Subdivision Surface Modeling Technology

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Foreword

Subdivision surface is a popular modeling technique in the field of computer-aided design (CAD) and computer graphics (CG) for its strong modeling capabilities for meshes of any topology. This book makes a comprehensive introduction to subdivision modeling technologies, the focus of which lies in not only fundamental theories but also practical applications. In theory aspect, this book seeks to make readers understand the contacts between spline surfaces and subdivision surfaces and makes the readers master the analysis techniques of subdivision surfaces. In application aspect, it introduces some typical modeling techniques, such as interpolation, fitting, fairing, intersection, trimming and interactive edit. By studying this book, readers can grasp the main technologies of subdivision surface modeling and use them in software development. This knowledge also benefits understanding of CAD/CG software operations.

Due to flexible topology adaptivity and strong modeling capability, subdivision surface modeling technology has developed quickly in the field of CAD, CG, and geometric modeling since its appearance during the 1970s. Many famous 3D modeling software, such as 3DMax, Maya, and Meshlab, has involved subdivision surface as a modeling tool. Subdivision modeling technology has been successfully applied in making characters of games and special effects of movies. As the saying goes, “Give a man a fish; you have fed him for today. Teach a man to fish; and you have fed him for a lifetime.” On the one hand, the book has done a detailed exposition to the basic theory of subdivision surfaces and strives to make readers to achieve the mastery. On the other hand, although the contents of the book are limited, we make a remarks about the main topic at the end of each chapter and list the closely related references for readers to self-improve. We believe that by learning through this book, readers will have a capability of researching and developing with subdivision surfaces independently.

The book was planned by Prof. Wenhe Liao, and most materials came from doctorial dissertations supervised by him. Associate Professor Hao Liu complied this book and wrote Chaps. 1–6 and Sects. 10.1 and 10.2. Dr. Tao Li arranged the rest of the book. He wrote Chap. 8, and Sects. 7.3, 10.3 and revised Sects. 7.1, 7.2 and Chaps. 9, 11. Chapter 9 was taken from Gang He’s doctorial dissertation, and

Dr. He revised the English manuscript of this chapter. Sections 7.1 and 7.2 and Chap. 11 came from Xiangyu Zhang's doctorial dissertation, and Dr. Zhang revised the corresponding English manuscript. Graduate Wei Fan made a lot of work for the final proof. The authors thank Dr. He, Dr. Zhang and Graduate Wei Fan for their contributions to this book.

This book is suitable for graduate students, teachers, and technical personals majoring in CAGD, CAD/CG, and other related fields as a reference book on surface modeling. Due to the limitation of our knowledge, there are inevitably some drawbacks in this book. If any flaw found, we are grateful for your contact with us (liuhao-01@nuaa.edu.cn).

Table of Symbols

Notation

Our general approach to notation is to accord to traditional convention meanwhile precisely to express our intentions. Consequently, we try to use traditional notation as far as possible. At the same time, some special notation is introduced; for example, $\boldsymbol{M}[i,j]$ denotes the entry of a matrix $\boldsymbol{M}$ in the ith row and in the jth row. The highlights of this notation are the following:

- Function application is denoted using parentheses (), for example $\boldsymbol{p}(u)$, $\boldsymbol{p}(u,v)$; a combinational number is also be denoted using parentheses (), for example $\begin{pmatrix} 5 \\ 3 \end{pmatrix} = \frac{5!}{3!(5-3)!}$. Diploid or triple is also denoted using parentheses (), for example $K = (V, E, F)$.
- Vectors and matrices are created by enclosing their members in square brackets, for example $\boldsymbol{U} = [u_0.u_1, u_2, \cdots]$, $\boldsymbol{M} = \begin{bmatrix} v & e & f \\ 0 & 0 & e \\ 0 & e & 0 \end{bmatrix}$. These members are scalars. Conversely, the ith entry of the vector $\boldsymbol{U}$ is denoted by $\boldsymbol{U}[i]$. The entry in the ith row and in the jth row is denoted by $\boldsymbol{M}[i,j]$. If members of a vector are also vectors, we especially denote the vector as $\overrightarrow{\bullet}$. For example $\overrightarrow{\boldsymbol{E}} = \begin{bmatrix} \boldsymbol{e}_0 \\ \boldsymbol{e}_1 \\ \vdots \\ \boldsymbol{e}_{n-1} \end{bmatrix}$. When a vector denotes a coordinate of a point, we also directly use name of components. For example for $\boldsymbol{R} = [x,y,z]$, $\boldsymbol{R}[x]$ denote the x component. We also use a vector to denote a form of a Lave tiling, for example [4,6,12] Lave tiling.
- Sets are created by enclosing their members in curly brackets $\{\}$. We arrange that indices of members of a vector, matrix, or a set start from 0.

- The expression $\frac{\partial^s \partial^t \boldsymbol{p}(\boldsymbol{x},\boldsymbol{y})}{\partial^s \boldsymbol{x}[i] \partial^t \boldsymbol{y}[j]}$ denotes the sth partial derivative with respect to $\boldsymbol{x}[i]$ and the tth partial derivative with respect to $\boldsymbol{y}[i]$ of the function $\boldsymbol{p}(\boldsymbol{x},\boldsymbol{y})$. For convenience, we also use $f_u(u,v)$ to denote the partial derivative of $f(u,v)$ with respect to u; $f'(u)$ denote derivative of the function $f(u)$ with respect to the variable u.

We also follow several important stylistic rules when assigning variable names. We assign any variable name to be denoted by italics, for example $\boldsymbol{U}$, $\boldsymbol{M}$, a, b, λ. Bold italics denote vectors or matrices, while italics denote scalar variables or names of geometric shapes. Often, a same letter probably has both a bold italic version and an italic version denoting different meanings. For example, M denotes a mesh, while $\boldsymbol{M}$ denotes a matrix.

