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Abstract. The wide adoption of social media platforms has brought about numerous benefits for communication and information sharing. However, it has also led to the rapid spread of misinformation, causing significant harm to individuals, communities, and society at large. Consequently, there has been a growing interest in devising efficient and effective strategies to contain the spread of misinformation. One popular countermeasure is blocking edges in the underlying network.

We model the spread of misinformation using the classical Independent Cascade model and study the problem of minimizing the spread by blocking a given number of edges. We prove that this problem is computationally hard, but we propose an intuitive community-based algorithm, which aims to detect well-connected communities in the network and disconnect the inter-community edges. Our experiments on various realworld social networks demonstrate that the proposed algorithm significantly outperforms the prior methods, which mostly rely on centrality measures.

Keywords: Social Networks · Misinformation Spreading · Countermeasure · Edge Blocking · Community Detection.

1 Introduction

In recent years, social media has revolutionized the way individuals connect with each other, share information, and express themselves. It has created new opportunities for political engagement, social activism, and community building, and has enabled individuals to access information and resources that were previously unavailable. Social media has also transformed the way businesses and organizations interact with customers and stakeholders, providing new channels for marketing, customer service, and public relations.

However, the widespread adoption of social media platforms has undeniably resulted in a significant increase in the dissemination of misinformation. This issue permeates various domains such as politics, economics, and sociology [9]. For example, following the breach of The Associated Press Twitter account, a fabricated announcement circulated, stating that "Breaking: Two Explosions in the White House and Barack Obama is injured." As a consequence, this false

information led to a staggering loss of 10 billion USD in just a few hours and triggered a rapid crash in the US stock market, cf. [33].

The far-reaching consequences of misinformation spreading in different contexts cannot be understated, as they possess the potential to shape public opinion, influence decision-making processes, and even impact social cohesion. The need to address the challenges posed by the rampant spread of misinformation across diverse topics has emerged as a critical concern in today's interconnected digital age.

The spread of misinformation on social media is a complex and multifaceted phenomenon that involves a range of factors, including the structure of social networks, the psychology of misinformation diffusion, and the role of technology in shaping information sharing. Understanding these factors is critical for developing effective interventions to minimize the spread of misinformation.

In order to mitigate the spread of misinformation in social networks, several approaches have been proposed in the past. Reducing the spread of misinformation can be achieved by some form of blocking, where a set of nodes or edges are identified and blocked from the network, under some budget constraints. Blocking a node implies that the account is either removed or banned and all its connections with other nodes are suspended, while blocking an edge implies that the connection between the two nodes connected by the edge is suspended, for example by not exposing posts from one user to another.

To effectively combat the spread of misinformation within social networks, the initial step involves promptly identifying the misinformation as it emerges, this can be done using various misinformation detection techniques, cf. [45,54]. However, it is important to recognize that detected misinformation may resurface in modified forms, highlighting the significance of monitoring subsequent posts associated with a piece of misinformation. As a result, to minimize the spread of misinformation within a network, two popular approaches can be used:

- Source-Aware Approach of misinformation contamination relies on identifying the sources responsible for propagating the misinformation and the users who have accepted and disseminated it within a social network. Then, containment strategies can be implemented to minimize their influence and curb the spreading ability of the misinformation.
- Source-Agnostic Approach focuses on mitigating the flow of misinformation within the network, without the prior knowledge of the specific sources of the misinformation. Through the implementation of a containment strategy, the aim is to reduce the overall dissemination of misinformation without specifically targeting its sources.

The source-aware approach is generally more powerful since it benefits from some extra source of information. However, there are two fundamental issues. Firstly, the identification of misinformation sources involves complexities such as data collection and network evolution, and source identification algorithms could be inaccurate. (Shelke and Attar [38] provide a compressive overview of various source detection approaches and the challenges faced with these methods.) Furthermore, the source-aware approach disregards the anonymity and privacy of individuals within the social networks, up to a large extent. Consequently, there has been a growing interest, cf. [26,49], in devising effective source-agnostic strategies that aim to minimize the flow of misinformation in the network without explicitly targeting individual sources. The present work also falls under the umbrella of this line of research.

