Judgement and the Epistemic Foundation of Logic

LOGIC, EPISTEMOLOGY, AND THE UNITY OF SCIENCE

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Logic, Epistemology, and the Unity of Science aims to reconsider the question of the unity of science in light of recent developments in logic. At present, no single logical, semantical or methodological framework dominates the philosophy of science. However, the editors of this series believe that formal techniques like, for example, independence friendly logic, dialogical logics, multimodal logics, game theoretic semantics and linear logics, have the potential to cast new light on basic issues in the discussion of the unity of science.

This series provides a venue where philosophers and logicians can apply specific technical insights to fundamental philosophical problems. While the series is open to a wide variety of perspectives, including the study and analysis of argumentation and the critical discussion of the relationship between logic and the philosophy of science, the aim is to provide an integrated picture of the scientific enterprise in all its diversity.

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Preface

In September 2009, the authors of this book came together in Leiden for a workshop called *Days of Judgement*. The majority of the chapters presented here are based upon a selection of the talks held at the Leiden workshop. Right from the start, the idea was to show the importance of the *history* of the notion of judgement for philosophy today. As one may learn from Wayne Martin's book *Theories of Judgment*, Cambridge University Press, 2006, the field of *judgement* is broad, and one needs to give a direction to the topic. The general idea of both the workshop and the book presented here is to take Per Martin-Löf's constructive type theory as a starting point, because the notion of judgement plays a central role there. Our logical system is not only in need of propositions; it also needs judgements in which propositions are asserted to be true and known. According to Martin-Löf, one is entitled to make a judgement if one has a ground for it. It is thus that the notion of judgement is related to the notions of truth, knowledge and ground. It is precisely the relation between these notions that has given a focus to the topic of the book presented here.

The book starts with two chapters that were not part of the workshop. In the first chapter, Martin-Löf gives a clear explanation of the way he understands the notion of judgement, and he relates his position to that of the logical positivists. The first part of the chapter is a reprint from the paper "Verificationism Then and Now", published in *The Foundational Debate* (W. DePauli-Schimanovich et al. (eds.). Dordrecht: Kluwer, 1995, 187–196). Martin-Löf has added to the paper a *postscript*, in which he makes an amendment to the paper. Göran Sundholm was asked to write an afterword to his paper "Constructions, Proofs and the Meaning of the Logical Constants", which appeared in 1983 in the *Journal of Philosophical Logic* (volume 11: 151–172). In this afterword, the second chapter here, Sundholm gives an overview of the history of constructive type theory of the last 30 years, focusing on the notions of construction, demonstration and judgement and the ambiguities in these notions. This afterword may help the reader to find the important literature that appeared on the notion of judgement within constructive type theory.

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Introduction

Maria van der Schaar

Assertion is a much discussed topic in philosophy today. Interest in the speech act of assertion seems to be new in philosophy. Assertion, though, is the linguistic counterpart to the notion of judgement, which has been a central notion in the history of philosophy and logic. As a first explanation of the notion of judgement, one may take Frege's understanding of it: acknowledging the truth of a thought or proposition. The aim of this book is (1) to give a historical introduction to the notion of judgement, in such a way that it becomes clear how the traditional theory of judgement relates to modern discussions on assertion, and (2) to understand how traditional theories of judgement can be used to give an epistemic foundation for logic. The aim of the introduction is to show how the notion of judgement could have disappeared from logic and how the notion can be brought back in order to give an epistemic foundation of logic.