Roman

Notation of points and vertices does most probably cause confusions. The highlights of these familiar letter notations are the following:

- $\boldsymbol{p}$, $\boldsymbol{q}$, point on continuous curves or surfaces.
- $\boldsymbol{V}$, $\boldsymbol{E}$, $\boldsymbol{F}$, vertex of polygon or mesh or grid. Note that $\boldsymbol{V}$ also denotes a knot vector for spline surfaces;
- $\boldsymbol{v}$, $\boldsymbol{e}$, $\boldsymbol{f}$, vertex of subpolygon or submesh.
- $\vec{\boldsymbol{V}}$, $\vec{\boldsymbol{E}}$, $\vec{\boldsymbol{F}}$, $\vec{\boldsymbol{e}}$, $\vec{\boldsymbol{f}}$, vector formed by vertices.
- $\tilde{\boldsymbol{v}}$, $\tilde{\boldsymbol{e}}$, $\tilde{\boldsymbol{f}}$, vertex after Fourier transformation for $\boldsymbol{v}$, $\boldsymbol{e}$, $\boldsymbol{f}$.

For other some familiar notations, we give their meanings as the following:

- i, j, integer indices
- u, v, continuous parameter variables
- $\boldsymbol{U} = [u_0, u_1, \ldots]$, $\boldsymbol{V} = [v_0, v_1, \ldots]$, knot vector for spline curves or knot vectors for spline surfaces.
- r, s, t, k, l, temporary variables; k usually used as level of subdivision; degree of polynomial; degree of continuity; size of a generation vector. r usually used as multiplicity of a knot u_i in a knot vector $\boldsymbol{U}$ or variable for integer index in a sequence;
- $\boldsymbol{s}(u,v)$ or $\boldsymbol{s}(s,t)$, a part of a subdivision surface
- C^k, G^k, k degree derivative continuity and k degree geometric continuity
- m, n, size of grid, mesh, polygon, matrix, or vector; n also denotes valence of vertex.
- M, M^k, mesh and mesh on kth level of subdivision;
- $N_{i,k}(u)$, $N_i(u)$, B-spline basic function
- d_i, e_i, parameters of vertices or edges in polygons or meshes
- $f()$, $g()$, $p()$, $q()$, $h()$, scalar functions
- d, differential operator
- T, subdivision operator

- x, y, z continuous domain variables. Usually denote coordinates of points or vertices
- $\boldsymbol{s}_i$, $\boldsymbol{t}_i$. generation vector for a grid. It is a unit vector
- D^k, generation vector group formed by generation vectors. It is a set.
- G_D^k, grid formed the vector group D^k
- $\boldsymbol{i} = (i_0,i_1)$, or $\boldsymbol{i} = (i_0,i_1,i_2)$, integer coordinates in a grid
- $\boldsymbol{x}$, $\boldsymbol{y}$, real coordinates in a grid
- $N_{D^k}(\boldsymbol{x})$, box spline basic function
- $C(z)$, generating function for $N_D(\boldsymbol{x})$:
- X,$\overset{s}{X}$, $\overset{s+}{X}$, $\overline{\overset{s+}{X}}$, parameter mesh of a manifold patch and sth side of X, extension of $\overset{s}{X}$, rectangular mesh mapped from $\overset{s+}{X}$.
- $c_{i,j}^s$, a chart in the parameter mesh $\overset{s}{X}$,
- w, weight for vertex in a mesh
- T_C, the non-uniform subdivision operator
- T, the skirt-removed operator
- T_{RC}, non-uniform skirt-removed scheme
- T_{CT}, non-uniform subdivision operator constructing subdivision surface interpolating mesh corner vertices
- $C(\bullet)$ denotes the operator taking the continuity degree
- $\boldsymbol{E}$, energy of curve or surface
- $\boldsymbol{M}$, subdivision matrix
- $\boldsymbol{K}$, picking matrices
- S, a usual name for a surface
- $\boldsymbol{S}(\cdot,\cdot)$, $\boldsymbol{S}(\cdot)$ or $\boldsymbol{S}$, a surface equation or any a point on a surface or a mapping from parameter region to a space or a set formed by all points in the surface S.

Greek Letters

α, β, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, coefficients or vector formed by coefficients
γ, aperture factor
φ, ϕ, functions or mappings
$\mu(y)$, a mapping constructing the basic curve in parameter regions
κ slope of line
Θ^s A set formed by all related vertices of $\boldsymbol{x}$ in X^s
ε, temporary variables, usually denote very little real number
ζ, a given vector function that represent imposed loads
ξ, eigenvector
Ξ denotes a matrix formed by eigenvectors
Σ, plane
Ω, parameter region

Miscellaneous

D
Z, Z^2, $Z^2/2$, set of integers, set of integer pairs. Set of integer pairs divided by 2
R, R^2, real number space, and two-dimension real number space
$\otimes$, convolution operator of two functions or Kronecker product of matrices
k, curvature

Functions

$supp(D^k)$, support region of a vector group D^k
$O(supp(D^k))$ or $O(n^k)$, an open set which is the inner region of $supp(D^k)$ or a polynomial of n^k
$span(D^k)$, space spanned by D^k
$U(x, \varepsilon)$ a ε neighborhood *of* $\boldsymbol{p}$
$edges(M)$ set of a mesh M
$max\{\bullet\}$, the maximum element of a set denoted by $\bullet$
$min\{\bullet\}$, the minimum element of a set denoted by $\bullet$
$|\bullet|$, the valence of a vertex or element number of a set or a vector
$a\%b$, the remainder after a divided by b

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