Two commonly employed strategies to contain the misinformation spreading are node and edge blocking. Edge blocking has garnered greater attention recently, cf. [48,40,26,49], since it is less intrusive (i.e., disrupts the original functionality and flow of the network less aggressively) and provides controlling power in a more granular level (note that usually blocking a node is equivalent to blocking all its adjacent edges).

In the present work, we focus on designing an effective and efficient sourceagnostic edge-blocking strategy. To model the spread of misinformation, we exploit the popular Independent Cascade model [19]. We investigate the problem of minimizing the expected number of nodes that will be exposed to a piece of misinformation when we are allowed to block k edges for some given integer k. We show that this problem is NP-hard. (It is worth stressing that while this problem has been extensively studied by prior work and several approximation approaches were proposed [49], we are the first to formally prove that the problem is computationally hard.)

We propose an intuitive community-based algorithm, which first uses a community detection algorithm such as Louvain community detection algorithm [2], to partition the nodes into communities (i.e., subsets of well-connected nodes). Then, we try to slow down the flow of misinformation between these communities by disconnecting the inter-community edges. The idea is that stopping the spread of misinformation inside a community is hard since it requires blocking a significant number of edges. However, there are substantially less edges between communities whose blocking could drastically reduce the extent that the misinformation travels.

We provide our experimental findings on several real-world graph data. We observe that our proposed algorithm consistently and significantly outperforms the existing algorithms.

Outline. The rest of the paper is structured as follows. Section 1.1 covers the preliminaries. Then, in Section 1.2 we give an overview of related previous work. The hardness results are provided in Section 2. Our proposed algorithm is presented in Section 3. Finally, our experimental findings and comparison of algorithms are provided in Section 4.

1.1 Preliminaries

Graph Definitions. Let $G = (V, E, \omega)$ be a weighted graph, where function $\omega : E \to [0, 1]$ assigns a value between 0 and 1 to each edge in the graph. Let us define n := |V| and m := |E|. For a node $v \in V$, $N(v) := \{v' \in V : (v, v') \in E\}$ is the *neighborhood* of v. Furthermore, $\hat{N}(v) := N(v) \cup \{v\}$ is the *closed neighborhood* of v. Let d(v) := |N(v)| be the *degree* of v in G. The girth of a

graph G is the length of the shortest cycle contained in the graph. If G has no cycle, then the girth is defined to be infinity.

Independent Cascade Model [19,12]. Each node can have one of the following three states:

- Ignorant (white): A node which has not heard of the misinformation.
- *Spreader (red)*: A spreader is a node who has heard the misinformation and spreads it.
- *Stifler (orange)*: A node who has heard the misinformation but does not spread it.

Let a coloring C be a function $C: V \to \{w, r, o\}$, where w, r, and o correspond to white, red, and orange, respectively. The process starts from an initial coloring C_0 . Then, in each round $t \in N$, all nodes simultaneously update their state according to following updating rules:

- A white node v becomes red with probability:

$$p^{*}(v) := 1 - \prod_{v' \in N(v) \& \mathcal{C}_{t-1}(v') = r} \left(1 - \omega \left((v, v') \right) \right).$$
(1)

- A red node becomes orange.
- An orange node remains orange.

More precisely, we have:

$$\mathcal{C}_t(v) = \begin{cases} r & \text{if } \mathcal{C}_{t-1}(v) = w \text{ with probability } p^*(v) \\ w & \text{if } \mathcal{C}_{t-1}(v) = w \text{ with probability } 1 - p^*(v) \\ o & \text{if } \mathcal{C}_{t-1}(v) = o \lor \mathcal{C}_{t-1}(v) = r. \end{cases}$$

Let $(v, v') \in E$, where v is white and v' is red. Then, v' makes v red with probability $\omega((v, v'))$. This explains the choice of probability $p^*(v)$ in Equation (1). Furthermore, a red node has one chance to spread, and then it becomes orange (stifler) and remains orange forever. This model is usually known as the *Independent Cascade* (IC) model, cf. [12,19].

