$
$F_2 = \{c_1, c_2, c_3, v_1, v_2\}$	$C_2 = \{c_3\}$	$V_2 = \{v_1, v_2\}$
$F_3 = \{c_1, c_2, c_3, c_4, v_1, v_3\}$	$C_3 = \{c_4\}$	$V_3 = \{v_1, v_3\}$
$F_4 = \{c_1, c_2, c_3, c_4, c_5\}$	$C_4 = \{c_5\}$	$V_4 = \emptyset$

After the sets of cumulative and volatile clauses have been identified for each F_i , the clause sets can be incrementally constructed by means of the following operations on the clause stack: adding a set C of clauses permanently to a formula by add(C), pushing a set C of clauses on the stack by push(C), and popping a set of clauses from the stack by pop(). The sequence $\sigma = (F_1, \ldots, F_4)$ from the example above is generated incrementally by executing the following stack operations:

	$\mathtt{add}(C_1)$	$\mathtt{push}(V_1)$
pop()	$\mathtt{add}(C_2)$	$\mathtt{push}(V_2)$
pop()	$\mathtt{add}(C_3)$	$\mathtt{push}(V_3)$
pop()	$\mathtt{add}(C_4)$	$\mathtt{push}(V_4)$

Note that the above schema of stack operations generalises to *arbitrary* sequences of clause sets, i.e., we need at most one push, one add, and one pop operation in each step, provided that the clauses have been classified as volatile or cumulative before.

The algorithm for identifying cumulative and volatile clauses in a sequence of clause sets appears as Algorithm 1. For SAT solvers supporting the IPASIR API, stack frames for volatile clauses pushed on the clause stack are implemented by selector variables. Our current implementation of the benchmarking program includes DepQBF as the only incremental QBF solver which supports push/pop operations natively via its API [10]. Note that the relevant part of the input that potentially limits scalability of Algorithm 1 is the number of variables and clauses in the formulas. The number of formulas is usually relatively low. The operations on clause sets are implemented such that set intersection

Input : Clause sets $F_1, F_2, ..., F_n$ (at least two sets are required) **Output**: $C_1, ..., C_n$ (sets of cumulative clauses to be added) $V_1, ..., V_n$ (sets of volatile clauses to be pushed or popped)

1 $V_1 \longleftarrow F_1 \setminus F_2; \quad C_1 \longleftarrow F_1 \setminus V_1;$ 2 for $i \leftarrow 2$ to n - 1 do $V_i \longleftarrow F_i \setminus F_{i+1};$ 3 $C_i \longleftarrow (F_i \setminus F_{i-1}) \setminus V_i;$ 4 foreach $c \in V_i \cap F_{i-1}$ do 5 for $j \leftarrow 1$ to i - 1 do 6 if $c \in C_i$ then 7 $C_j \longleftarrow C_j \setminus \{c\};$ for k = j to i - 1 do $\ V_k \longleftarrow V_k \cup \{c\};$ 8 9 10 break; 11 12 $C_n \longleftarrow F_n \setminus F_{n-1}; \quad V_n \longleftarrow \emptyset;$

Algorithm 1: Identifying cumulative and volatile clauses.

and difference are in $O(m \cdot \log m)$, searching an element is in O(m), and adding or deleting elements are in O(1), where m is the maximal number of clauses in any formula.

Analyzing PCNFs. For sequences of QBFs, additionally the differences between quantifier prefixes must be identified. Two quantified sets Q and Q' are matching iff $Q \cap Q' \neq \emptyset$. Prefix R is update-compatible to prefix S iff all of the following conditions hold: (i) for any quantified set of R, there is at most one matching quantified set in S; (ii) if P is a quantified set of R and Q is a matching quantified set in S, then quant(P) = quant(Q); and (iii) for any two quantified sets P_1 and P_2 in S with matching quantified sets Q_1 and Q_2 in R, respectively, if the nesting level of P_1 is less than the nesting level P_2 , then the nesting level of Q_1 is less than the nesting level of Q_2 .