Three moments in the history of logic have made it possible that the notion of judgement could disappear from modern logic. First, Bolzano in his *Wissenschaftslehre* (1837) accounts for the truth and falsity of judgements in terms of the truth and falsity of objective, Platonic propositions. The truth-bearer is no longer conceived as a product of an act of judgement, but as something that is independent of a judging and thinking mind. This is an important step away from term logic in the direction of propositional logic, but it comes with a price. For, how do we have epistemic access to these objective propositions that are supposed to be the contents of our judgements? Furthermore, Bolzano explains the validity of inferences in terms of relations of consequence between objective propositions. A non-epistemic foundation of logic is thus proposed, in which the act of judgement plays only a secondary role. Finally, Bolzano's explanation of validity and analyticity in terms of semantic variation has been understood as replacing the explanation of these notions in terms of the

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epistemic notion of containment, as we can still find it in Kant (see Sundholm's paper on analyticity). The knowing subject and the act of judgement no longer play a role in the explanation of fundamental logical notions. This is an important step in the direction of modern logic and the modern concept of analyticity as truth come what may, but again it has another side to it. For, we need both the purely formal and the epistemic notion in order to explain how an act of inference may bring us from known premises to new judgements known and to explain that an analytic truth may be known.

Second, Hilbert's idea that formal systems are objects of study for metamathematical research and his idea that meanings can arbitrarily be given to formal systems and their axioms have had a great influence on the model-theoretical tradition. On Hilbert's account, as soon as an axiom system is consistent, it specifies a class of models. For Hilbert, the question whether the axioms are judgements made or whether they are true does not arise. In this sense, his idea of axiom radically differs from Frege's. Whereas for Hilbert, the proof of the consistency of a set of axioms is essential, for Frege, the consistency of the axioms follows from the fact that these are true: a consistency proof is thus not required. In answer to Brouwer, Hilbert developed a finitary point of view of mathematics in the 1920s. There is an epistemic privileged part of mathematics, which relies only on a purely intuitive basis of signs. But, Gödel's incompleteness theorems showed that a finitary consistency proof of Hilbert's system cannot be given. Although the discussion whether mathematics can be given an epistemic ground on Hilbertian terms has not ended, one can safely conclude that the model-theoretical tradition that originated with Hilbert excludes epistemology from logic. The modern notion of axiom as a non-epistemic starting point of a system has replaced the traditional concept of axiom as judgement made whose truth can neither be proved, nor is in need of proof, because its truth is understood upon apprehension of the concepts involved. And the idea of a formal structure that can be given different interpretations, enables one to speak about different models in which a certain sentence or proposition is true or false. The notion of truth in a model is completely unrelated to the notions of judgement and knowledge and is therefore not the kind of truth Frege was speaking of. Hilbert's metamathematical approach also gave rise to the idea that language is not a universal medium, as Frege held, but is rather a calculus, to which an endless variety of interpretations can be given.

Third, the logical positivists were strongly influenced by Hilbert's early conception of mathematics. In the *Logical Syntax of Language* (1934), Carnap conceives of the language of logic as a calculus (§ 2, § 46): a symbol or expression in logic does not have any meaning, and the notion of judgement that still played a central role in Frege's *Begriffsschrift* is now replaced by that of a sentence without meaning.[1]

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[1] "But the development of logic during the past 10 years has shown clearly that it can only be studied with any degree of accuracy when it is based, not on judgments (thoughts, or the content of thoughts) but rather on linguistic expressions, of which sentences are the most important, because only for them is it possible to lay down sharply defined rules" (Carnap 1937, 1).

The idea of the priority of syntax, the Hilbertian concept of axioms and Carnap's aim to solve the conflict with respect to the foundations of mathematics gave rise to Carnap's famous principle of tolerance, relating to mathematics and logic: "let any postulates and any rules of inference be chosen arbitrarily; then this choice, whatever it may be, will determine what meaning is to be assigned to the fundamental logical symbols." (Carnap 1937, xv). The criterion what mathematics or logic one will use in a certain situation is for Carnap a pragmatic one; for, justification can be given only within a system. "The first attempts to cast the ship of logic off from the *terra firma* of the classical forms were certainly bold ones, considered from the historical point of view. But they were hampered by the striving after 'correctness'. Now, however, that impediment has been overcome, and before us lies the boundless ocean of unlimited possibilities." (Carnap 1937, xv).

