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Ronald de Haan

Parameterized Complexity in the Polynomial Hierarchy

Extending Parameterized Complexity Theory
to Higher Levels of the Hierarchy

Author

Ronald de Haan
Institute for Logic, Language
and Computation
University of Amsterdam
Amsterdam, The Netherlands

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“Parameterized Complexity in the Polynomial Hierarchy” was co-recipient of the E. W. Beth Dissertation Prize 2017 for outstanding dissertations in the fields of logic, language, and information. This work extends the theory of parameterized complexity to higher levels of the Polynomial Hierarchy (PH). For problems at higher levels of the PH, a promising solving approach is to develop fixed-parameter tractable reductions to SAT, and to subsequently use a SAT solving algorithm to solve the problem. In this dissertation, a theoretical toolbox is developed that can be used to classify in which cases this is possible. The use of this toolbox is illustrated by applying it to analyze a wide range of problems from various areas of computer science and artificial intelligence.

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Preface

This book came about during my PhD studies at the Technische Universität Wien, from 2012 until 2016.

It originally appeared as my PhD dissertation in 2016, for which I was awarded the E. W. Beth Dissertation Prize for outstanding contributions in the domains of logic, language, and information by the Association for Logic, Language, and Information (FoLLI) in 2017.

As a result, this book is now being published in the FoLLI Publications on Logic, Language, and Information, in Springer's *Lecture Notes in Computer Science* series.

The theme of this book is “The Extension of Parameterized Complexity Theory to Higher Levels of the Polynomial Hierarchy.”

This theme allows for theoretical explorations that are motivated by the aim to develop theoretical tools that can be used to classify whether computational problems can be solved by the approach of first reducing them to SAT in fixed-parameter tractable time, and then solving them by invoking a SAT solving algorithm.

This toolbox is relevant for computational problems from all kinds of domains, in fact, many of the theoretical developments and applications in this book are driven by concrete, natural problems from a variety of areas of computer science and artificial intelligence.

I hope that this book will serve as a starting point for future theoretical investigations on this topic.

I am grateful to all the people that supported me throughout my PhD studies; helping me directly or indirectly in writing this book.

In particular, I owe many thanks to my PhD supervisor Stefan Szeider, for showing me how to do theoretical research, and for giving me the freedom to discover, as well as become an expert in an area of research that fits my interests.

September 2019

Ronald de Haan

Contents

1	Introduction	1
1.1	Context: Intractability and a New Way of Coping	2
1.2	Structured Complexity Investigation	3
1.2.1	Problem Statement	4
1.3	Contributions	5
1.3.1	Main Contributions	8
1.3.2	Overview of the Parameterized Complexity Framework	9
1.3.3	Research Impact	9
1.4	Roadmap	9
1.5	Reflecting on the Theoretical Paradigm	13
1.5.1	Black Box Algorithms for NP-complete Problems	13
1.5.2	Focus on the Second Level of the Polynomial Hierarchy	15
1.5.3	Complexity-Theoretic Assumptions	16
1.5.4	Worst-Case Behavior of Fpt-Reductions	17
 Foundations		
2	Complexity Theory and Non-determinism	21
2.1	Basics of Complexity Theory: P, NP	21
2.1.1	Traditional Tractability: P	22
2.1.2	Traditional Intractability: NP and NP-Completeness	24
2.2	The Polynomial Hierarchy and Polynomial Space	26
2.2.1	Polynomial Hierarchy	26
2.2.2	Polynomial Space	27
2.2.3	Alternating Turing Machines	28
2.3	Bounded Query Complexity	29
2.4	Non-uniform Complexity	31
3	Parameterized Complexity Theory	33
3.1	Fixed-Parameter Tractability	33
3.1.1	Polynomial-Time Solvability for Constant Parameter Values	34
3.1.2	Integers as Parameter Values	35
3.1.3	Alternative Characterizations of FPT	36

