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Information Fusion in Distributed Sensor Networks with Byzantines



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About This Book

Every day, we share our personal information through digital systems which are constantly exposed to threats. For this reason, security-oriented disciplines of signal processing have received increasing attention in the last decades: multimedia forensics, digital watermarking, biometrics, network monitoring, steganography and steganalysis are just a few examples. Even though each of these fields has its own peculiarities, they all have to deal with a common problem: the presence of one or more adversaries aiming at making the system fail. Adversarial Signal Processing lays the basis of a general theory that takes into account the impact that the presence of an adversary has on the design of effective signal processing tools.

By focusing on the application side of Adversarial Signal Processing, namely adversarial information fusion in distributed sensor networks, and adopting a game-theoretic approach, this book presents the recent advances in the field and how several issues have been addressed. First, a heuristic decision fusion setup is presented together with the corresponding soft isolation defense scheme that protects the network from adversaries, specifically, Byzantines. Second, the development of an optimum decision fusion strategy in the presence of Byzantines is outlined. Finally, a technique to reduce the complexity of the optimum fusion by relying on a novel nearly optimum message passing algorithm based on factor graphs is presented.

Contents

1	Intr	oductio	n	1
	1.1	Motiva	ation	1
	1.2	Goal a	and Summary	5
		1.2.1	Goal	5
		1.2.2	Summary of the Book	6
	Refe	erences		7
2	Basi	c Notio	ons of Distributed Detection, Information	
	Fusi	on and	Game Theory	9
	2.1	Introd	uction	9
	2.2	Detect	ion Theory	10
		2.2.1	Bayesian Detection	11
		2.2.2	Detection Performance Metrics	13
		2.2.3	Neyman-Pearson Detection	14
		2.2.4	Sequential Detection	15
	2.3	Inform	nation Fusion Rules	16
		2.3.1	Simple Fusion Rules	17
		2.3.2	Advanced Fusion Rules	19
	2.4	Game	Theory in a Nutshell	21
		2.4.1	Nash Equilibirium	23
		2.4.2	Dominance Solvable Games	24
	2.5	Conclu	usion	26
	Refe	erences		26
3	Secu	rity At	ttacks and Defenses in Distributed	
	Sens	or Net	works	29
	3.1	Introdu	uction	29
	3.2	Attack	s to Distributed Sensor Networks	29
		3.2.1	Attacks to the Observations	31

		3.2.2	Attacks to the Sensors	33
		3.2.3	Attacks to the Reports	34
	3.3	Defens	ses Against Attacks to Distributed Sensor	
		Netwo	vrks	34
		3.3.1	Defenses Against Attacks to the Observations	34
		3.3.2	Defenses Against Attacks to Sensors	37
		3.3.3	Defenses Against Attacks to Reports	40
	3.4	Conclu	usion	40
	Refe	rences		41
4	Adv	ersaria	l Decision Fusion: A Heuristic	
	Арр	roach		45
	4.1	Introdu	uction	45
	4.2	Decisi	on Fusion with Isolation of Byzantines	46
		4.2.1	Problem Formulation	46
		4.2.2	Byzantine Identification: Hard Reputation	
			Measure	48
	4.3	Decisi	on Fusion with Soft Identification of Malicious Nodes	48
	4.4	A Gan	ne-Theoretical Approach to the Decision Fusion	
		Proble	m	50
		4.4.1	The Decision Fusion Game	50
		4.4.2	Equilibrium Point Analysis of the Decision	
			Fusion Game	51
	4.5	Perfor	mance Analysis	52
	4.6	Conclu	usions	55
	Refe	rences		55
5	A G	ame-Tł	neoretic Framework for Optimum Decision	
-	Fusi	on in t	he Presence of Byzantines	57
	5.1	Introdu	uction	57
	5.2	Optim	um Fusion Rule	58
		5.2.1	Unconstrained Maximum Entropy Distribution	61
		5.2.2	Constrained Maximum Entropy Distributions	61
		5.2.3	Fixed Number of Byzantines	64
	5.3	An Ef	ficient Implementation Based on Dynamic	
		Progra	mming	64
	5.4	Optim	um Decision Fusion Game	66
	5.5	Simula	ation Results and Discussion	68
		5.5.1	Equilibrium Point of the DF_{Bvz} game	68
		5.5.2	Performance at the Equilibrium	76
		5.5.3	Assumptions Validation and Discussion	78
	5.6	Conclu	usions	80
	Refe	rences		80

