Approximation of Euclidean Metric by Digital Distances

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To my father Shree Dulal Krishna Mukherjee
with deep gratitude, love, and respect.

Preface

In this monograph, different types of distance functions in an n-D integral space are discussed to consider their usefulness in approximating Euclidean metric. It discusses the properties of these distance functions and presents various approaches to error analysis in approximating Euclidean metrics. The main emphasis of this book is to present the mathematical treatises for performing error analysis of a digital metric with reference to the Euclidean metric in an integral coordinate space of arbitrary dimension. I hope that the monograph will be useful to researchers and postgraduate students in areas of digital geometry, pattern recognition, and image processing. The theory and results on the properties of different distance functions presented may have applications in various pattern recognition techniques. Analytical approaches discussed in the book would be useful in solving related problems in digital and distance geometry. As a prerequisite, the author expects that the readers have gone through first-level courses on vector algebra, coordinate geometry, and functional analysis.

There are six chapters in this book. In Chap. 1, the mathematical background of metrics, norms, distance functions, and spaces is presented with a brief discussion on the motivation behind the efforts in approximating Euclidean metrics by digital distances. Chapter 2 discusses digital distances, their classes, and hierarchies. In recent works, it has been shown that generalization of a family of distance functions is possible, and many of the results derived previously can be shown as special cases of the properties of the general class of distance functions. Chapter 3 presents analytical approaches for the analysis of errors of approximating Euclidean metrics by digital metrics. Chapter 4 considers the same with geometric approaches. In this regard, the properties of hyperspheres are also discussed. In Chap. 5 linear combinations of digital metrics for approximating Euclidean metrics are considered. Finally, in the concluding chapter, a few good digital distances in different dimensions of integral coordinate spaces are summarized. The chapter also concludes by highlighting a few open problems in this regard.

About four years back, I ventured into writing a monograph on this topic. But due to personal reasons, I could not proceed and a sort of inertia gripped me. I am thankful to the publisher, who showed interest in reviving the project and motivated me to complete it. I take this opportunity to express my gratitude to my supervisor Prof. B. N. Chatterji, who had provided immense support in my early research days, and encouraged me on exploring my independent research thoughts, even though they appeared simple and naive. My friend Prof. P. P. Das, who has contributed significantly to the development of the theory of digital distances, introduced me to this area of research. I always wondered how he could obtain all those amazing equations and expressions of digital metrics and their properties. It was always a pleasure to work with him and learn from him. My son Rudrabha is always curious about what his father is doing on the computer. He himself has become quite busy with his own research work. Even then, whenever needed, he provided all sorts of assistance in resolving technical glitches in composing this manuscript during this period. My wife Jhuma has to bear with my long hours of engagement before the computer terminals. As a doctor, she has all the worries and concerns about my health. I am fortunate to receive her love and care. I lost my mother five years back. I am quite unfortunate to miss her blessings on this occasion. From my early years, my parents instilled in me the dream of pursuing academic excellence. With deep gratitude, love, and respect, I dedicate this book to my father.

Kharagpur, India Jayanta Mukhopadhyay
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About the Author

Dr. Jayanta Mukhopadhyay (Mukherjee) received his B.Tech., M.Tech., and Ph.D. degrees in Electronics and Electrical Communication Engineering from the Indian Institute of Technology (IIT), Kharagpur, in 1985, 1987, and 1990, respectively. He joined the faculty of the Department of Electronics and Electrical Communication Engineering at IIT, Kharagpur, in 1990, and later moved to the Department of Computer Science and Engineering where he is presently a Professor. He served as the Head of the Computer and Informatics Center at IIT, Kharagpur, from September, 2004 to July, 2007. He also served as the Head of the Department of Computer Science and Engineering and the School of Information and Technology from April, 2010 to March, 2013.

