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Proof, Computation and Agency

Logic at the Crossroads

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Foreword

This book is all about new links in logic. The "First Indian Conference on Logic and its Relationship with Other Disciplines" took place in Mumbai at IIT Bombay, from January 8 to 12, 2005, as an initiative of the Mumbai logic circle, bringing together Indian logicians from various disciplinary backgrounds and different locations with a group of open-minded and active international colleagues.

The conference took place over 6 days: two of them devoted to tutorials and four to advanced talks. Tutorials as well as advanced talks were given by Indian logicians and by visitors from abroad. The visitors responding to the Call for the Mumbai event came from Australia, the Czech Republic, Finland, Great Britain, Italy, Israel, Japan, the Netherlands, the USA, and other nations. Together, they formed a distinguished galaxy seldom found even with conferences in the West. The papers emphasized novel approaches and perspectives emanating from research by some of the masters of logic and its interfaces with surrounding disciplines.

The Conference venue were the outstanding logistical and scenic facilities at IIT Bombay, with smooth organizational efforts and crucial support provided by faculty, and especially also by student volunteers. The geese, the pond, and the view of the lake provided most people with the needed inspiration to think of deeper matters.

Talks ranged from reflections on the range of mathematical proof and definability to recent developments in computational logic, as well as new interfaces between logic, information dynamics, and games. In addition, there was a wide range of presentations on schools of Indian logic. One term used nowadays for this broad view of logic is "intelligent interaction". The Mumbai Conference took this term in the double sense of both information exchange and community formation, and indeed both processes were in evidence. Accordingly, the "Second Indian Conference on Logic and its Relationship with Other Disciplines" was held from January 9 to 11 in 2007 at IIT Bombay, Mumbai, with equal success. Besides these two Conferences, a first Indian Winter School with the same topic and title was held at IIT Bombay, Mumbai from January 1 to 15, 2006, primarily for students and researchers.

The current volume contains a broad selection of material from the first Conference, while further volumes will document the follow-up. We see this publication venture as providing information, but also as a means of shaping a community. Our

aim is to present an Indian audience with what we see as a representative sample of modern logic, in the hands of some of its best practitioners. This volume is not a textbook, however, accessible throughout to absolute beginners. But we do expect that the papers collected here will give a sense of vitality, strength and direction, and that, through the story-lines and references, our chapters will open further doors to the field as the reader gets inspired.

We also hope that this volume will help the Indian logic community take shape and flight, and indeed, there are some very encouraging signs. Since this book was put together, an 'Association for Logic in India' has been formed that has taken the IIT Bombay initiative as well as other congenial events in Kolkata 2007 under its aegis. The website http://ali.cmi.ac.in/ shows its lively past and current activities. Thus the Mumbai founding events have found a natural continuation in a national series of conferences and schools.

Finally, on a still larger scale, we hope that the present publication initiative will maintain the ties between the Indian community and its friends and colleagues worldwide. In particular, we see the papers on Indian Logic in this volume as reaching out to more classical communities in the field while informing their modern colleagues. One day, we envision that logic will have one history, with Gotama and Gangesa along with Aristotle and Boole. Be that as it may be, our book may be a first gentle push toward joining forces between historically different, but maybe in the final analysis, not all that different strands in our fascinating field.

The editors of this volume have had an easy task in some ways. The positive response and generous cooperation from both Indian and international colleagues has been overwhelming. Editing is easy with authors of this energy and quality, while serious referees were easy to find. Nevertheless, producing this volume has also meant solving a host of non-trivial logistic problems, which could only have been done by our Mumbai experts. Special thanks are due to Mr. Abhisekh Sankaran, who has provided vital technical support to our efforts in putting the act together. We must not fail to register here our appreciation to Allied Publishers, especially Mr. Sharad Gupta, for their concern, persistence and endurance to see the project through in its first Indian instalment. We are grateful that we now have a chance to bring this book to an even wider audience through the medium of Springer Publishers.

We would like to record here the support and advice given to the organizers of the First Conference by several faculty members from various departments and authorities at IIT Bombay, Mumbai, as well as the Institute of Mathematical Sciences, Chennai and TIFR, Mumbai. Furthermore, generous financial support was received, for the Conference and this publication, from Infosys Technologies Ltd., the Department of Science and Technology, Government of India, the Indian Council of Philosophical Research, New Delhi and Microsoft, India. All this is hereby gratefully acknowledged.

Mumbai — *Amitabha Gupta*
New York — *Rohit Parikh*
Amsterdam — *Johan van Benthem*

Preface

Logic: A View from the Cross-roads

Logic stands at the cross-roads of many disciplines and cultures. The discovery that human reasoning and argumentation contains stable patterns that can be studied as such has been made independently in Antiquity in the Greek, Indian, and Chinese traditions, showing a remarkable unity to human thought. Ever since those days, logic has had a special relationship with disciplines where pure reasoning is of the essence, in particular, philosophy and mathematics.