Furthermore, early logical empiricists, such as Moritz Schlick, were critical of the Kantian use of pure intuition in mathematics and of the notion of immediate evidence that is traditionally conceived as the characterizing mark of foundational truths. The general idea of the logical positivists that no epistemic foundation of logic can be given, made it possible that logic and epistemology became two separated fields. The basic notion in logic became either the sentence or the objective proposition, rather than the notion of judgement, because the traditional notion of judgement was thought to be infected with psychological elements. In the logical empiricist's conception of formal system, there is no place for the notion of judgement, that is, for the acknowledgement of the truth of a proposition or sentence.

As a result of these three moments in the history of logic, logic is now considered as either having no foundation at all, or as founded on a Platonic realm of propositions. In both cases, the notion of judgement and the epistemic agent have disappeared from logic. On such an account, a logical system cannot be used to provide proofs for a judging agent, either because the system concerns nothing but relations between abstract notions in a Platonic realm, whose accessibility by a judging agent is not understood, or because there is only an arbitrary relation between the formulas and the interpretation given. An important question that arises for logic today is therefore how to relate logic to epistemology. One way in which logic and epistemology can be related is by giving the notion of assertion or judgement a proper place in logic: axioms and theorems are judgements and assertions made.

Recent discussions of the notion of assertion show two different ways in which the notion of assertion can be explained. The condition under which one is entitled to make an assertion may either be understood in epistemic or in non-epistemic terms. With respect to judgement, one may defend either an epistemic account of judgement, or a non-epistemic account. On a non-epistemic account, one is entitled to judge, precisely if the thought contained is true, where truth is explained in a non-epistemic way. On an epistemic account of judgement, one is, for example, entitled to judge precisely if one has a ground or reason for one's judgement. It is precisely such an epistemic understanding of the notion of judgement that is needed when we rethink the relation between logic and epistemology. What may count as a ground for a judgement is determined by the judgemental content, or, if one prefers, by the meaning of the sentence by means of which the judgement is made manifest.

There is thus an internal relation between a judgement and its possible ground. In order to understand this relation between judgement and ground, this volume will investigate the rationalist tradition up to Kant, as we will see below.

Such an epistemic notion of judgement can already be found in Brouwer and his pupil Arend Heyting. In his paper on the reliability of logical principles from 1908, Brouwer explains that the law of excluded middle demands that a thesis is either correct or incorrect. For mathematics, this amounts to the thesis that we can either give a construction such that we are entitled to affirm the thesis or we can show that the thesis leads to absurdity, which means that we are entitled to the denial of the thesis. Such a strict epistemic demand on affirmation and denial shows that the law of excluded middle does not hold for mathematics insofar as it is concerned with infinite totalities. For Brouwer, the question of the validity of the law of excluded middle is directly related to Hilbert's thesis that there exist no unsolvable problems in mathematics. "It follows that the question of the validity of the *principium tertii exclusi* is equivalent to the question whether unsolvable mathematical problems can exist. There is not a shred of proof for the conviction, which has sometimes been put forward, that there exist no unsolvable mathematical problems" (Brouwer 1908, 156; 1975, 109). Brouwer should have been sensitive here, as he was elsewhere, to the distinction between the thesis that every mathematical problem is solvable, which is equivalent to the law of excluded middle, and the weaker claim that there are no unsolvable mathematical problems, which is correct on an intuitionistic account.[2] Per Martin-Löf will take up this topic in the first paper of this volume, where it is discussed in relation to Schlick's thesis that there are no unanswerable questions.