The main focus of the present paper is to devise an effective edge-blocking strategy to minimize the spread of misinformation, simulated by the Independent Cascade model. An exact formulation of the problem is given in Section 2.

1.2 Prior Work

Information Spreading Models. A plethora of (mis)-information spreading models have been developed and studied in recent years, cf. [19,53,10,50,52,51]. Here, we focus on the most fundamental and relevant models.

- Independent Cascade Model [12,19]. As described in Section 1.1, this is a model of information diffusion that assumes that information spreads through a network of individuals in a series of steps. In every step, it considers each node to be in one of the three states, red (spreader), white (ignorant) and orange (stifler). Then, each red node gets one chance (before becoming orange) to make its white neighbors red (i.e., inform them). Motivated by viral marketing, the main focus in this model is to develop algorithms for finding subsets of nodes that maximize the spread of the red color, mostly exploiting monotonicity and submodularity properties [30]. Different variants of the Independent Cascade model have also been studied, for example, where there is a forgetting mechanism in place [53] or when there are more than one pieces of (mis)-information spreading [28]
- Linear Threshold Model [19]. This model of information diffusion assumes that individuals are more likely to be exposed to some (mis)-information if a larger fraction of their neighbors have been exposed to it. More precisely, each node v has a threshold τ_v chosen randomly from the interval [0, 1]. Then, a white node v becomes red once at least τ_v fraction of its neighbors are red, and then remains red forever. In the Linear Threshold model also, motivated by viral marketing, the problem of finding a set of seed nodes which can maximize the final number of red nodes has been extensively studied, cf. [19,35]. Generalized variants of threshold model have been considered too, cf. [25].
- Susceptible-Infected-Recovered (SIR) Model [6]. The SIR model is a commonly used epidemiological model that describes the spread of infectious diseases in a population. It divides the population into three compartments: Susceptible, Infected, and Recovered. Then, each node can change its state following a predefined stochastic updating rule relying on some model parameters, infection and recovery factor. While the model was originally introduced to emulate the spread of diseases, it has gained some popularity in modeling (mis)-information spreading. The original model assumes that the homogeneous mixing condition holds, that is, the nodes are connected via a clique. However, network based variants of the model have been studied as well, cf. [20].

Countermeasures. Reducing the propagation of misinformation is a significant challenge in the field of social network analysis and has garnered considerable interest. Various approaches have been proposed to tackle this issue and mitigate the spread of false information in social networks.

Edge Blocking Countermeasure: One countermeasure which has gained significant popularity is edge blocking, cf. [48,40,26,49,31,53]. Holme et al. [17] considered four different edge blocking strategies: blocking by the descending order of the degree and the betweenness centrality, calculated for either the initial network or the resulting network during the blocking procedure. It is observed edges blocked in order of betweenness show more efficient misinformation mitigation as compared to edges blocked in decreasing order of degree. Kimura et al. [24]

introduced a method of efficiently estimating the influence of nodes using bond percolation. This bond percolation method then was used in [22,23] to identify a set of edges which, when blocked, maximize the contamination degree of the network. Yan et al. [46] proposed a greedy method to identify the most critical edges among a set of candidate edges to minimize the spread of a misinformation. Pagerank centrality [3] is used in [46] as a criterion for blocking the edges to minimize the spread of misinformation. The susceptibility of a graph to diffusion is defined in [21] as the sum of the expected influence of each node when it is the single source for a cascade. Further, a greedy method is proposed that minimizes the spread susceptibility of the network. Tong et al. [41] provided an approach where the edges blocked depend on the eigenvalue of the adjacency matrix of the network. Finally, in a very recent work, Zareie and Sakellariou [49] took into account additional features of edges (beyond centrality), such as entropy, to determine what edges to block. Some more results on edge blocking problem are discussed in, [4,47].