Modern logic as we have today began with developments in the 19th century due to Frege, Boole, Peirce, Cantor and others concerned with logic as well as with the foundations of mathematics. They were followed by early 20th century figures like Russell, Wittgenstein, and Ramsey. A sense of crisis provoked by Russell's and other paradoxes eventually led to Hilbert's program and to deep technical results in the first half of the 20th century on provability, computability, truth, and definability found by Hilbert, Herbrand, Gödel, Gentzen, Skolem, Tarski, Turing, and others. Since that time, mathematical notions and techniques have pervaded the field, and this book contains several chapters devoted to the great themes from that mathematics-oriented tradition.

But modern logic is also a meeting ground of many further disciplines. The study of decidability and undecidability of mathematical questions led to the foundations of computing and eventually, to the birth of computers. Thus, nowadays, there is a wide family of interfaces between logic and computer science, ranging from the study of programming languages to the design of databases or intelligent multi-agent systems, and sometimes even straight into practical matters of information technology. This trend, too, is well-represented in various chapters of this volume, and 'computational logic' broadly conceived may indeed be the bulk of all logic research today.

Still, this is by no means the complete picture of current developments. Logic is being used extensively in the study of topics beyond narrower proof or computation, with a particular emphasis on the behaviour of rational agents which have knowledge, beliefs, preferences, and obligations, and need to adjust these in a world

that changes over time. All this has traditionally been the domain of "philosophical logic" at the interface of logic and philosophy, with ties to epistemology, philosophy of action, philosophy of science, and ethics. But over the last decades, this stream of research has started converging with computational logic, as computer systems become more and more sophisticated, alone or in networked societies, thus becoming more and more like us. This is why AI, in particular, has been a large scale "consumer" of logic.

Finally, logic in the last century has had extensive contacts with linguistics, since its account of meaning and definability provides intriguing interfaces with natural language in all its facets, from syntax to semantics and pragmatics. Frege, with his distinction between "sense" and "reference" already gave us the morning star – evening star problem central to both semantics and the philosophy of language. But more recent influences have been those of Lewis with his somewhat controversial notion of common knowledge, and Grice with his notion of implicature which has made pragmatics deeply embedded into semantics. Another important figure has been Saul Kripke whose semantics of modal logic and whose notion of rigid designator have played central roles in recent developments.

Right now, it seems fair to say that the discipline of logic also finds *itself* at a crossroads in the other sense, that of choosing its future directions. This book documents what are arguably the major ones.

First, the study of rational agency as a broader goal beyond computation and deduction naturally leads to the logical study of intelligent interaction and social behaviour. Let's not forget that logic probably arose in the first place as a study of legal, philosophical and political debate! An inspiring paradigm for this is "Social Software", the study of patterns of social interaction by means of techniques from logic and computer science. In this way, logic also joins forces with the social sciences such as game theory in economics, or social choice theory, or even the Law, as our most famous historical example of rational social engineering.

Next, even though logic has traditionally viewed itself as normative, and not as a description of actual behaviour, connections between logic and empirical psychology and cognitive science are on the rise, as more facts about actual behaviour and even about the human brain are becoming available for inspection. Logical theories suddenly find themselves at the forefront of theorizing about human reasoning skills. In a sense, this was already implicit in earlier studies of logic and natural language, where actual facts of meaning and inference in human practice have long served as a constraint on theory.

Finally, the future is sometimes opened up by paying proper attention to one's past. There is a rising interest in logic in other traditions than the Western one, including the independent origins in the Indian and Chinese traditions (a fact which by itself should suffice for shaking up some received cultural stereotypes), but also, its transformation in the golden age of early medieval Arabic culture. Some first explorations have already exposed fascinating vistas.

It was in this lively current setting that a group of Indian logicians decided to start a series of meetings in Mumbai devoted to the full width of logic today, and its modern and ancient roots in Indian cultural life. One purpose of these meetings was

to restore India to its rightful place in the logical world. While India has been one of the traditional homes of logic, in recent days, Indian contributions in logic have remained a small part of Indian contributions to science or even to mathematics and philosophy. One could say that the Indian genius has been inadequately harnessed in this important area.

As has already been pointed out that the current volume contains a representative selection of material from the first Conference, while the planned further volumes in this book series will document the follow-up.

Here is a brief guide to the chapters in this volume.

