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# Nonsmooth Modeling and Simulation for Switched Circuits



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# Preface

The major aim of this monograph is to show that the nonsmooth dynamics framework (involving keywords like complementarity problems, piecewise-linear characteristics, inclusions into normal cones, variational inequalities, multivalued characteristics) is a convenient and efficient way to handle *analog* switched circuits. It has been long known in the circuits community that such nonsmooth switched systems are difficult to simulate numerically, for various reasons that will be recalled. In parallel the simulation of nonsmooth mechanical systems (*i.e.* mainly mechanical systems with nonsmooth interface or contact laws, like unilateral constraints, impacts, Coulomb's friction and its many extensions, *etc.*) has been the object of a lot of research studies (see for instance the recent monographs Acary and Brogliato 2008 and Studer 2009). This field has now reached a certain degree of maturity, and has proved to be a quite useful and efficient approach in many areas of mechanics. Here we would like to show that the tools that have been employed in the contact mechanics context, can be successfully extended to the simulation of analog switched circuits. To the best of our knowledge it is the first time that such extensive numerical simulations using the nonsmooth dynamics framework for analog switched circuits are presented and published.

Montbonnot

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# Contents

## Part I Theoretical Framework

<b>1</b>	<b>Introduction to Switched Circuits</b>	<b>3</b>
1.1	Simple Examples of Switched Circuits	3
1.1.1	Diode Modeling	3
1.1.2	An RCD Circuit	5
1.1.3	An RLZD Circuit	7
1.1.4	An RCZD Circuit	10
1.1.5	An RLD Circuit	12
1.1.6	More Examples: Order-Two and Order-Three Circuits	19
1.1.7	A Circuit with an Ideal Switch	23
1.2	A Unified Dynamical Framework: Lur'e Dynamical Systems	26
1.3	An Aside on Nonsmooth Mechanics: The Bouncing Ball	27
1.4	Conclusions	29
1.5	Historical Summary	30
<b>2</b>	<b>Mathematical Background</b>	<b>33</b>
2.1	Basics from Convex and Nonsmooth Analysis	33
2.1.1	Convex Sets and Functions	33
2.1.2	Multivalued Functions	46
2.2	Non Convex Sets	51
2.3	Basics from Complementarity Theory	51
2.3.1	Definitions	51
2.3.2	Complementarity Problems: Existence and Uniqueness of Solutions	53
2.3.3	Links with Inclusions into Normal Cones	56
2.3.4	Links with Variational Inequalities	57
2.3.5	Links with Optimization	58
2.4	Mathematical Formalisms	59
2.4.1	Moreau's Sweeping Process, Measure Differential Inclusions	60
2.4.2	Dynamical Variational Inequalities	65
2.4.3	Complementarity Dynamical Systems	66

2.4.4	Filippov's Inclusions . . . . .	73
2.4.5	Maximal Monotone Inclusions, Unilateral Differential Inclusions . . . . .	81
2.4.6	Equivalences Between the Formalisms . . . . .	82
2.5	The Dynamics of the Simple Circuits . . . . .	84
2.5.1	The Ideal Diode Voltage/Current Law . . . . .	85
2.5.2	The Piecewise-Linear Diode Voltage/Current Law . . . . .	85
2.5.3	A Mixed Nonlinear/Unilateral Diode . . . . .	85
2.5.4	From Smooth to Nonsmooth Electrical Powers . . . . .	86
2.5.5	The RLD Circuit in (1.16) . . . . .	88
2.5.6	The RCD Circuit in (1.3) . . . . .	90
2.5.7	The RLZD Circuit in (1.7) . . . . .	91
2.5.8	Coulomb's Friction and Zener Diodes . . . . .	91
2.5.9	The RCZD Circuit in (1.11) . . . . .	93
2.5.10	The Circuit in (1.41) . . . . .	94
2.5.11	The Switched Circuit in (1.52) . . . . .	96
2.5.12	Well-Posedness of the OSNSP in (1.45) . . . . .	97
2.5.13	The Bouncing Ball . . . . .	98
2.6	Time-Discretization Schemes . . . . .	99
2.6.1	Maximal Monotone Differential Inclusions . . . . .	100
2.6.2	Linear Complementarity Systems . . . . .	100
2.6.3	Moreau's Sweeping Process . . . . .	102
2.7	Conclusions and Recapitulation . . . . .	104