The instructions to update quantifier prefixes are adding a quantified set at a given nesting level or adding a variable to a quantified set at a given nesting level. Update compatibility between prefixes R and S guarantees that there is a sequence of instructions to turn R into S after unused variables and empty quantified sets have been deleted by the QBF solver. In particular, Condition (i) guarantees that there is no ambiguity when mapping quantified sets from the prefixes, (ii) expresses that quantifiers cannot change, and (iii) states that quantified sets cannot be swapped. The algorithm to generate update instructions first checks if two quantifier prefixes R and S are update-compatible. If this is the case, then update instructions are computed as illustrated by Algorithm 2.

4 Case Studies

In this section, we showcase our approach using different hardware verification problems for both SAT and QBF solvers. Benchmark problems consist of sequences of formulas

Input : Prefix R and S (R has to be update-compatible to S) **Output** : Instructions to update R to S

1 $n \leftarrow 0; \quad m \leftarrow 0;$ 2 foreach quantified set Q in S from left to right do if Q has a matching quantified set M in R then 3 $m \leftarrow n + \text{nesting level of } M \text{ in } R;$ 4 **print** "Add literals $Q \setminus M$ to quantified set at nesting level m."; 5 else 6 $n \longleftarrow n+1;$ 7 $m \longleftarrow m + 1;$ 8 print "Add quantified set Q at nesting level m."; 9

Algorithm 2: Generating update instructions for quantifier prefixes.

Table 1. Summary of different SAT solvers on hardware verification problems.

	#problems	MiniSAT	PicoSAT	Lingeling	DepQBF
BMC problems unrolled by 50 steps	11	284/7	216/3	276/7	190/1
BMC problems unrolled by 100 steps	28	905 / 14	754 / 4	872 / 19	491/2

that were either generated by a model-checking tool or that were taken from existing benchmark collections where the original application is not available.

SAT: Bounded-Model Checking for Hardware Verification. We consider benchmarks used for the single safety property track of the last Hardware Model Checking Competition (HWMCC 2014)³. Based on the CNFs generated by the BMC-based model checker $aigbmc^4$, we use our tools to generate incremental solver calls and compare different SAT solvers that implement the IPASIR interface. We used the SAT solvers MiniSAT (v.220) [5], PicoSAT (v.961) [2], and Lingeling (v.ayv) [3] as well as the QBF solver DepQBF (v.4) for the considered problems. All experiments were performed on an AMD Opteron 6238 at 2.6 GHz under 64-bit Linux with a time limit of 3600 seconds and a memory limit of 7 GB.

Table 1 summarises the results. For each solver and problem class, numbers m/n mean that m formulas in total were solved within the time limit, and n is the number of problems where the maximal number of formulas among all other solvers could be solved. For example, the first line summarises the results for BMC problems that were unrolled by 50 steps. There are 11 problems in this class, thus 550 formulas in total. From these formulas, MiniSAT could solve 284 formulas, and for 7 out of 11 problems, no other solver could solve more formulas than MiniSAT. Not surprisingly, all SAT solvers outperform the QBF solver DepQBF but there are few cases where DepQBF can compete. MiniSAT solves most formulas in total while Lingeling dominates on most benchmarks. More detailed experimental results can be found in the appendix. The average time for our analyzing algorithm was 522 seconds. The number of clauses in the original sequences ranged from 2.3 to 56.3 million with an average of around

6

³ http://fmv.jku.at/hwmccl4cav/

⁴ Part of the AIGER package (http://fmv.jku.at/aiger/)

19 million clauses. The inputs for the benchmarking program that represent only the update instructions comprise only 1.2 million clauses on average which shows that we obtain a quite compact representation of incremental benchmarks. We have submitted all problems from Table 1 to the Incremental Library Track of the SAT Race 2015.

Table 2. QBF solvers on incomplete design problems.

Banchmark	Ŀ	non-incremental		incremental			
Deneminark	n	QuBE	DepQBF	QuBE (fwd)	QuBE (bwd)	DepQBF	
enc04	17	3	3	3	2	1	
enc09	17	7	5	7	4	3	
enc01	33	31	17	28	24	5	
enc03	33	33	16	289	28	27	
enc05	33	64	24	61	46	7	
enc06	33	29	26	28	24	10	
enc07	33	75	16	76	69	5	
enc08	33	108	16	110	79	5	
enc02	65	271	106	ТО	269	175	
tlc01	132	26	68	133	130	17	
tlc03	132	24	160	8	8	17	
tlc04	132	769	2196	1204	27	25	
tlc05	152	1330	4201	2057	38	34	
tlc02	258	MO	ТО	MO	98	1908	