3.2	Fixed-Parameter Intractability	36
3.2.1	The Classes para-K	37
3.2.2	The Weft Hierarchy	38
3.2.3	The A-Hierarchy and First-Order Logic	39
3.2.4	The Classes XNP, Xco-NP, $X\Sigma_i^p$, and $X\Pi_i^p$	40

Beyond Para-NP

4	Fpt-Reducibility to SAT.	45
4.1	Modern SAT Solvers.	47
4.1.1	The Success of SAT Solvers	48
4.1.2	Algorithmic Techniques.	49
4.1.3	Successful Algorithms for Other NP-complete Problems	50
4.2	Various Fpt-Reductions to SAT	52
4.2.1	Quantified Boolean Satisfiability.	52
4.2.2	Backdoors for Abductive Reasoning	58
4.2.3	Backdoors for Disjunctive Answer Set Programming.	59
4.2.4	Bounded Model Checking	60
4.3	Sneak Preview: Fpt-Time Turing Reductions to SAT.	60
4.4	Irreducibility	62
4.4.1	Hardness for para- Σ_2^p	62
4.4.2	Hardness for A[2]	63
5	The Need for a New Completeness Theory	71
5.1	Running Example: Disjunctive Answer Set Programming.	72
5.1.1	Parameterized Variants Where Known Theory Suffices	75
5.2	Motivating New Theory.	76
5.2.1	Hardness for para-NP and para-co-NP.	78
5.2.2	Membership in XNP and Xco-NP.	78
6	A New Completeness Theory	85
6.1	New Parameterized Complexity Classes.	86
6.1.1	The Hierarchies $\Sigma_2^p[k*, t]$ and $\Sigma_2^p[*k, t]$	86
6.1.2	Another Hierarchy	88
6.2	The Parameterized Complexity Class $\Sigma_2^p[k*]$	89
6.2.1	Collapse	89
6.2.2	Answer Set Programming and Completeness for $\Sigma_2^p[k*]$	90
6.2.3	Additional Characterizations of $\Sigma_2^p[k*]$	94
6.3	The $\Sigma_2^p[*k, t]$ Hierarchy	115
6.3.1	A Normalization Result for $\Sigma_2^p[*k, 1]$	116
6.3.2	A Normalization Result for $\Sigma_2^p[*k, P]$	118
6.3.3	Answer Set Programming and $\Sigma_2^p[*k, P]$ -Completeness	122
6.3.4	Alternating Turing Machine Characterization	127

6.4	Relation to Known Parameterized Complexity Classes.	130
6.4.1	Relation of $\Sigma_2^P[k*]$ to Other Classes.	130
6.4.2	Relation of $\Sigma_2^P[*k, t]$ to Other Classes	132
6.4.3	Relation Between $\Sigma_2^P[k*]$ and $\Sigma_2^P[*k, t]$	134
7	Fpt-Algorithms with Access to a SAT Oracle	137
7.1	Known Parameterized Complexity Classes	138
7.2	The Parameterized Complexity Class $FPT^{NP}[\text{few}]$	141
7.2.1	A Parameterized Variant of the Boolean Hierarchy	142
7.2.2	Satisfiability Problems Complete for $FPT^{NP}[\text{few}]$	144
7.3	Lower Bounds on the Number of Oracle Queries	146
7.4	Bounded Optimization Problems.	147
7.5	Witness-Producing SAT Oracles	151
7.5.1	Comparing the Oracle Models for Decision Problems	153
7.5.2	Comparing the Oracle Models for Search Problems.	155
 Applying the Theory		
8	Problems in Knowledge Representation and Reasoning	161
8.1	Disjunctive Answer Set Programming	162
8.2	Abductive Reasoning.	162
8.3	Robust Constraint Satisfaction	173
9	Model Checking for Temporal Logics	181
9.1	The (Parameterized) Complexity of Model Checking.	184
9.1.1	Temporal Logics.	184
9.1.2	(Parameterized) Complexity Results	186
9.2	Symbolically Represented Kripke Structures.	187
9.2.1	PSPACE-hardness for Symbolic Model Checking.	187
9.2.2	An Fpt-Reduction to SAT for LTLAU,X.	191
9.3	Another Parameterized Variant of the Polynomial Hierarchy.	193
9.3.1	Alternative Characterizations	194
9.3.2	Relation to Other Parameterized Variants of the PH	196
9.4	Completeness for PH(level) and para-PSPACE.	196
10	Problems Related to Propositional Satisfiability	205
10.1	Minimization of DNF Formulas and Implicants	206
10.1.1	Minimizing Implicants.	206
10.1.2	Minimizing DNF Formulas	209
10.2	Inconsistency Repair	214
11	Problems in Judgment Aggregation	219
11.1	Judgment Aggregation.	221
11.1.1	Formula-Based Judgment Aggregation.	222
11.1.2	Constraint-Based Judgment Aggregation	223