## Part II Dynamics Generation and Numerical Algorithms

3	<b>Conventional Circuit Equation Formulation and Simulation</b> . . . . .	109
3.1	Circuit Topology and Kirchhoff's Laws . . . . .	109
3.1.1	The Circuit Network as a Connected Oriented Graph . . . . .	109
3.1.2	The Incidence Matrix $A$ and Kirchhoff's Current Laws . . . . .	110
3.1.3	The Loop Matrix $B$ and Kirchhoff's Voltage Laws . . . . .	111
3.1.4	KVL in Terms of Nodes Voltages . . . . .	111
3.2	The Sparse Tableau Analysis (STA) . . . . .	112
3.3	The Modified Nodal Analysis . . . . .	112
3.3.1	Classification of the Branches . . . . .	113
3.3.2	Standard Resistive, Capacitive and Inductive Branches . . . . .	113
3.4	The Charge/Flux Oriented MNA . . . . .	115
3.5	Standard DAEs Stemming from the MNA . . . . .	115
3.5.1	Various Forms of DAEs . . . . .	116
3.5.2	Index and Solvability . . . . .	118
3.6	Semi-Explicit DAE Forms . . . . .	124
3.6.1	A First Naive Attempt . . . . .	124
3.6.2	A Second Attempt . . . . .	126
3.6.3	The Proposed Solution . . . . .	129
3.7	Basics on Standard Circuit Simulation . . . . .	131
3.7.1	Computation of the Initial Conditions . . . . .	131

3.7.2	Time-Discretization of the MNA . . . . .	132
3.7.3	Solving Nonlinear Systems . . . . .	132
3.7.4	Implementation Details and the Stamping Method . . . . .	134
<b>4</b>	<b>Nonsmooth Modeling of Electrical Components . . . . .</b>	<b>137</b>
4.1	General Nonsmooth Electrical Element . . . . .	137
4.2	Nonsmooth Elements as Inclusions into the Subdifferential of Convex Functions and Variational Inequality (VI) . . . . .	138
4.3	Nonsmooth Elements as Inclusions into Normal Cones and Variational Inequalities . . . . .	140
4.4	Complementarity Problems . . . . .	141
4.5	The Linear Input/Output Relation Case . . . . .	142
4.5.1	Some Instances of Linear Nonsmooth Components . . . . .	142
4.6	Generic Piecewise-Linear Components . . . . .	143
4.6.1	The First Model Description of van Bokhoven . . . . .	143
4.6.2	The Second Model Description of van Bokhoven . . . . .	144
4.7	Special Instances of Nonsmooth Components . . . . .	145
4.7.1	Ideal Diode . . . . .	145
4.7.2	Zener Diode . . . . .	145
4.7.3	Ideal Switch . . . . .	146
4.7.4	Explicit Ideal Switch. Glockner's Model . . . . .	147
4.7.5	MOSFET Transistor . . . . .	148
4.7.6	Nonlinear and Nonsmooth MOS Transistor . . . . .	152
4.7.7	Comparator Component . . . . .	153
<b>5</b>	<b>Time-Stepping Schemes and One Step Solvers . . . . .</b>	<b>155</b>
5.1	Summary of the Mathematical Formalisms . . . . .	155
5.1.1	Nonsmooth DAE Formulation. Differential Generalized Equation (DGE) . . . . .	155
5.1.2	The Semi-Explicit Nonsmooth DAE: Semi-Explicit DGE .	157
5.2	Principles of the Numerical Time-Integration Scheme . . . . .	158
5.2.1	Time-Stepping Solutions for a Solution of Class $C^1$ . . . . .	160
5.2.2	Time-Stepping Schemes for an Absolutely Continuous Solution . . . . .	161
5.2.3	Time-Stepping Solutions for a Solution of Bounded Variations . . . . .	166
5.2.4	Illustrations of Wrong Discretizations . . . . .	170
5.2.5	How to Choose a Scheme in Practice? . . . . .	173
5.2.6	Newton's Method for the Nonlinear Dynamics . . . . .	177
5.3	Time-Discretization of the General Cases . . . . .	179
5.4	One-Step NonSmooth Problems (OSNSP) Solvers . . . . .	180
5.4.1	$K$ is a Finite Representable Convex Set . . . . .	181
5.4.2	$K$ is a Generalized Box . . . . .	181