He was a Humboldt Research Fellow at the Technical University of Munich, in Germany, for one year in 2002. He has also held short-term visiting positions at the University of California, Santa Barbara, University of Southern California, and the National University of Singapore. His research interests are in image processing, pattern recognition, computer graphics, multimedia systems, and medical informatics. He has supervised 25 doctoral students and published more than 300 research papers in journals and conference proceedings in these areas. He has authored a book on "Image and Video Processing in the compressed domain", and co-authored a book on "Digital Geometry in Image Processing".

Dr. Mukhopadhyay is a senior member of the IEEE. He also holds life membership of various professional societies in his areas of expertise such Indian Association of Medical Informatics (IAMI), Telemedicine Society of India (TSI), Indian Unit of Pattern Recognition and Artificial Intelligence (IUPRAI, India). He has served as a member of technical program committees of several national and international conferences, and served as Program Co-Chairs of Indian Conference on Computer Vision, Graphics and Image Processing (ICVGIP) in 2000 and 2008. He also served as Program Chairs of the International Workshop on Recent Advances in Medical Informatics in 2013 and 2014. He is serving as a member of the editorial boards of Journal of Visual Communication and Image Representation, and Pattern Recognition Letters published by Elsevier.

He received the Young Scientist Award from the Indian National Science Academy in 1992, and is a Fellow of the Indian National Academy of Engineering (INAE).

Acronyms

ANAE	Average normalized absolute error
ARE	Average relative error
CMID	Chamfer mask induced distance
CWD	Chamfering weighted distance
EARE	Empirical average relative error
EARES	Empirical average relative error on sampling
ECM	Equivalent chamfer mask
EDT	Euclidean distance transform
EMNAE	Empirical maximum normalized absolute error
EMRE	Empirical maximum relative error
EMRES	Empirical maximum relative error on sampling
ENAE	Empirical normalized average error
ERB	Equivalent rational ball
ERM	Equivalent rational mask
HOD	Hyperoctagonal distance
LSE	Least squares estimation
LWD	Linear combination form of weighted distance
MAE	Maximum absolute error
MAT	Medial axis transform
MNAE	Maximum normalized absolute error
MRE	Maximum relative error
MSE	Mean squared error
NAE	Normalized absolute error
OEN	Overestimated norm
RMSE	Root mean squared error
UEN	Underestimated norm
UMSE	Unbiased mean squared error

WGNS	Weighted generalized neighborhood sequence
WtCWD	Linear combination of weighted t-cost and chamfering weighted distance
WtD	Weighted t-cost distance

Symbols and Notations

$\mathscr{R}$	Set of real numbers.
$\mathscr{P}$	Set of nonnegative real numbers.
$\mathscr{R}^+$	Set of positive real numbers.
$\mathscr{Z}$	Set of integers.
$\mathscr{N}$	Set of non-negative integers.
$\mathscr{Q}$	Set of rational numbers.
$\mathscr{Z}_{\mathscr{M}}$	A set of integers: $\{0, 1, 2, \ldots, M\}$
$\|S\|$	The cardinality of set S.
$\mathscr{R}^n$	n-D real space.
$\mathscr{Z}^n$	n-D integral space.
$\mathscr{E}_S(x)$	Expectation of a random variable x over a set S.
$\bar{u}$	$\bar{u}$ is a point in $\mathscr{R}^n$ (or $\mathscr{Z}^n$), and also denoted as $\bar{u} = (u_1, u_2, \ldots, u_n)$, $\forall i$, $u_i \in \mathscr{R}$ (or $u_i \in \mathscr{Z}$).
$\bar{0}$	$(0, 0, \ldots, 0)$.
$\|\|\bar{u}\|\|$	Magnitude of $\bar{u}$, .i.e. $\sqrt{u_1^2 + u_2^2 + \cdots + u_n^2}$.
$n!$	Factorial of a nonnegative integer n.
$\|x\|$	Absolute value or magnitude of x, where x is a number.
$\lceil x \rceil$	Ceiling of x, i.e. the smallest integer greater than or equal to $x \in \mathscr{R}$.
$\lfloor x \rfloor$	Floor of x i.e. the greatest integer smaller than or equal to $x \in \mathscr{R}$.
$u_{(j)}$	jth maximum magnitude of components of $\bar{u}$.
$d(\bar{x}, \bar{y})$, $d(\bar{u})$	If $d(\bar{u})$ denotes a norm in a space, its induced distance function is denoted by $d(\bar{x}, \bar{y})$.
$DT(p)$	Distance transform at a point p.
$FT(p)$	Feature transform at a point p.
$MRE(\rho)$	Maximum relative error of a distance function ρ, with respect to the Euclidean metric in the same dimensional space.