QSAT: Partial Design Problems. To illustrate our approach in the context of QBF solving, we consider the problem of verifying partial designs, i.e., sequential circuits where parts of the specification are black-boxed. In recent work [11,12], the question whether a given safety property can be violated regardless of the implementation of a blackbox has been translated to OBFs which are solved incrementally by a version of the QBF solver QuBE [8]. Benchmarks are available from QBFLIB,5 however neither the solver used in [11,12] nor the application program used to generate sequences of QBFs

are publicly available. Marin et al. [11] introduced two encoding strategies: forward incremental and backward incremental reasoning. In a nutshell, the quantifier prefix is always extended to the right in the former approach, while it is extended to the left in the latter approach. Both strategies yield the same sequences of formulas up to renaming [11]. We used the publicly available instances from the forward-incremental encoding without preprocessing to evaluate DepQBF. Instances from the backward-incremental approach are not publicly available.

Table 2 shows the comparison between QuBE and DepQBF. Runtimes are in seconds, k is the index of the first satisfiable formula, TO and MO refer to a timeout and memout, respectively. The maximal runtime of Algorithm 1 and 2 was 95 seconds. Runtimes for QuBE in Table 2 are the ones reported in [11]. There, experiments were carried out on an AMD Opteron 252 processor running at 2.6 GHz with 4GB of main memory and a timeout of 7200 seconds. Experiments for DepQBF were performed on a 2.53 GHz Intel Core 2 Duo processor with 4GB of main memory with OS X 10.9.5 installed. Thus runtimes are not directly comparable because experiments were carried out on different machines, they give, however, a rough picture of how the solvers relate. Like QuBE, DepQBF benefits from the incremental strategy on most instances. The backward-incremental strategy is clearly the dominating strategy for QuBE. A quite eye-catching observation is that forward-incremental solving, while hardly improving the performance of QuBE compared to the non-incremental approach, works quite well for DepQBF.

⁵ http://www.qbflib.org

5 Conclusion

We presented an approach to automated benchmarking of incremental SAT and QBF solvers by translating sequences of formulas into API calls of incremental SAT and QBF solvers executed by a benchmarking program. Several incremental solvers may be tightly integrated into the benchmarking program by linking them as libraries. Thus, we decouple the generation of formulas by an application from the solving process which is particularly relevant when application programs are not available. Additionally, we make sequences of formulas which already exist in public benchmark collections available for benchmarking and testing. We illustrated our approach to automated benchmarking of incremental SAT and QBF solvers on instances from hardware verification problems. To improve the performance of incremental QBF solving on these problems, we want to integrate incremental preprocessing into DepQBF. As shown in [11,12], preprocessing potentially improves the performance of incremental workflows considerably.

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A Correctness of Algorithms 1 and 2

Theorem 1. Algorithm 1 is totally correct with respect to the precondition that $\sigma = (F_1, \ldots, F_n)$ is a sequence of sets of clauses with $n \ge 2$ and the postcondition that any C_i , $1 \le i \le n$, contains the cumulative clauses of F_i in σ , and any V_i , $1 \le i \le n$, contains the volatile clauses of F_i in σ .

Proof. Clearly, Algorithm 1 terminates on each input.

We show that the condition that any C_j , $1 \le j < i$, contains the cumulative clauses of F_j in the subsequence $\sigma_i = (F_1, \ldots, F_i)$ of σ , and any V_j , $1 \le j < i$, contains the volatile clauses of F_j in σ_i is an invariant of the main loop (at Line 2). The invariant together with i = n implies the postcondition as C_n always contains those clauses that are in F_n but not in F_{n-1} , and V_n always equals the empty set (Line 12). Likewise, the precondition implies the invariant since after Line 1, V_1 contains all clauses of F_1 that are not in F_2 and which are thus volatile in F_1 in the sequence F_1 , F_2 , and C_1 contains all clauses which are in F_1 and F_2 and which are hence cumulative in F_1 in the sequence F_1 , F_2 .