In a paper by Heyting from 1930, we find an explanation of the intuitionistic notion of the meaning of a sentence, and of the notion of assertion: "A proposition [declarative sentence] p like, for example, 'Euler's constant is rational', expresses a problem, or better yet, a certain expectation (that of finding two integers a and b such that $C = a/b$), which can be fulfilled [réalisée] or disappointed [déçue]" (Heyting 1930, 307). Heyting's explanation of the intuitionistic notion of assertion or judgement is clearly epistemic: "To satisfy the intuitionistic demands, the assertion must be the observation of an empirical fact, that is, of the realization of the expectation expressed by the proposition p. Here, then, is the Brouwerian assertion of p: *It is known how to prove p* [by construction]" (idem). Martin-Löf's explanation of judgement, presented in the paper below, essentially captures this notion of assertion and judgement.

We are in need of a logic that reconsiders its relation to epistemology by means of an epistemic notion of judgement. Per Martin-Löf has developed a new conception of logic, in which the concepts of knowledge and judgement are brought in right from the start. For him, logic is a demonstrative science, in which the epistemic act

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[2] In the Brouwer archive, one may find a note from the writings for the 1907 dissertation: "Can one ever demonstrate of a question, that it can never be decided? No, because one would have to do so by *reductio ad absurdum*. So one would have to say: assume that the proposition has been decided in the sentence a, and from that deduce a contradiction. But then it would have been proved that not a is true, and the question *is* decided" (Van Dalen 2001, 174 note a, my translation).

of judgement plays a fundamental role. Acts of inference, which are acts of judgement based upon already known judgements, result in knowledge of theorems. And the axioms result from acts of immediate insight, which are also acts of judgement. Thus, going back to the conception of axioms before Hilbert, and Carnap's *Logical Syntax of Language*.

The papers can be understood as presenting a historical context for the notions that play a central role in Martin-Löf's logic, such as the notion of judgement, judgemental content, analyticity, the *a priori*, sufficient ground and the assertion sign. The epistemic conception of logic in constructive type theory goes back to Husserl's phenomenology, on the one hand, and to Frege's idea of logic, on the other hand. Husserl's elucidation of a cognitive act in terms of intention and fulfilment, in the sixth of the *Logical Investigations*, has been of importance for Heyting's explanation of a proposition as an expectation that can be fulfilled or disappointed and for the epistemic notion of judgement that he gives. In Frege's *Begriffsschrift*, the judgement stroke is not only a sign that the judgemental content is judged to be true, it is also a sign that the content is true and known. For Frege, axioms are pieces of knowledge and the logical truths that can be derived from them give the most general kind of knowledge of the world. All axioms and theorems in the *Begriffsschrift* are thus preceded by the judgement stroke. Logic, for Frege, is not a mere calculus; it is a language with meaning, and the most universal science there is. In Frege's writings, the notion of judgement has a fundamental role in logic without making logic a psychological enterprise, although a subjective or personal element does play a role in the *Begriffsschrift*. Because the judgemental stroke precedes only what Frege has shown to be true and known, the axioms and theorems in the *Begriffsschrift* are the judgements made by Frege. One can also make the point in a less subjective way. The judgements made in the ideal *Begriffsschrift* are those made by an ideal judger. The judgement stroke plays an important role in the early *Begriffsschrift*, but seems to lose its importance on the more Platonic conception of logic that we find in Frege's later writings. Nevertheless, judgement plays a role in Frege's later conception of logic insofar as he asserts that the assertive force contains the clearest indication of the essence of logic.[3]

No doubt we cannot simply go back to Frege's idea of logic because his logicist project is inconsistent and because his later theory of judgement presupposes a Platonism with respect to judgemental contents. We are thus in need of a new conception of judgement that is to play a role in a theory of inference. Such a notion can also be used in mathematics and in science in general. For, science does not consist of sentences or abstract propositions but of assertions and judgements made.

Knowledge of Kant's theory of judgement and of the Kantian tradition makes it possible to understand how the notions of judgement and judging agent can be used in logic and philosophy in general without reducing logic and philosophy to psychology.