$\Delta_{max}^{(n)}(\rho_1, \rho_2)$	Maximum absolute deviation between two norms ρ_1 and ρ_2 in a bounded region in n-D.
$E_{v\pi}^{(n)}(\rho)$	Volume-π error of a norm ρ in n-D.
$E_{s\pi}^{(n)}(\rho)$	Surface-π error of a norm ρ in n-D.
$E_{\psi\pi}^{(n)}(\rho)$	Shape-π error of a norm ρ in n-D.
κ_{opt}	Optimum scale for a distance function providing minimum MRE.
$E^{(n)}(\bar{u})$	Euclidean norm at $\bar{u} \in \mathscr{R}^n$.
$L_p(\bar{u})$	L_p norm at $\bar{u} \in \mathscr{R}^n$, i.e. $(\sum_{i=1}^{n} \|u_i\|^p)^{\frac{1}{p}}$.
type-m or $O(m)$-neighbor	$\bar{v}$ is a *type-m* or $O(m)$ neighbor if the coordinate values of at most m number of components of $\bar{v}$ differ by at most 1 from those of $\bar{u}$. This implies $\sum_{i=1}^{n} \|u_i - v_i\| \leq m$ and $\forall i, \|u_i - v_i\| \leq 1$. When equality condition holds the $\bar{v}$ is a *strict type-m* or a *strict* $O(m)$ neighbor of $\bar{u}$.
B	B is a cyclic or periodic neighborhood sequence in n-D such that $B = \{b(1), b(2), \ldots, b(p)\}$, where $b(i) \in \{1, 2, \ldots, n\}$ is a neighborhood type. B is also used to denote the hyperoctagonal distance defined by it.
$CPERM(B)$	The set of cyclic permutations of the periodic neighborhood sequence B.
$[\omega_1, \omega_2, \ldots, \omega_n]$	A sorted neighborhood sequence B, with a fixed vector representation such that *type-i* neighborhood occurs ω_i times in the sequence.
$\mathbb{N}$	A *generalized neighborhood* of a point so that $\bar{v} \in \mathscr{Z}^n$ is a neighbor of $\bar{u} \in \mathscr{Z}^n$, if $\exists \bar{p} \in \mathbb{N}$, such that $\bar{v} = \bar{u} + \bar{p}$. $\mathbb{N}$ maintains 2^n-Symmetry.
$\mathbb{P}$	A *weighted generalized neighborhood* of a point given by $(\mathbb{N}, W(\cdot))$, where $W : \mathbb{N} \to \mathscr{R}^+$ to $\mathbb{N}$, such that W is centrally symmetric.
$\mathbb{B}$	A *weighted generalized neighborhood sequence* (WGNS) $\mathbb{B} = \{\mathbb{P}_1, \mathbb{P}_2, \ldots, \mathbb{P}_p\}$ with the length of period p, where $\mathbb{P}_i$ defines a weighted generalized neighborhood.
$d_4(\bar{u})$	City block or 4-neighbor norm at $\bar{u} \in \mathscr{Z}^2$.
$d_8(\bar{u})$	Chess board or 8-neighbor norm at $\bar{u} \in \mathscr{Z}^2$.
$d_{oct}(\bar{u})$	Octagonal norm at $\bar{u} \in \mathscr{Z}^2$.
$d_6(\bar{u})$	6-neighbor norm at $\bar{u} \in \mathscr{Z}^3$.
$d_{18}(\bar{u})$	18-neighbor norm at $\bar{u} \in \mathscr{Z}^3$.
$d_{26}(\bar{u})$	26-neighbor norm at $\bar{u} \in \mathscr{Z}^3$.

