A Guide to Graph Algorithms

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Preface

A series of summerschools organized by the University of Science and Technology in China germinated this book. It is a collection of texts that are either 'trend-setters' or just good examples. A first draft of this text was prepared in 2016 and 2017. It was substantially updated in subsequent years.

This book provides an introduction to the research area of graph algorithms and reviews the development over the last decade. The contents is divided into parts titled 'Graphs,' 'Algorithms,' 'Problem Formulations,' and 'Recent Trends.' The first part reviews some graph - theoretic concepts. The second part presents a few early highlights in graph algorithms. The third part is a very short introduction into graph algebras and monadic second order - logic. The last part of the book uses 'treewidth' as a stepping stone and talks through a wide variety of recent trends.

The book assumes familiarity with a few basic concepts and programming techniques that are taught during a first year computer science course in algorithms. It provides a smooth introduction for those who want to dive deep into this fascinating research area. The book contains a lot of exercises, many up at present - day research - level; they will keep the reader pleasantly entertained for many hours.

About the authors

Dr. Mingyu Xiao is a professor in the school of computer science and engineering, University of Electronic Science and Technology of China, Chengdu, China. He received his PhD in Computer Science from the Chinese University of Hong Kong in 2008.

Dr. Ton Kloks is a researcher of graph algorithms. He studied mathematics at Eindhoven University in the Netherlands during the 1980s. He received his PhD in Computer Science from Utrecht University in The Netherlands in 1993 for his thesis on treewidth.

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This text originated in 2016 – 2017, and was substantially updated during the summers of 2018, 2019, 2020 and 2021.