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[3] "Now the thing that indicates most clearly the essence of logic is the assertoric force with which a sentence [the German has 'ein Gedanke'] is uttered [the German has 'ausgesprochen', which can also be translated as 'expressed']" (Frege 1915, 252).

Ground or reason is a central notion in the Kantian account of judgement, and it is thus that the rationalist tradition and its notion of sufficient ground become relevant for the notion of judgement. The notion of judgement that needs to be brought back to logic stands in a long tradition, in which the notion of judgement is essentially related to the notion of ground or reason, and the rationalist tradition may therefore play an inspiring role in bringing back the notion of judgement to logic.

In Part I, the stage is set by Per Martin-Löf and Göran Sundholm. Martin-Löf's paper on the verification principle explains what a judgement is from a constructivist point of view. The meaning of a judgement is fixed by laying down what it is that you must know in order to have the right to make the judgement in question. Starting with one of the basic judgemental forms *A is true*, where *A* is a proposition, we can say that *A* is true if there exists a verification of *A*, that is, if a proof of *A* has been constructed. We thus have obtained a verification principle of truth. We can see now in what sense both the idea of a judging agent and that of an objective reason or ground play a central role in Martin-Löf's theory. What one has to know in order to be entitled to make the judgement is a ground for the judgement. On the one hand, what counts as a ground is given by the explanation of the judgement in question, or, if one prefers, it is given by the meaning of the sentence that one uses to make the judgement. The ground for the judgement, and thereby the judgement itself, is in this sense objective. On the other hand, the ground has to be known to the person who makes the judgement. To put the point in terms of proofs and propositions, the judging agent needs to construct a proof for the relevant proposition, or to understand that something counts as a proof for the proposition, in order to be entitled to judge that the proposition *A* is true. In this sense, the notion of judging agent cannot be neglected. Because there is thus a strict conception of judgement and assertion, the law of excluded middle in its positive formulation does not hold. Instead, a negative formulation of the law can be defended. The constructivist's thesis that there are no propositions of which neither the truth nor the falsity can be known leaves open the possibility that there are many propositions which we do not know how to decide whether they are true or false. To put it in terms of the solvability of problems: although we can say that there is no proposition that we know to be undecidable, we are not allowed to assert that every proposition is decidable. We are thus not entitled to assert that every question can in principle be answered, and it is in this sense that the constructivist's position differs from that of a logical positivist such as Schlick, who says: "Whenever there is a meaningful problem, one can, in theory, always show the way that leads to its solution."[4]

Martin-Löf also gives a constructivist interpretation for the logical positivist's thesis that the meaning of a proposition is the method of its verification, that is, of the verification principle of meaning. For a constructivist, the meaning of a proposition is its method*s* of verification. And a method of verification can be understood as a proof for a proposition. The constructivist explanation of a proposition in terms of

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[4] "Wo immer ein sinnvolles Problem vorliegt, kann man theoretisch stets auch den Weg angeben, der zu seiner Auflösung führt" (Schlick 1930, 7).

proofs thus expresses a verification principle of meaning. Martin-Löf's paper is a reprint from 1995, and an amendment is added for this occasion as postscript.

Sundholm's afterword to the paper on the explanation of the logical constants in terms of constructions and proofs gives a history of constructive type theory since the paper appeared in 1983.[5] The afterword may serve as a guideline to the central notions in constructive type theory, and indicates where the reader can find the relevant literature. The paper from 1983 gives an analysis of Arend Heyting's distinction between assertion (*Behauptung*) and proposition (*Aussage*). Whereas a proposition expresses a certain expectation or intention, the assertion of the proposition signifies the fulfilment of the intention by a certain construction. Heyting is thus already making the distinction between judgement and proposition, which is so essential to an epistemic conception of logic. Not unimportant for a wider understanding of the constructivist project, Heyting uses the distinction between intention and fulfilment that Husserl introduced in his elucidation of the cognitive act in the sixth *Logical Investigation*.