C	on	ter	nts
Š	\mathbf{u}	w	103

6	An]	Efficient Nearly-Optimum Decision Fusion Technique	
	Base	d on Message Passing	83
	6.1	Introduction	83
	6.2	Notation and Problem Formulation	84
	6.3	A Decision Fusion Algorithm Based on Message	
		Passing	86
		6.3.1 Introduction to Sum-Product Message Passing	86
		6.3.2 Nearly-Optimal Data Fusion by Means of Message	
		Passing	88
	6.4	Simulation Results and Discussion	93
		6.4.1 Complexity Assessment	94
		6.4.2 Performance Evaluation	94
	6.5	Conclusions	100
	Refe	rences	101
7	Con	clusion	103
	7.1	Open Issues	103
	Refe	rence	104
Bi	bliogi	raphy	105
In	dex .	~ ~	107

Symbols

H_0	Null hypothesis
H_1	Alternative hypothesis
n	Number of nodes in the network
\mathbf{X}_i	Observation vectors available to sensor <i>i</i>
S_i	The system state under hypothesis $H_i, i \in \{0, 1\}$
$P(H_0)$	A-priori probability that the system is in state S_0
$P(H_1)$	A-priori probability that the system is in state S_1
$P(x H_j)$	The observation probability density conditioned to hypothesis H_j
$S^* \in \{0,1\}$	The global decision at the fusion center regarding S
C_{ij}	Cost of deciding H_i when H_j is true
C	Average cost or risk function for Bayesian detection
$\Lambda(x)$	Likelihood ratio regarding the observation x
λ	Decision threshold
P_{fa}	Probability of false alarm
P_{md}	Probability of missed detection
P_d	Probability of correct detection
P _{null}	Probability to decide H_0 when H_0 is true
P_e	Probability of error
λ_{NP}	Local Neyman-Pearson likelihood decision threshold
α_{NP}	Acceptable false alarm for Neyman-Pearson detector
${\mathcal F}$	Lagrange function for Neyman-Pearson detector optimization
$\lambda_i, i \in \{0, 1\}$	Decision threshold for hypothesis H_i for local SPRT detector
α_{ST}	Local SPRT detector constraint on false alarm probability
β_{ST}	Local SPRT detector constraint on missed detection
	probability
u_i	Information sent by sensor i to the FC
P_{d_i}	Local probability of correct detection at node <i>i</i>
P_{fa_i}	Local probability of false alarm at node <i>i</i>
P_{md_i}	Local probability of missed detection at node <i>i</i>
Q_D	Global probability of correct detection at the FC

