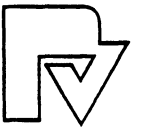
Fuzzy Graphs and Fuzzy Hypergraphs



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FOREWORD

In the course of fuzzy technological development, fuzzy graph theory was identified quite early on for its importance in making things work. Two very important and useful concepts are those of granularity and of nonlinear approximations. The concept of granularity has evolved as a cornerstone of Lotfi A.Zadeh's theory of perception, while the concept of nonlinear approximation is the driving force behind the success of the consumer electronics products manufacturing.

It is fair to say fuzzy graph theory paved the way for engineers to build many rule-based expert systems. In the open literature, there are many papers written on the subject of fuzzy graph theory. However, there are relatively books available on the very same topic. Professors' Mordeson and Nair have made a real contribution in putting together a very comprehensive book on fuzzy graphs and fuzzy hypergraphs. In particular, the discussion on hypergraphs certainly is an innovative idea.

For an experienced engineer who has spent a great deal of time in the laboratory, it is usually a good idea to revisit the theory. Professors Mordeson and Nair have created such a volume which enables engineers and designers to benefit from referencing in one place. In addition, this volume is a testament to the numerous contributions Professor John N. Mordeson and his associates have made to the mathematical studies in so many different topics of fuzzy mathematics.

The Center for Research in Fuzzy Mathematics and Computer Science, under the direction of Dr. John N. Mordeson, is one of the earliest of these establishments in the world. The scholarly and academic products that

have grown out from the center certainly are very impressive indeed, both in terms of quality as well as quantity.

In a sense, fuzzy mathematics is a generalization of traditional mathematics. In this regard, I have no doubt that Professor John N. Mordeson and his associates will be recognized as important leading researchers and the Center for Research in Fuzzy Mathematics and Computer Science will have its place in the annals of fuzzy theory as an important innovation and institution.

Paul P. Wang
Duke University

PREFACE

In 1965, L. A. Zadeh introduced the concept of a fuzzy subset of a set as a way for representing uncertainty. Zadeh's ideas stirred the interest of researchers worldwide. His ideas have been applied to a wide range of scientific areas. Theoretical mathematics has also been touched by the notion of a fuzzy subset. We consider two areas of mathematics here.

The book deals with fuzzy graph theory and fuzzy hypergraph theory. The book is based on papers that have appeared in journals and conference proceedings. The purpose of this book is to present an up to date account of results from these two areas and to give applications of the results. The book should be of interest to research mathematicians and to engineers and computer scientists interested in applications. For the purpose of a comprehensive presentation of fuzzy graph theory, we include not only much of what appears in volume 20 of this series, but also a greatly expanded version.

In Chapter 1, basic concepts of fuzzy subset theory are given. The notion of a fuzzy relation and its basic properties are presented. The concept of a fuzzy relation is fundamental to many applications given, e. g., cluster analysis and pattern classification. Chapter 1 is based primarily on the work of Rosenfeld and Yeh and Bang.

Chapter 2 presents many concepts and theoretical results of fuzzy graphs. The material from this chapter is the result of the work of many authors including that of the authors of this book. However much of the work is an outgrowth of the ideas of Rosenfeld. We acknowledge the authors at the beginning of each section. This chapter deals with the fuzzification of such concepts as paths, connectedness, bridges, cut vertices, trees, forests,

cut sets, chords, cotrees, twigs, 1-chains, cocycles, line graphs, intersection graphs, and interval graphs.

In Chapter 3, applications of fuzzy graph theory are presented. Here again many of the results of this chapter are based on the work of Rosenfeld and Yeh and Bang. Applications of fuzzy graphs to cluster analysis and database theory are presented. Applications of fuzzy graphs to the problem concerning group structure are also given.

In Chapter 4, we present theoretical aspects of fuzzy hypergraph theory with applications to portfolio management, managerial decision making with an example to waste management, and to neural cell-assemblies. The results of this chapter are taken mainly from the work of Goetschel and his coauthors. We have reorganized Goetschel's work and added some examples. This chapter deals with the concepts of fuzzy transversals of fuzzy hypergraphs, colorings of fuzzy hypergraphs, and intersecting fuzzy hypergraphs. In 1982, Z. Pawlak introduced the idea of a rough set in order to provide a systematic approach for the study of indiscernibility of objects. We show how (fuzzy) hypergraphs and rough sets are related in such a way that ideas may be carried back and forth between the two areas.

John N. Mordeson
Premchand S. Nair

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