Sundholm's paper on analyticity shows how the concept of analyticity is fundamental to logic and that the two ways in which analyticity may be explained – as epistemic containment and as variation – have determined two developments in logic from Aristotle on. This means that right from the start, one can find both an epistemic and a purely formal conception of logic and that the former account needs a place besides the latter.

Part II deals with Descartes and Spinoza. In the seventeenth century, the epistemic account of logic and judgement is developed within a rationalist conception of knowledge and truth. In Descartes, judgement in accordance with certain epistemic rules is the key to science (as we see in the paper by Elodie Cassan). And the principle of sufficient reason plays an important role in the development of a rationalist conception of judgement in Spinoza (as Michael Della Rocca shows). Spinoza's idea that there is an essential connection between reason and judgement has had a great influence on Leibniz, and thereby on philosophers and logicians that were influenced by Leibniz.

In Part III, Wolff is the mediating figure between the rationalist and the Kantian approach to judgement. For Wolff, a logical analysis of judgements into condition and statement is central to his account of judgement. The condition of the judgement is to be understood as the (sufficient) ground for its truth. Such a ground of the judgement provides a demonstration for the judgement in question and thereby a possibility that it can be known. The younger Kant broadens this notion of condition in such a way that it does not consist in a sufficient reason for the judgement but in the epistemic source of sensibility and understanding of the concepts united in the judgement. Kant is thereby able to use the logical analysis of judgement for his criticism of rationalist metaphysics, while changing the rationalist and metaphysical notion of sufficient ground into an epistemic and logical notion (see the paper by Johan Blok).

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[5] The reader is kindly invited to read the paper 'Constructions, Proofs and the Meaning of the Logical Constants', *Journal of Philosophical Logic* (volume 11, 1983: 151–172).

In the nineteenth century, we see two developments – on the one hand, a line developing Kant's conception of judgement. The neo-Kantian Windelband transforms the Kantian thesis that judgement is at the same time a logical and an epistemic notion by claiming that judgement is an epistemic assessment ('Beurteilung'; see the paper by Arnaud Dewalque) and thereby not a value-free process. Judgement is understood as assessing the truth-value of a propositional content. On the other hand, the rationalist metaphysics of Spinoza, Leibniz and Wolff is transformed by Bolzano in a rather anti-Kantian way. Whereas Kant explains the epistemic distinction between *a priori* and *a posteriori* judgements in epistemic terms, Bolzano explains this distinction in terms of non-epistemic, semantic properties of propositions (see the paper by Stefan Roski). In a similar way, Bolzano explains logical validity in terms of the possibility of semantic variation in objective propositions, thereby making the judging subject irrelevant to the objectivity of logic. It is the tension between these two lines of thinking about the notion of judgement, the Kantian and the Bolzanian one, which determines the account of judgement and logic given by Husserl, Frege and Russell.

In Part IV, we see the influence of Bolzano, Lotze and the neo-Kantians on Husserl, Frege and Russell. They give an account of the relation between the act of judgement, the propositional content and the object of judgement, in such a way that an objective foundation of logic can be given without neglecting the judging and knowing subject (for Husserl, this is shown in the paper by Robin Rollinger). Jeremy Kelly's account of judgemental force in Frege and early Russell shows that this notion can be interpreted in non-psychological, logical terms. The syntactical difference between the finite form of the verb and the participial form, so essential to the distinction between asserted and unasserted propositions in Russell's *Principles of Mathematics*, is thereby given a logical interpretation, thus allowing for the objectivity of logic without losing sight of the idea of judgemental force.

We thus see that one does not have to go back far in the history of logic to understand how the notion of judgement can play a role in logic. By going farther back into the rationalist and Kantian tradition, it is also possible to understand from which broader backgrounds these theories have emerged, while both traditions on their own are still of value for the development of a notion of judgement and assertion in which the notion of ground or reason plays a central role.

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