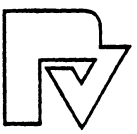
Fuzzy Classifier Design



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Preface

Fuzzy sets were first proposed by Lotfi Zadeh in his seminal paper [366] in 1965, and ever since have been a center of many discussions, fervently admired and condemned. Both proponents and opponents consider the arguments pointless because none of them would step back from their territory. And still, discussions burst out from a single sparkle like a conference paper or a message on some fuzzy-mail newsgroup. Here is an excerpt from an e-mail message posted in 1993 to fuzzy-mail@vexpert.dbai.twvien.ac.at, by somebody who signed "Dave".

> '... Why then the "logic" in "fuzzy logic"? I don't think anyone has successfully used fuzzy sets for logical inference, nor do I think anyone will. In my admittedly neophyte opinion, "fuzzy logic" is a misnomer, an oxymoron. (I would be delighted to be proven wrong on that.)... I came to the fuzzy literature with an open mind (and open wallet), high hopes and keen interest. I am very much disillusioned with "fuzzy" per se, but I did happen across some extremely interesting things along the way."

Dave, thanks for the nice quote! Enthusiastic on the surface, are not many of us suspicious deep down? In some books and journals the word *fuzzy* is religiously avoided: fuzzy set theory is viewed as a second-hand cheap trick whose aim is nothing else but to devalue good classical theories and open up the way to lazy ignorants and newcomers. This view is sometimes attributed to the conservatism of the "western culture" compared to the enterprising and practical "eastern culture" that took fuzzy sets on board right from the very start [342]. The other face of this coin is the "fuzzy" euphoria going under the motto "fuzzify the unfuzzifiable", sometimes with little or no rationale. Despite the unfinished philosophic debate there are a remarkable number of research papers and engineering applications involving fuzzy sets – some good and some bad, like in any other branch of science.

What are fuzzy classifiers and where do they stand in this stream? Do we need fuzzy classifiers or are we satisfied with pattern recognition "classics"? Are fuzzy classifiers better in some sense? In which cases and by how much are they better? Are fuzzy classifiers just "fuzzified" versions of non-fuzzy classifiers or are they an original contribution?

The difficulty in writing a coherent text on fuzzy classifiers roots in the diversity of the field itself. A paragraph taken from the Introduction by John Hartigan to the book "Clustering and Classification" [20], 1996, reads:

> 'Let us agree that classification does not fit easily into any of the academic disciplines in which its practitioners live. We remain too diffuse and dispersed in our methods and applications to form a coherent academic discipline ourselves.'

The author speaks about clustering and classification, but insert *fuzzy* in front of these two words, and the statement is enhanced by orders of magnitude.

Fuzzy Classifier Design is an attempt to address and answer some of the above questions without giving a comprehensive account of the area. My intention was not to take sides in the pro-con argument but to highlight some issues which would let you, my reader, make your own educated choice about fuzzy classifier design.

Bangor, Wales
February, 2000

Ludmila Ilieva Kuncheva

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