11.2	Agenda Safety for the Majority Rule	225
11.2.1	CNF Formulas	225
11.2.2	Syntactic Restrictions	227
11.2.3	Bounded Treewidth.	233
11.2.4	Small Counterexamples	237
11.3	Computing Outcomes for the Kemeny Rule	239
11.3.1	Upper Bounds for the Formula-Based Framework.	241
11.3.2	Lower Bounds for the Formula-Based Framework	242
11.3.3	Upper Bounds for the Constraint-Based Framework	247
11.3.4	Lower Bounds for the Constraint-Based Framework	248
11.3.5	Overview.	250
12	Planning Problems	251
12.1	SAS ⁺ Planning	251
12.2	Planning with Uncertainty	252
12.3	Soft Goal Optimization	257
13	Graph Problems	261
13.1	Extending Graph Colorings	261
13.2	Extending Cliques.	263
13.2.1	Π_2^P -Completeness for CLIQUE-EXTENSION	267
 Relation to Other Topics in Complexity Theory		
14	Subexponential-Time Reductions	271
14.1	A Known Separation Result.	272
14.2	More Separation Results	273
14.2.1	Separation Results for $A[2]$	274
14.2.2	Separation Results for $\Sigma_2^P[k*]$ and $\Sigma_2^P[*k, t]$	278
14.3	Relating $\Sigma_2^P[k*]$ and $\Sigma_2^P[*k, t]$ to Each Other	282
15	Non-uniform Parameterized Complexity.	285
15.1	Non-uniform Parameterized Complexity Classes	286
15.1.1	Fpt-Size and Xp-Size Advice	287
15.1.2	Slice-Wise Advice	287
15.1.3	Poly-size and Kernel-Size Advice	288
15.1.4	Slice-Wise Non-uniformity.	289
15.2	Basic Results	289
15.2.1	Alternative Characterizations	289
15.2.2	Separations	293
15.3	Relation to Parameterized Knowledge Compilation	301
15.3.1	Parameterized Knowledge Compilation	301
15.3.2	(Conditional) Incomparability Results.	305
15.3.3	Restricting the Instance Space	311

15.3.4	The Parameterized Compilability of Finding Small Cliques.	321
15.3.5	Other Parameterized Compilation Problems	326
15.4	Parameterized Variants of the Karp-Lipton Theorem	330
Conclusions		
16	Open Problems and Future Research Directions	335
16.1	Improving Solving Methods	336
16.2	Open Theoretical Problems	337
16.2.1	Further Study of the $\Sigma_2^P[*k, t]$ Hierarchy.	337
16.2.2	Further Study of the Relation Between Classes.	338
16.2.3	Other Topics	339
16.2.4	Generalizing to Higher Levels of the PH.	340
16.3	Limited Non-determinism.	340
16.4	Witness Oracles	342
16.5	Non-deterministic Kernelization	343
17	Conclusion	349
17.1	Summary	349
17.2	Research Impact	352
Appendix A: Compendium of Parameterized Problems		355
Appendix B: Generalization to Higher Levels of the Polynomial Hierarchy		375
References		387
Index of Parameterized Problems		397