Q_{FA}	Global probability of false alarm at the FC
$Q_{D_{AND}}$	Global probability of correct detection for the AND rule
$Q_{FA_{AND}}$	Global probability of false alarm for the AND rule
$Q_{D_{OR}}$	Global probability of correct detection for the OR rule
$Q_{FA_{OR}}$	Global probability of false alarm for the OR rule
$Q_{D_{lm}}$	Global probability of correct detection for the <i>k</i> -out-of- <i>n</i> rule
$O_{FA_{lm}}$	Global probability of false alarm for the k-out-of-n rule
U_{SLC}	Square Law Combining information fusion result
U_{MRC}	Maximum Ratio Combining information fusion result
U_{SC}	Selection Combining information fusion result
ζ	Decision threshold of the soft combination rules
$\Upsilon_i, i \in \{0, 1\}$	Decision threshold for hypothesis H_i for global SPRT detector
α_{FC}	Global SPRT detector constraint on false alarm probability
β_{FC}	Global SPRT detector constraint on missed detection
110	probability
\mathcal{X}_{M}^{2}	Chi-square distribution with M degrees of freedom
$\Gamma(.)$	The incomplete gamma function
Q(.)	The generalized Marcum Q-function
\mathcal{S}_i	Strategy set available to player <i>i</i>
v_l	Payoff (or utility) of player l
$G(N, \mathcal{S}, \mathbf{v})$	Game with N players, strategy set S and payoff vector v
$\Pi(\mathcal{Z})$	Set of all probability distributions over the set \mathcal{Z}
r_i	Report sent by node <i>i</i> to the FC
α	Fraction of nodes (or links) under attack or the probability that
	a node (or link) is under attack
r _{ij}	Report by node <i>i</i> at instant <i>j</i>
т	Observation window size
P _{mal}	Node malicious probability or crossover probability of the attacked links
u_{ii}	Decision by node <i>i</i> at instant <i>j</i>
Γ_i	Hard reputation score of node <i>i</i>
$d_{int}(j)$	Intermediate decision at instant <i>j</i> at the FC
η	Isolation threshold
\dot{R}_{ii}	Soft reputation score of node i at instant j
$DF(\mathcal{S}_{FC}, \mathcal{S}_{FC}, v)$	Decision fusion game with S_{FC} the strategy set for the FC, S_B
(,	the strategy set for Byzantines, and payoff v
P_{ear}	Probability of error after removal of Byzantines
P^B_{ISO}	Probability of correct isolation of Byzantines
P_{ISO}^{H}	Probability of erroneous isolation of honest nodes
P_{mal}^{FC}	The FC guess of P_{mal}
$P_X(x)$	Probability mass function of the random variable x
S^m	Sequence of system states random variable with instantiation $\frac{m}{2}$
$\mathbf{P}_{i}(\mathbf{i}) \mathbf{i} \in [0, 1]$	S Probability that a system is in state S at time <i>i</i>
$\boldsymbol{r}_{S_j}(l), l \in \{0, 1\}$	Probability that a system is in state S_j at time <i>i</i>

Symbols

U_{ij}	Random variable for the local decision of node i at instant j with instantiation u_i
$A^n = (A, A)$	Ringry random sequence for Byzantine positions with a^n its
$A = (A_1, \ldots, A_n)$	binary random sequence for byzantine positions with a , its instantiation
$\mathbf{P} = \{\mathbf{P}_{i}\}$	Random matrix of all received reports by EC with $\mathbf{P} = \{\mathbf{R}_n\}$
$\mathbf{K} = \{\mathbf{K}_{ij}\}$	Random matrix of an received reports by FC with $\mathbf{R} = \{R_{ij}\}$ as its instantiation
$P(a^n)$	Probability of Byzantine sequence
r (u)	Local decision error at the nodes
δ	The probability that the FC receives a wrong report
(<i>i</i>)	The number of instants at which the report is equal to the
mèq	system state for node <i>i</i>
$E[N_B]$	Expected number of Byzantines
\mathcal{U}_{Λ}	Expected value of A_i
$H(A^n)$	Entropy distribution of Byzantines
$h(\mu_{A_{\perp}})$	Binary entropy function for the expected value of A_i
h	The FC expected maximum number of Byzantines
$\mathcal{I} = \{1, \dots, n\}$	Indexing set of size <i>n</i>
\mathcal{I}_k	Set of all k-subsets of \mathcal{I}
I	Random variable with indexes of Byzantine nodes
P(I)	Equivalent to the probability of a Byzantine sequence $P(a^n)$
n _B	Fixed number of Byzantines in the network known to the FC
$DF_{Byz}(\mathcal{S}_B, \mathcal{S}_{FC}, v)$	Decision fusion game with S_B the strategy set of Byzantines,
	S_{FC} the strategy set of the FC, and v the payoff
P^B_{mal}	Malicious probability strategy of the Byzantines
$\mathcal{S}^q_{\scriptscriptstyle B}$	Quantized Byzantines' strategy set
$\mathcal{S}_{FC}^{\overline{q}}$	Quantized FC's strategy set with $\mathbf{r} = \{r_{ij}\}$ as its instantiation
V	Payoff matrix for each pair of strategies
P_e^*	Probability of error at the equilibrium
$P(P_{mal}^B)$	Probability assigned by Byzantines to a strategy in mixed
(mai)	strategy Nash equilibrium
$P(P_{mal}^{FC})$	Probability assigned by FC to a strategy in mixed strategy
(mai)	Nash equilibrium
ρ	State transition probability in a two-state Markov model
$m_{vf^{(l)}}$	Variable-to-function message for factor l
$m_{c}^{(l)}$	Function-to-variable message for factor l
fv	-