## Part III Numerical Simulations

<b>6 The Automatic Circuit Equations Formulation (ACEF) Module and the SICONOS Software . . . . .</b>	185
6.1 An Insight into SICONOS . . . . .	185
6.1.1 Step 1. Building a Nonsmooth Dynamical System . . . . .	186
6.1.2 Step 2. Simulation Strategy Definition . . . . .	189
6.2 SICONOS Software . . . . .	190
6.2.1 General Principles of Modeling and Simulation . . . . .	190
6.2.2 NSDS Related Components . . . . .	193
6.2.3 Simulation-Related Components . . . . .	195
6.2.4 SICONOS software design . . . . .	196
6.3 The ACEF Module and Algorithms . . . . .	198
6.3.1 A Module Able to Read a Circuit File: A Parser . . . . .	200
6.3.2 Build the Vector of Unknowns $I_{NS}$ . . . . .	201
6.3.3 An Algorithm to Choose the Unknowns . . . . .	201
6.3.4 Building the System $N(x, t)\dot{x} = f(x, z, t)$ of (3.70) . . . . .	202
6.3.5 Building the Relation $0 = g(x, z, t)$ of (3.70) . . . . .	203
6.3.6 The Stamp Method for Nonsmooth Components . . . . .	204
6.3.7 Some Stamp Examples . . . . .	205
6.3.8 The ACEF Global Execution Algorithm . . . . .	206
6.3.9 An Example of the Stamp Method with Nonsmooth Component . . . . .	206
<b>7 Simple Circuits . . . . .</b>	215
7.1 Maffezzoni's Example . . . . .	215
7.1.1 The Dynamical Model . . . . .	216
7.1.2 Simulation Results: Failure of the Newton-Raphson Algorithm . . . . .	217
7.1.3 Numerical Results with SICONOS . . . . .	219
7.1.4 Numerical Results with ELD0 . . . . .	219
7.2 A First Diode-Bridge Wave Rectifier . . . . .	221
7.2.1 Dynamical Equations . . . . .	221
7.2.2 Simulation Results . . . . .	222
7.3 A Second Diode-Bridge Wave Rectifier . . . . .	224
7.4 The Ćuk Converter . . . . .	225
7.5 A Circuit Exhibiting Sliding Modes . . . . .	231
7.5.1 Models and Dynamical System . . . . .	234
7.5.2 Simulation and Comparisons . . . . .	235
<b>8 Buck and Delta-Sigma Converters . . . . .</b>	239
8.1 The Buck Converter with Load Resistor . . . . .	239
8.1.1 Dynamical Equations . . . . .	240
8.1.2 Numerical Results with SICONOS . . . . .	241
8.1.3 Comparisons and Discussions . . . . .	244

