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Preface

This is a new type of calculus book: Students who master this text will be well versed in calculus and, in addition, possess a useful working knowledge of one of the most important mathematical software systems, namely, MACSYMA. This will equip them with the mathematical competence they need for science and engineering and the competitive workplace. The choice of MACSYMA is not essential for the didactic goal of the book. In fact, any of the other major mathematical software systems, e.g., AXIOM, MATHEMATICA, MAPLE, DERIVE, or REDUCE, could have been taken for the examples and for acquiring the skill in using these systems for doing mathematics on computers. The symbolic and numerical calculations described in this book will be easily performed in any of these systems by slight modification of the syntax as soon as the student understands and masters the MACSYMA examples in this book. What is important, however, is that the student gets all the information necessary to design and execute the calculations in at least one concrete implementation language as this is done in this book and also that the use of the mathematical software system is completely integrated with the text.

In these times of globalization, firms which are unable to hire adequately trained technology experts will not prosper. For corporations which depend heavily on science and engineering, remaining competitive in the global economy will require hiring employees having had a traditionally rigorous mathematical education. Educators do students an injustice when they offer oversimplified material under the deceptive guise of calculus reform. We direct this book to those teachers and students who share our commitment to excellence in mathematical preparation.

The instructor who uses this book should schedule student work on the computer at least one day per week. Lectures should include either live demonstrations (using the MACSYMA code given in this book or equivalent code in any other mathematical software system) or interactive sessions, with both input and output lines projected on a screen for student viewing.

When we were teaching with this text, we have typically combined three lectures with one laboratory session and ensured students access to the computer laboratory for special projects.

By selecting the appropriate sections of the book, the text can be used for a broad spectrum of course schedules in the various curricula of mathematics, computer science, engineering, business, health sciences, etc.

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Advice to the student on using this book

Begin with the introduction to MACSYMA in Appendix A, which contains enough information to get you started (but is no substitute for the MACSYMA User Guide).

Use MACSYMA to plot the graphs of all functions encountered here (you will find in this book fewer drawings than in traditional calculus texts because we expect you to draw your own). Use MACSYMA to compute and to check your hand calculations, but do not rely on it. Remember that software is no substitute for thinking.

This text has many problems and exercises, some routine (testing your knowledge of basic concepts) and other more challenging. Do as many as you can of each type.

Do not believe anything you read, even in a calculus book. This is why you should demand to see proofs. Simple proofs are given throughout the book, but complicated proofs are kept in separate and easily avoidable sections.

Equations are numbered on the right side.

Computer work lends itself to working in teams where members compare notes, check each other's work, and suggest new ideas. A congenial study group is a good start.