

Beyond Planar Graphs

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Editors

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Communications of NII Shonan Meetings

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Preface

Most real-world data sets are relational, which can be modeled as graphs, consisting of vertices and edges. Planar graphs are fundamental for both Graph Theory and Graph Algorithms, and extensively studied: structural properties and fundamental algorithms for planar graphs have been discovered. However, most real-world graphs, such as social networks and biological networks, are *non-planar*. To analyze and visualize such real-world networks, we need to solve fundamental mathematical and algorithmic research questions on sparse non-planar graphs, called *beyond planar graphs*.

Recently, research topics in topological graph theory generalize the notion of planarity to beyond-planar graphs, i.e., non-planar graphs with topological constraints such as specific types of crossings, or with forbidden crossing patterns. Examples include:

- *k-planar graphs*, which can be embedded with at most k crossings per edge;
- *k-quasi-planar graphs*, which can be embedded without k mutually crossing edges.

Consequently, combinatorics (such as edge density), algorithmics (such as testing/embedding algorithms), and geometric representations (such as straight-line drawings) of beyond-planar graphs have emerged as new research directions.

The NII (National Institute of Informatics) Shonan Meeting No-089 *Algorithmics on Beyond Planar Graphs* was held on November 27–December 1, 2016 in Shonan, Japan, to bring world-renowned researchers on Graph Algorithm, Graph Drawing, Computational Geometry, Graph Theory, and Combinatorial Optimization.

The main aim of the workshop was to identify research opportunities on Beyond Planar Graphs and collaboratively develop innovative theory and algorithms for sparse non-planar topological graphs with specific applications to large and complex network visualization.

The workshop had 26 participants from 7 countries, and consisted of 7 invited talks, open problem sessions, discussion sessions, and report sessions from each working group. Outcomes of the workshop include the Shonan Meeting Report,

research articles as well as invited contributions to the book chapters from the participants.

This book contains a selection of book chapters initiated from the Shonan Workshop No-089 on Beyond Planar Graphs. More specifically, it consists of 13 chapters that represent recent advances in various areas of beyond planar graph research. Each book chapter was peer-reviewed according to the book standards.

The main aims and objectives of this book include:

- timely provide the state-of-the-art survey and a bibliography on beyond planar graphs;
- set the research agenda on beyond planar graphs by identifying fundamental research questions and new research directions;
- foster cross-disciplinary research collaboration between Computer Science (Graph Drawing and Computational Geometry) and Mathematics (Graph Theory and Combinatorics).

This book is the first general and extensive review of the algorithmic and mathematical results of beyond planar graphs. New algorithms for beyond planar graphs will be in high demand by practitioners in various application domains to solve complex visualization problems. As such, this book will be a valuable resource for researchers in Graph theory, Algorithms, and Theoretical Computer Science, and will stimulate further deep scientific investigations into many areas of beyond planar graphs.

We wish to thank all the authors for contributing their chapters to this book. We also thank all the participants of the Shonan Workshop No-089 for their valuable contribution and participation during the workshop, which greatly helped to improve many aspects of the chapters published in this book.

Finally, we would like to thank NII for the opportunity to organize a successful meeting to enable these exciting initiatives, and Springer for the opportunity to edit this book, with dedicated assistance and support to make this book possible.

Sydney, Australia
Sanda, Japan
March, 2020

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