8.2	The Buck Converter Loaded by a Resistor and an Inverter Chain . . . . .	253
8.2.1	Simulation as a Nonsmooth Dynamical System with SICONOS . . . . .	253
8.2.2	Simulation with PLECS . . . . .	254
8.3	The Delta-Sigma Converter . . . . .	254
8.3.1	Dynamical Equations . . . . .	259
8.3.2	Numerical Results with SICONOS . . . . .	264
8.3.3	Comparisons and Discussions . . . . .	265
8.4	Conclusions . . . . .	265
<b>Erratum</b>	. . . . .	E1
<b>Appendix A Some Facts in Real Analysis</b>	. . . . .	267
A.1	Absolutely Continuous Functions and Sets . . . . .	267
A.2	Lipschitz Continuous Functions and Sets . . . . .	268
A.3	Functions of Bounded Variations in Time . . . . .	268
A.4	Multifunctions of Bounded Variation in Time . . . . .	270
A.5	Differential Measures . . . . .	270
A.6	Measure Differential Inclusion (MDI) . . . . .	271
<b>References</b>	. . . . .	273
<b>Index</b>	. . . . .	281

# List of Abbreviations

ACEF	Automatic Circuit Equations Formulation
API	Application Programming Interface
BCE	Branch Constitutive Equation
BDF	Backward Differentiation Formulas
CCP	Cone Complementarity Problem
DAE	Differential Algebraic Equation
KCL	Kirchhoff's Current Laws
KVL	Kirchhoff's Voltage Laws
LCP	Linear Complementarity Problem
LTI	Linear Time Invariant
MCP	Mixed Complementarity Problem
MDE	Measure Differential Equation
MDI	Measure Differential Inclusion
MLCP	Mixed Linear Complementarity Problem
MNA	Modified Nodal Analysis
NCP	Nonlinear Complementarity Problem
NSDS	NonSmooth Dynamical Systems
ODE	Ordinary Differential Equation
OSNSP	Onestep NonSmooth Problem
STA	Sparse Tableau Analysis

# List of Algorithms

Algorithm 1	Build $I_{NS}$ . . . . .	201
Algorithm 2	Build $x$ and $z$ . . . . .	202
Algorithm 3	How build the system $N(x, t)x' = f(x, z, t)$ with $N(x, t)$ a regular matrix? . . . . .	203
Algorithm 4	How to build the system $0 = g(x, z, t)?$ . . . . .	204
Algorithm 5	Resistor stamp algorithm . . . . .	205
Algorithm 6	Inductor stamp algorithm . . . . .	205
Algorithm 7	Capacitor stamp algorithm . . . . .	206
Algorithm 8	Capacitor stamp after inversion algorithm . . . . .	207
Algorithm 9	Diode stamp algorithm . . . . .	207
Algorithm 10	Linear and nonsmooth MOS stamp algorithm . . . . .	208
Algorithm 11	Nonlinear and nonsmooth MOS stamp algorithm . . . . .	209
Algorithm 12	ACEF global algorithm . . . . .	210

# List of Figures

Fig. 1.1	Diode symbol . . . . .	4
Fig. 1.2	Four models of diodes . . . . .	4
Fig. 1.3	The ideal diode voltage/current law . . . . .	5
Fig. 1.4	A circuit with an ideal diode, a resistor, a capacitor and a voltage source . . . . .	6
Fig. 1.5	A circuit with an ideal Zener diode, a resistor, an inductor and a voltage source . . . . .	7
Fig. 1.6	Two models of Zener diode . . . . .	8
Fig. 1.7	Iterations of the backward Euler method for (1.7) . . . . .	9
Fig. 1.8	Zener diode voltage/current law . . . . .	11
Fig. 1.9	Solving the generalized equation (1.12) . . . . .	11
Fig. 1.10	A circuit with an ideal diode, a resistor, an inductor and a current source . . . . .	12
Fig. 1.11	Solutions of (1.34): the state $x_k$ vs. time $t_k$ . . . . .	18
Fig. 1.12	Solutions of (1.34): the multiplier $v_k$ vs. time $t_k$ . . . . .	19
Fig. 1.13	Solutions of (1.34): the impulse $\sigma_k$ vs. time $t_k$ . . . . .	20
Fig. 1.14	RLC circuits with an ideal diode . . . . .	20
Fig. 1.15	RLC circuits with an ideal diode . . . . .	21
Fig. 1.16	A circuit with Zener and ideal diodes . . . . .	22
Fig. 1.17	A circuit with an ideal switch . . . . .	23
Fig. 1.18	Circuits as feedback systems . . . . .	26
Fig. 1.19	Discretized circuits as feedback systems . . . . .	27
Fig. 1.20	A piecewise-nonlinear DIAC model (Diode for Alternative Current) . . . . .	27
Fig. 1.21	The bouncing-ball . . . . .	28
Fig. 2.1	Planar convex and non-convex sets . . . . .	34
Fig. 2.2	Convex and non-convex cones . . . . .	35
Fig. 2.3	Convex cones and their polar cones . . . . .	35
Fig. 2.4	Epigraph of the absolute value function . . . . .	38
Fig. 2.5	Epigraph of the indicator of $C$ . . . . .	38
Fig. 2.6	Lower and upper semi-continuous functions . . . . .	39
Fig. 2.7	Normal cones . . . . .	41
Fig. 2.8	Tangent cones . . . . .	41