$d_m^{(n)}$	m-neighbor distance in $\mathscr{Z}^n$, $1 \leq m \leq n$.
$\delta_m^{(n)}$	Real m-neighbor distance in $\mathscr{R}^n$, $m \in \mathscr{R}$.
$d_B^{(n)}$	Hyperoctagonal distance defined by the neighborhood sequence B.
$d_B^{(2)}(\cdot\vert q,p)$	A simple octagonal distance in $\mathscr{Z}^2$ such that $m = \frac{q}{p} \in [1,2]$.
$D_t^{(n)}$	A t-cost norm in n-D, $1 \leq t \leq n$.
$WtD^{(n)}(\bar{u};W)$	Weighted t-cost norm [42] in n-D with the ordered set of weights W, where the distance value is in $\mathscr{R}$.
$WtD_{sub}^{(n)}$	Simple upper bound optimized weighted t-cost distance in n-D.
$WtD_{isr}^{(n)}$	Inverse square root weighted t-cost distance in n-D.
$LWD^{(n)}(\bar{u};\Gamma)$	Linear combination form of weighted distance (LWD) norm in n-D with the ordered set of weights Γ, where the distance value is in $\mathscr{R}$.
$CWD^{(n)}(\bar{u};\Delta)$	Chamfering weighted distance (CWD) norm in n-D with the ordered set of weights Δ, where the distance value is in $\mathscr{R}$.
$CWD_{eu}^{(n)}$	The CWD in n-D with the set of weights $\{\sqrt{i}\vert 1 \leq i \leq n\}$.
$CWD_{euopt}^{(n)}$	The $CWD_{eu}^{(n)}$ scaled by an optimum scale κ_{opt} providing minimum MRE.
$<\delta_1, \delta_2, \ldots, \delta_n>$	The CWD with the ordered set of weights as $\{\delta_1, \delta_2, \ldots, \delta_n\}$.
$CWD_{3\_4\_gen}^{(n)}$	The CWD with the set of weights as $<3, 4, \ldots, n+2>$.
$CWD_{eu\_int}^{(n)}$	The CWD with a set of integral weights approximating $CWD_{eu}^{(n)}$ (Eq. (6.1)).
$WtCWD^{(n)}(\bar{u};W,\Delta,a,b)$	A linear combination of $WtD^{(n)}(\bar{u};W)$ and $CWD^{(n)}(\bar{u};\Delta)$, such that $WtCWD^{(n)}(\bar{u};W,\Delta,a,b) = a \cdot WtD^{(n)}(\bar{u};W) + b \cdot CWD^{(n)}(\bar{u},\Delta)$.
$WtCWD_{isr\_eu}^{(n)}$	A linear combination of $WtD_{isr}^{(n)}$ and $CWD_{eu}^{(n)}$.
$WtCWD_{sub\_rec}^{(n)}$	A linear combination of $WtD_{sub}^{(n)}$ and the CWD with reciprocal sets of weights.
$\mathscr{M}$	A chamfer mask.
$\mathscr{M}_g$	The generator of a chamfer mask.
$\mathscr{M}_{55}(a,b,c)$	A chamfer mask of size 5×5 in 2D, with parameters a, b and c.
$\mathscr{M}_{555}(a,b,c,d,e,f)$	A chamfer mask of size $5 \times 5 \times 5$ in 3D, with parameters a, b, c, d, e, and f.

$\mathscr{M}_{\mathbb{B}}$	The equivalent chamfer mask of a WGNS $\mathbb{B}$.
$\phi(\bar{u})$	The set of points in n-D formed by all possible signed permutation of coordinate values of $\bar{u}$, i.e. $\phi(\bar{u}) = \{\bar{v}|\bar{v}$ is formed by permuting $s_i u_i$s where s_i is either +1 or -1 $\}$.