List of Figures

Fig. 2.1	Parallel Topology	11
Fig. 2.2	ROC curve example	14
Fig. 2.3	Neyman-Pearson Setup	15
Fig. 2.4	SPRT detector.	16
Fig. 3.1	Classification of attacks to distributed sensor networks	30
Fig. 4.1	Decision fusion under adversarial conditions	46
Fig. 4.2	Error probability $P_{e,ar}$ at the equilibrium for $P_d = 0.8$ (a)	
	and $P_d = 0.9$ (b)	54
Fig. 4.3	P_{iso}^H versus P_{iso}^B at $P_{mal} = 1.0$, for $\alpha = 0.46$ and $P_d = 0.8$.	
	For the soft scheme, 10 thresholds are taken	55
Fig. 5.1	Sketch of the adversarial decision fusion scheme	58
Fig. 5.2	Efficient implementation of the function in (5.18) based	
	on dynamic programming. The figure depicts the tree	
	with the iterations for the case $k < n - k \dots$	66
Fig. 6.1	Markovian model for system states. When $\rho = 0.5$ subsequent	
	states are independent	85
Fig. 6.2	Node-to-factor message passing	88
Fig. 6.3	Factor-to-node message passing	88
Fig. 6.4	End of message passing for node z_i	89
Fig. 6.5	Factor graph for the problem in Eq. (6.10)	89
Fig. 6.6	Factor graph for the problem at hand with the illustration	
	of all the exchanged messages	91
Fig. 6.7	Number of operations required for different $n, m = 10$	
	and 5 message passing local iterations for message passing	
	and optimal schemes	95
Fig. 6.8	Number of operations required for different $m, n = 20$	
	and 5 message passing local iterations for message passing	
	and optimal schemes.	95

Fig. 6.9	Error probability as a function of α for the following setting:	
	$n = 20$, independent Sequence of States $\rho = 0.5$, $\varepsilon = 0.15$,	
	$m = 10$ and $P_{mal} = 1.0$	96
Fig. 6.10	Error probability as a function of α for the following setting:	
	$n = 20$, Markovian Sequence of States $\rho = 0.95$, $\varepsilon = 0.15$,	
	$m = 10$ and $P_{mal} = 1.0$	97
Fig. 6.11	Error probability as a function of α for the following setting:	
	$n = 20$, Markovian Sequence of States $\rho = 0.95$, $\varepsilon = 0.15$,	
	$m = 30$ and $P_{mal} = 1.0$	98
Fig. 6.12	Error probability as a function of α for the following setting:	
	$n = 20$, Markovian Sequence of States $\rho = 0.95$, $\varepsilon = 0.15$,	
	$m = 30$ and $P_{mal} = 0.5$	98
Fig. 6.13	Error probability as a function of <i>m</i> for the following settings:	
	$n = 20$, Markovian Sequence of States $\rho = 0.95$, $\varepsilon = 0.15$	
	and $\alpha = 0.45$	99
Fig. 6.14	Error probability as a function of <i>m</i> for the following settings:	
	$n = 20$, independent Sequence of States $\rho = 0.5$, $\varepsilon = 0.15$	
	and $\alpha = 0.45$	99
Fig. 6.15	Comparison between the case of independent and Markovian	
	system states ($n = 20$, $\rho = \{0.5, 0.95\}$, $\varepsilon = 0.15$, $m = 10$,	
	$P_{\rm mal} = 1.0$)	100