Fig. 2.9	Normal cones to a finitely represented set . . . . .	44
Fig. 2.10	Normal cone to the epigraph . . . . .	45
Fig. 2.11	Conjugating, subdifferentiating and inverting . . . . .	46
Fig. 2.12	Multivalued functions . . . . .	48
Fig. 2.13	Monotone mappings . . . . .	49
Fig. 2.14	Cone complementarity problem . . . . .	52
Fig. 2.15	A moving convex set $C(t)$ (the normal cones are depicted with dashed lines) . . . . .	61
Fig. 2.16	An RLC circuit with a controlled voltage source . . . . .	64
Fig. 2.17	Electrical circuit with capacitors, resistors and ideal diodes . . . . .	72
Fig. 2.18	A 4-diode bridge wave rectifier . . . . .	72
Fig. 2.19	A diode with a mixed nonlinear/unilateral behaviour . . . . .	86
Fig. 2.20	A circuit with a mixed diode . . . . .	87
Fig. 2.21	From smooth to nonsmooth powers . . . . .	87
Fig. 2.22	Subdifferentials . . . . .	88
Fig. 2.23	Two Zener diodes mounted in series . . . . .	91
Fig. 2.24	The sum of the two Zener voltage/current laws . . . . .	92
Fig. 2.25	Circuit with Zener diodes . . . . .	93
Fig. 2.26	Bouncing-ball feedback interconnection with the corner law . . . . .	99
Fig. 2.27	Iterations for (2.143) . . . . .	102
Fig. 2.28	The catching-up algorithm . . . . .	103
Fig. 3.1	Simple circuits illustrating DAE formulations . . . . .	125
Fig. 3.2	Newton's method failure . . . . .	133
Fig. 4.1	The relay with dead-zone multivalued mapping . . . . .	140
Fig. 4.2	The relay multivalued mapping . . . . .	141
Fig. 4.3	Multivalued mapping. Example of Leenaerts and Van Bokhoven (1998) . . . . .	144
Fig. 4.4	Ideal diode with residual current and voltage . . . . .	145
Fig. 4.5	The Zener diode . . . . .	146
Fig. 4.6	Ideal switch . . . . .	147
Fig. 4.7	nMOS transistor symbol . . . . .	149
Fig. 4.8	Piecewise-linear approximation of $f(\cdot)$ . . . . .	151
Fig. 4.9	Static characteristic of an nMOS transistor with a simple piecewise-linear model and SPICE level 1 model . . . . .	151
Fig. 4.10	The comparator component . . . . .	153
Fig. 5.1	Solution of (1.39) with the time-stepping scheme (5.22). (1) Exact solution $x(t_k)$ . (2) $x_k$ with $\theta = 1, \gamma = 1$ . (3) $x_k$ with $\theta = 1/2, \gamma = 1$ . (4) $x_k$ with $\theta = 1/2, \gamma = 1/2$ . . . . .	162
Fig. 5.2	Simulation of (5.35) with the time-stepping scheme (5.28). <b>(a)</b> $\theta = 1$ <b>(b)</b> $\theta = 1/2$ . . . . .	165
Fig. 5.3	Simulation of (5.35) with the time-stepping scheme (5.22). <b>(a)</b> $\theta = 1/2, \gamma = 1$ <b>(b)</b> $\theta = 1/2, \gamma = 1/2$ . . . . .	165
Fig. 5.4	Simulation of system (1.5). (1) scheme (5.46). (2) scheme (5.50) with $\theta = 1/2$ . (3) scheme (5.22) with $\theta = 1/2, \gamma = 1/2$ . . . . .	168
Fig. 5.5	Two configurations of the 4-diode bridge rectifier . . . . .	170