It remains to show that if the invariant holds for some $i, 2 \le i \le n-1$ at Line 3, then it holds for i+1 after executing Lines 3–11. After Line 3, V_i contains all the clauses that are volatile in F_i in σ_{i+1} . Likewise, after Line 4, C_i contains all the clauses that are in F_i but not in F_{i-1} and which are not volatile in F_i , that is, which are cumulative in F_i in σ_{i+1} . Note that if a clause c is volatile in some $F_j, j < i$, in σ_i , then c is also volatile in F_j in σ_{i+1} . On the other hand, if a clause is cumulative in F_j , it can be the case that c becomes volatile in σ_{i+1} if $c \notin F_{i+1}$. Hence, it is possible that clauses that were previously classified as cumulative need to be reclassified.

We make use of the following claim: After Line 4, a clause c is in $V_i \cap F_{i-1}$ iff, for some $j < i, C_j$ contains a clause c that is volatile in σ_{i+1} . This claim is proven as follows: Assume that for some $j < i, C_j$ contains a clause c that is volatile in σ_{i+1} . As the invariant holds for $i, c \in F_k$, for all $j \le k \le i$ but $c \notin F_{i+1}$ and thus $c \in V_i$. Clearly, $c \in V_i \cap F_{i-1}$. On the other hand, assume some clause c is in $V_i \cap F_{i-1}$. Clearly, $c \in V_i$ implies $c \in F_i$. Hence, as the invariant holds for $i, c \in C_j$, for some j < i, and, since $c \in V_i$, c is volatile in F_j in σ_{i+1} .

By virtue of the above claim, $V_i \cap F_{i-1}$ contains precisely those clauses which need to be reclassified as volatile. After Lines 6 – 11, for each $c \in V_i \cap F_{i-1}$, the first (and only) C_j with $c \in C_j$ is found, and c is removed from C_j and added to all V_l , $j \leq l \leq i - 1$. Hence, after Lines 3 – 11, the invariant holds for i + 1.

Algorithm 2 works as follows. For each quantified set q of S, either it has one matching set q' in R (Lines 4 and 5) or it does not have a matching quantified set in R (Lines 7,8, and 9). In the former case, we need to add the atoms in q to q' if they are not already there (Line 5). In the latter case, we need to add the entire set q to the prefix (Line 9). Adding atoms and quantified sets is always done at the right nesting level m. We store in n the number of new quantified sets that have been added. At Line 5, when adding atoms to a matching quantified set, m is the nesting level of the matching quantified set in R plus the number n of previously added unmatched quantified sets. At Line 9, when adding an entire quantified set, m is the nesting level of the quantified set that was modified last plus one.

 Table 3. Detailed results for Table 1: SAT solvers on hardware verification problems.

	MiniSAT	PicoSAT	Lingeling	DepQBF
6s393r	15	15	14	10
6s394r	27	23	22	16
6s514r	17	16	16	14
arbi0s08	12	12	13	11
arbi0s16	17	17	17	17
arbixs08	9	10	10	10
cuabq2f	28	22	40	19
cuabq2mf	77	46	65	20
cuabq4f	21	22	26	15
cuabq4mf	25	23	40	16
cuabq8f	22	21	26	19
cubak	83	43	55	28
cufq2	101	88	83	21
cugbak	38	32	42	21
cuhanoi10	35	34	35	17
cujc12	47	46	38	15
cunim1	22	21	23	19
cunim2	22	21	22	18
cuom1	14	13	14	10
cuom2	14	13	15	10

cuom3	15	14	16	10
cupts14	34	32	37	25
cupts15	35	32	37	27
cupts16	35	34	37	24
cutarb16	52	37	48	30
cutf1	44	32	35	21
pdtfifo1to0	16	16	16	13
pdtpmsdc16	28	19	30	15
BMC pro	blems u	nrolled	by 100 ste	ps.
6s188	39	34	36	33
6s24	25	23	27	20
6s270b1	51	13	51	11
arbi0s32p03	32	32	33	32
arbixs16p03	16	16	16	16
bmhan1f1	29	21	29	19
bobpcihm	17	15	16	11
bobsmvhd3	14	11	14	10
cufq1	45	33	37	23
cujc128	7	8	7	7
cuic32	9	10	10	8

MiniSAT PicoSAT Lingeling DepQBF

BMC problems unrolled by 100 steps. BMC problems unrolled by 50 steps.