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Perspectives on Enclosure
Methods

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*To Götz Alefeld
on the occasion of his
60th birthday.*

Preface

Interval mathematics and enclosure methods have been developed to a high standard during the last decades together with their side effects on the arithmetic of computers, on programming languages and compilers, on the elementary functions and the run time system. A good number of meetings, the SCAN meetings in particular, have been devoted to this area. The latest of these meetings: scan2000 – GAMM-IMACS International Symposium on Scientific Computing, Computer Arithmetic and Validated Numerics was held at Universität Karlsruhe (TH), September 19-22, 2000 jointly with INTERVAL 2000 – International Conference on Interval Methods in Science and Engineering. The conference attracted 193 participants from 33 countries from all over the world. 12 invited lectures and 153 contributed talks were given at the symposium.

This volume contains selected papers on enclosure methods and validated numerics. The majority of contributions is based on invited lectures delivered at the scan2000 conference. These contributions are supplemented by a few selected additional papers. All papers represent original work following the common goal to push the limits of enclosure methods forward. We wish to thank all authors for their contributions and for adapting their manuscripts to the goals of this volume.

The editors and authors dedicate this volume to Prof. Götz Alefeld on the occasion of his 60th birthday. He has been among the leaders in the field for decades. The development of enclosure methods owes many outstanding contributions to him. His work shaped our current understanding of interval and enclosure methods in particular for linear and nonlinear systems of equations. He participated at the scan2000 conference as an author and as the president of the Gesellschaft für Angewandte Mathematik und Mechanik (GAMM).

Karlsruhe, Germany,
February 2001

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Rudolf Lohner
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