Fig. 5.6	Simulation of the configuration (Fig. 5.5(a)) with the scheme (5.22). Timestep $h = 10^{-6}$ . (1) $\theta = 1, \gamma = 1$ (2) $\theta = 1/2, \gamma = 1/2$	171
Fig. 5.7	Simulation of the configuration (Fig. 5.5(b)) with the scheme (5.28). Timestep $h = 10^{-6}$ . (1) $\theta = 1$ (2) $\theta = 1/2$	172
Fig. 5.8	Simulation of the configuration (Fig. 5.5(b)) with the scheme (5.22). Timestep $h = 10^{-6}$ . (1) $\theta = 1, \gamma = 1$ (2) $\theta = 1/2, \gamma = 1/2$	173
Fig. 5.9	Simulation of the configuration (Fig. 5.5(b)) with the scheme (5.28). Timestep $h = 10^{-6}$ . (1) $\theta = 1$ (2) $\theta = 1/2$	174
Fig. 5.10	Simulation of the configuration (Fig. 5.5(b)) with the scheme (5.22). Timestep $h = 10^{-6}$ . (1) $\theta = 1, \gamma = 1$ (2) $\theta = 1/2, \gamma = 1/2$	175
Fig. 5.11	Simulation of the configuration (Fig. 5.5(b)) with the scheme (5.28). Timestep $h = 10^{-6}$ . $\theta = 1$	176
Fig. 6.1	A four-diode bridge wave rectifier	186
Fig. 6.2	SICONOS nonsmooth dynamical system modeling principle	191
Fig. 6.3	Some multivalued piecewise-linear laws: saturation, relay, relay with dead zone	195
Fig. 6.4	General design of SICONOS software	196
Fig. 6.5	Kernel components dependencies	198
Fig. 6.6	Simplified class diagram for Kernel modeling part	199
Fig. 6.7	Simplified class diagram for Kernel simulation part	200
Fig. 6.8	Circuit containing a loop of capacitors	207
Fig. 6.9	Circuit with a loop of capacitors and a nonsmooth electrical element	213
Fig. 7.1	A simple switched circuit	216
Fig. 7.2	Equivalent linear circuit	217
Fig. 7.3	Circuit state at $t = t_N$	218
Fig. 7.4	Equivalent linear model, first step	218
Fig. 7.5	Equivalent linear model, second step	218
Fig. 7.6	Equivalent linear model, third step	219
Fig. 7.7	Switched circuit simulations	220
Fig. 7.8	A 4-diode bridge wave rectifier	221
Fig. 7.9	Bridge wave rectifier simulation results	223
Fig. 7.10	Step size control using a $10^{-3}$ tolerance. 193 steps +95 rejected	224
Fig. 7.11	Filtered full wave rectifier	224
Fig. 7.12	SICONOS (a) and SMASH (b) simulations of the diode-bridge circuit, 0.1 $\mu$ s time step	226
Fig. 7.13	SICONOS (a) and SMASH (b) simulations of the diode-bridge circuit, 1 $\mu$ s time step	227
Fig. 7.14	SMASH (b) and SICONOS (a) simulations of the diode-bridge circuit, 10 $\mu$ s time step	228
Fig. 7.15	Simulation results of the diode-bridge circuit. (a) SICONOS with $h = 2 \mu$ s (b) SMASH with $h = 0.1 \mu$ s	229
Fig. 7.16	Simulation results of the diode-bridge circuit. (a) SICONOS with $h = 1 \mu$ s (b) SMASH with $h = 0.1 \mu$ s	230
Fig. 7.17	The Ćuk converter	231
Fig. 7.18	Regulation function of the switch $V_G(t)$	231