The first two chapters in Part I, entitled "Views of Logic Today," provide a grand setting to what is to follow. John Crossley gives a masterful survey of mathematical logic, which highlights the essential and far-ranging insights that have accumulated by now into proof, computation, and definability via formal systems. Rohit Parikh then follows up with an extended view of logic as a way of studying patterns and procedures in social agency, showing how the perspective of "social software" can systematize this field, while bringing out interesting aspects of different cultures.

Part II comprising a group of three papers on "Logic and Mathematics" demonstrates the continued vitality of mathematical logic and foundations of mathematics. John Crossley examines what is a mathematical proof, a question which continues to inspire new logical theory. Petr Hajek develops a rigorous treatment of fuzzy logic, once thought to be a "soft" subject, but by now fully respectable mathematically. Finally, Wilfrid Hodges takes us back to the formative years of model theory and recursion theory, and Tarski's seminal results joining expressive power and complexity in arithmetical settings.

As a counter-point to this grand tradition in logic, we have placed in Part III some papers from another grand tradition providing "Perspectives from Indian Logic". K. Ramasubramanian presents an overview that will help Western readers understand the rich tapestry of this field, while even teaching Indian readers a thing or two. Moreover, the same author adds a concrete sample, in the form of an exposition of the subtlety of a specific theme in Indian Logic, viz. the concept of *hetvabhasa* in the *Nyāya-sastra*. Finally, Sundar Sarukkai positions Indian logic in its connections with philosophical epistemology and the philosophy of science, showing how topics ran naturally into each other. This interface seems especially relevant in understanding logic/philosophy interfaces today.

The next groups of papers spread over three sections document some major streams in contemporary logic. We start with Part IV, viz., "Logic and Computation," an area which is richly represented in India today. John Crossley examines the entangled and complementary notions of "proof" and "program", and charts their marriage in current theories, including Combinatory Logic and Hoare Logic. Yuri Gurevich and Andreas Blass discuss the state of the art concerning Zero-One Laws, the mysterious statistical regularities underlying the expressive power of many logical systems that were first brought to light in the 1970s. Ron van der Meyden develops another strand, the penetration of systems from philosophical logic, such as epistemic and temporal logic, into the study of computational systems and network security. Daniele Mundici and and Ferdinando Cicalese then take up the topic

of coding theory, and its deep and surprising connections with many-valued logics which came originally from the Polish School in the early 20th century. Finally, Noson Yanofsky gives a masterful yet light-footed survey of the new paradigm of quantum computing, which both students and experts will find both informative and delightful.

It may be said with some exaggeration that modern computing is rational agency in societies of interacting agents, which pass information, correct themselves, and engage in purposeful strategic behaviour. Thus, connections between logic, philosophy of action, computer sciences, and social sciences come to the fore. The next part, i.e. Part V on "Logic, Agency, and Games" has some samples of this recent trend. Johan van Benthem presents an extensive survey of ways in which games have been used to model logical core activities such as model checking, model building, or argumentation, and then relates this interactive view of logic to standard notions from game theory such as strategies, equilibrium, and imperfect information. Eric Pacuit tackles one specific issue in social software, viz. the correctness and scope of the algorithm of "Adjusted Winner" in fair division, and shows how logical techniques provide general viewpoints and more reliable conclusions about what such procedures achieve. Next, Krister Segerberg discusses the more general phenomenon of belief revision in the light of dynamic logics of computation and action, and shows how this systematic stance can solve the problem of "iterated belief revision" which has plagued belief revision theory in AI and philosophy for a long time. Finally, G. Venkatesh develops systems of temporal preference logic which can describe players' goals and evaluations over time, as different parts of a game tree become available when successive moves are played. This points the way toward rich logics of games as a paradigm for intelligent interaction over time.

While social phenomena suggest some more careful attention to empirical facts than what has been the norm for logicians, game-theoretic analyses may still be normative flashes of brilliance, rather than the outcome of genuine painstaking observation of behaviour. Our final part, Part VI, is on "Logic, Language and Cognition". It moves closer to the realities of at least one major empirical phenomenon, the "sea of discourse" that we all live in, viz. Natural Language. D.B. Acharya and Shalini Joshi start by exploring the scope and limits of discrete mathematical models (most logical systems are) for the behavioral, cognitive, and social sciences. Then Wilfrid Hodges gives a masterful analysis of the Principle of Compositionality and the emergence of meaning from sentence understanding. This theme has been crucial to understanding the recursive learnability of language since, not just Frege and the modern semanticists, but even, as the author shows, all the way back into medieval Arabic Logic, another relevant historical and cultural tradition.

In their totality, these contributions offer a view of reasoning, computation, and rational agency in a wide modern sense, with logic acting as a common language, and indeed, an intellectual catalyst.

Amsterdam — *Johan van Benthem*
New York — *Rohit Parikh*
Mumbai — *Amitabha Gupta*

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