List of Tables

Table 2.1	Decision cases in binary detection	12
Table 2.2	Example of game representation in normal form.	
	The row player is player 1 and the column player is player 2.	
	The entries of the table are the payoffs of the game for each	
	pair of strategies.	22
Table 2.3	Example of removal of weakly dominated strategies	
	will cause the loss of some Nash equilibria. The row player	
	is player 1 and the column player is player 2	25
Table 4.1	Payoff of the DF_H game for $\alpha = 0.46$ and $P_d = 0.8$,	
	$P_{fa} = 0.2 \ldots$	53
Table 4.2	Payoff of the DF_S game for $\alpha = 0.46$ and $P_d = 0.8$,	
	$P_{fa} = 0.2 \dots \dots$	53
Table 5.1	Payoff of the DF_{Bvz} game $(10^3 \times P_e)$ with independent node	
	states with $\alpha = 0.3$, $m = 4$, $n = 20$, $\varepsilon = 0.1$. The equilibrium	
	point is highlighted in bold	71
Table 5.2	Payoff of the DF_{Rvz} game $(10^2 \times P_e)$ with independent node	
	states with $\alpha = 0.4$, $m = 4$, $n = 20$, $\varepsilon = 0.1$. The equilibrium	
	point is highlighted in bold	71
Table 5.3	Payoff of the DF_{Rvz} game $(10^2 \times P_e)$ with independent node	
	states with $\alpha = 0.45$, $m = 4$, $n = 20$, $\varepsilon = 0.1$.	
	The equilibrium point is highlighted in bold	71
Table 5.4	Payoff of the DF_{Byz} game $(10^4 \times P_z)$ with $n_B = 6, m = 4$.	
	$n = 20, \epsilon = 0.1$. The equilibrium point is highlighted	
	in bold	72
Table 5.5	Payoff of the $DF_{P_{NR}}$ game $(10^3 \times P_c)$ with $n_P = 8$, $m = 4$.	
	$n = 20, \epsilon = 0.1$. No pure strategy equilibrium exists	72
Table 5.6	Payoff of the DF_{Pm} game $(10^2 \times P_1)$ with $n_P = 9$ $m = 4$. –
1 4010 5.0	$n = 20$ $\varepsilon = 0.1$ The equilibrium point is highlighted	
	in hold	72
		. 4