Fig. 7.19	Simulation values versus time (s) for the Ćuk Converter. (1) $C_1$ voltage, (2) $C_1$ voltage, (3) $V_G$ gate voltage, (4) MOS switch drain voltage, (5) $L_1$ current and (6) $L_2$ current . . . . .	232
Fig. 7.20	Simulation values versus time (s) for the Ćuk Converter. (1) $C_1$ voltage, (2) $C_1$ voltage, (5) $L_1$ current and (6) $L_2$ current . . . . .	233
Fig. 7.21	RLC Zener-diodes circuit . . . . .	234
Fig. 7.22	Simulation of the circuit with a sliding mode . . . . .	236
Fig. 8.1	Buck converter . . . . .	240
Fig. 8.2	SICONOS buck converter simulation using standard parameters . . . . .	241
Fig. 8.3	SICONOS buck converter simulation using $L = 7 \mu\text{H}$ , $C = 15 \mu\text{F}$ , $R_{11} = 12\text{k}\Omega$ , $R_{21} = 6\text{M}\Omega$ , $C_{11} = 15 \text{ pF}$ . . . . .	242
Fig. 8.4	SICONOS buck converter simulation using sliding mode parameters . . . . .	242
Fig. 8.5	Focus on the pMOS component . . . . .	243
Fig. 8.6	Comparison of piecewise-linear and SPICE (tanh based) comparator models . . . . .	245
Fig. 8.7	SICONOS buck converter simulation using sliding mode parameters and multivalued comparator . . . . .	247
Fig. 8.8	ELDO buck converter simulation using sliding mode parameters and $V_{out} = 1.5(\tanh(10000V_{in}) + 1)$ for the comparator . . . . .	247
Fig. 8.9	PLECS circuit part of the buck converter with a load resistor . . . . .	249
Fig. 8.10	SIMULINK model of the buck converter with a load resistor . . . . .	250
Fig. 8.11	PLECS buck converter simulation using standard parameters. Time in seconds . . . . .	251
Fig. 8.12	PLECS buck converter simulation using standard parameters, zoom view of steady state. Time in $\mu\text{s}$ . . . . .	251
Fig. 8.13	Global error . . . . .	252
Fig. 8.14	Global error evaluation as a function of the number of hyperplanes . . . . .	252
Fig. 8.15	Buck converter supplying a resistor load and an inverter chain . . . . .	253
Fig. 8.16	SICONOS results, buck & load resistor & inverters (first 200 $\mu\text{s}$ ) . . . . .	255
Fig. 8.17	SICONOS results, buck & load resistor & inverters (zoom) . . . . .	256
Fig. 8.18	SICONOS results, buck & load resistor & inverters (zoom) . . . . .	257
Fig. 8.19	SICONOS simulation results, buck converter supplying a load resistor and a 100 inverters chain (first 100 $\mu\text{s}$ ) . . . . .	258
Fig. 8.20	SICONOS simulation results, buck converter supplying a load resistor and a 100 inverters chain; Zoom on the start-up of the inverter chain . . . . .	259
Fig. 8.21	SICONOS results, buck & load resistor & 100 inverters (zoom) . . . . .	260
Fig. 8.22	PLECS circuit part of the buck converter supplying a resistor and inverters . . . . .	261
Fig. 8.23	SIMULINK model of the buck converter supplying a resistor and inverters . . . . .	262
Fig. 8.24	Delta-Sigma converter . . . . .	263
Fig. 8.25	ELDO simulation . . . . .	264
Fig. 8.26	SICONOS simulation . . . . .	264

## List of Tables

Table 4.1 Parameters for Sah's model of nMOS transistor . . . . .	148
Table 8.1 Numerical comparisons on the buck converter example . . . . .	246