Table 5.7	Mixed strategies equilibrium for the DF_{Byz} game with	
	$n_B = 8, m = 4, n = 20, \varepsilon = 0.1, P_e$ indicates the error	70
T 11 C 0	probability at the equilibrium $\dots \dots \dots$	12
Table 5.8	Payoff of the DF_{Byz} game $(10^2 \times P_e)$ with $N_B < n/2$.	
	The other parameters of the game are set as follows: $m = 4$,	
	$n = 20, \varepsilon = 0.1$. The equilibrium point is highlighted	70
T 11 C 0		13
Table 5.9	Payoff of the DF_{Byz} game $(10^4 \times P_e)$ with $N_B < n/3$.	
	The other parameters of the game are set as follows: $m = 4$,	
	$n = 20, \varepsilon = 0.1$. The equilibrium point is highlighted	
	in bold	13
Table 5.10	Payoff of the DF_{Byz} game $(10^3 \times P_e)$ with independent node	
	states with $\alpha = 0.3$, $m = 10$, $n = 20$, $\varepsilon = 0.1$.	
	The equilibrium point is highlighted in bold	74
Table 5.11	Payoff of the DF_{Byz} game $(10^2 \times P_e)$ with independent node	
	states with $m = 10$, $n = 20$, $\alpha = 0.4$, $\varepsilon = 0.1$.	
	The equilibrium point is highlighted in bold	74
Table 5.12	Payoff of the DF_{Byz} game $(10^2 \times P_e)$ with independent node	
	states with $\alpha = 0.45$, $m = 10$, $n = 20$, $\varepsilon = 0.1$.	
	The equilibrium point is highlighted in bold	75
Table 5.13	Payoff of the DF_{Byz} game $(10^4 \times P_e)$ with $n_B = 6$,	
	$m = 10, n = 20, \varepsilon = 0.1$. The equilibrium point is	
	highlighted in bold.	75
Table 5.14	Payoff of the DF_{Byz} game $(10^4 \times P_e)$ with $n_B = 8$, $m = 10$,	
	$n = 20, \varepsilon = 0.1$. No pure strategy equilibrium exists	75
Table 5.15	Payoff of the DF_{Byz} game $(10^4 \times P_e)$ with $n_B = 9$, $m = 10$,	
	$n = 20, \varepsilon = 0.1$. No pure strategy equilibrium exists	75
Table 5.16	Payoff of the DF_{Byz} game $(10^4 \times P_e)$ with $N_B < n/2$.	
	The other parameters of the game are set as follows: $m = 10$,	
	$n = 20, \epsilon = 0.1$. No pure strategy equilibrium exists	76
Table 5.17	Payoff of the DF_{Byz} game $(10^4 \times P_e)$ with $N_B < n/3$ in the	
	following setup: $m = 10$, $n = 20$, $\varepsilon = 0.1$. The equilibrium	
	point is highlighted in bold	76
Table 5.18	Mixed strategies equilibrium for the DF_{Byz} game with	
	$n_B = 8, m = 10, n = 20, \varepsilon = 0.1. P_e^*$ indicates the error	
	probability at the equilibrium	76
Table 5.19	Mixed strategies equilibrium for the DF_{Byz} game with	
	$n_B = 9, m = 10, n = 20, \varepsilon = 0.1. P_e^*$ indicates the error	
	probability at the equilibrium	76
Table 5.20	Mixed strategies equilibrium for the DF_{Byz} game with	
	$N_B < n/2$ with $m = 10$, $n = 20$, $\varepsilon = 0.1$. P_e^* indicates	
	the error probability at the equilibrium	77

xviii

List of Tables

Table 5.21	Error probability at the equilibrium for various fusion	
	schemes. All the results have been obtained by letting $m = 4$,	
T 11 5 22	$n = 20, \varepsilon = 0.1$	//
Table 5.22	Error probability at the equilibrium for various fusion	
	schemes. All the results have been obtained by letting	
	$m = 10, n = 20, \varepsilon = 0.1$	78
Table 5.23	Payoff of the DF_{Byz} game with independent node states with	
	$\alpha_{FC} = 0.2, \alpha = 0.3, m = 4, n = 20, \varepsilon = 0.1$. The equilibrium	
	point is highlighted in bold	79
Table 5.24	Payoff of the DF_{Byz} game with independent node states with	
	$\alpha_{FC} = 0.2, \alpha = 0.4, m = 4, n = 20, \varepsilon = 0.1$. The equilibrium	
	point is highlighted in bold	79
Table 5.25	Payoff of the DF_{Byz} game with $N_{B_{FC}} < n/4$ in the following	
	setup: $m = 4$, $n = 20$, $\varepsilon = 0.1$, $N_B < n/2$. The equilibrium	
	point is highlighted in bold	79
Table 5.26	Payoff of the DF_{Bvz} game with $N_{B_{FC}} < n/6$ in the following	
	setup: $m = 4$, $n = 20$, $\varepsilon = 0.1$, $N_B < n/2$. The equilibrium	
	point is highlighted in bold	80
Table 6.1	Running Time (in seconds) for the Optimal and the	
	Message Passing Algorithms for: $m = 10$, $\varepsilon = 0.15$,	
	Number of Trials = 10^5 and Message Passing	
	Iterations = 5.	96