Other Countermeasures: Motivated by blocking accounts in real-world online social platforms, the countermeasure of blocking nodes has been widely studied. Various node blocking methods have been investigated in the literature, that use degree centrality, betweenness centrality and closeness centrality as a criterion to block nodes, cf. [16,44,15,7]. In [39], the authors proposed two heuristic algorithms for minimizing the spread of misinformation simulated by the Independent Cascade model via node blocking. Pham et al. [34] studied a variant of the problem with some time and budget constraints. Schneider et al. [37] considered the setup where the sum of the sizes of the connected large clusters in the network is considered as an information flow metric and nodes with high betweenness centrality are suggested to be blocked to minimize the sum of the sizes.

Some other countermeasures have also been considered. For example, the authors of [43,8] studied truth spreading as a misinformation mitigation method, where truth is spread as anti-misinformation. Zehmakan et al. [53] introduced a similar countermeasure, where a subset of nodes is selected to be "fact-checkers" whose role is to trigger the spread of truth once exposed to a piece of misinformation. See [11,53] for more examples of countermeasures.

2 Problem Formulation and Hardness Result

In this section, we aim to show that the problem of minimizing the spread of misinformation via blocking edges is computationally hard. It is worth emphasizing that while this problem has been studied extensively by prior work and several approximation algorithms have been put forward, cf. [48,40,26,49], this work is the first to analyze the computational complexity of this problem rigorously. Let us first provide a more concrete formulation of the problem.

EDGE BLOCKING PROBLEM.

Input: A weighted graph $G = (V_G, E_G, \omega)$, an integer k, a random distribution

over all possible colorings.

Output: The maximum expected final number of white nodes when k edges are blocked (i.e., removed), starting from an initial coloring C_0 chosen from the given distribution and following the Independent Cascade model.

Our hardness result is provided in Theorem 2, building on the DENSEST SUBGRAPH PROBLEM [29]. We first need to provide some basic definitions and lemmas.

DENSEST SUBGRAPH PROBLEM

Input: A connected undirected graph $H = (V_H, E_H)$ and an integer $k < |V_H|$. **Output**: The maximum number of edges in a subgraph induced by k nodes in H.

Theorem 1 ([29]). The DENSEST SUBGRAPH PROBLEM is NP-hard.

Remark. Note that to be precise, when talking about NP-hardness, we need to refer to the decision variant of the problem, where an integer a is also given as input and the problem is to determine whether there is a subgraph induced by k nodes which has a edges.

Lemma 1. The DENSEST SUBGRAPH PROBLEM is polynomial-time solvable if k is smaller than the girth of H.

Proof. Let $OPT_{DS}(H, k)$ be the optimal solution for the DENSEST SUBGRAPH PROBLEM for input H and k. Consider an arbitrary set of nodes of size k. The induced subgraph by this set contains at most k - 1 nodes. This is true because otherwise, the graph has a cycle of length k or smaller which is in contradiction with the assumption of the theorem. Therefore, we have $OPT_{DS}(H, k) \leq k - 1$. On the other hand, any set of k nodes which induces a connected subgraph in khas at least k - 1 edges, which implies $OPT_{DS}(H, k) \geq k - 1$. Hence, we conclude that $OPT_{DS}(H, k) = k - 1$. We can check whether the condition of the lemma is satisfied in polynomial time and return k - 1 as the answer. □

Definition 1 (Transformer). Consider a connected undirected graph $H = (V_H, E_H)$, where $V_H := \{v_1, \dots, v_{n_H}\}$ and $E_H := \{e_1, \dots, e_{m_H}\}$. Construct graph $G = (V_G, E_G, \omega)$ as follows:

 $\begin{aligned} - V_G &:= X \cup Y \cup \{z\} \text{ where } X &:= \{x_1, \cdots, x_{n_H}\} \text{ and } Y &:= \{y_1, \cdots, y_{m_H}\}.\\ - E_G &:= \{(y_j, x_i) : v_i \in e_j\} \cup \{(x_i, z) : 1 \le i \le n_H\}.\\ - \omega(e) &= 1 \text{ for } e \in E_G. \end{aligned}$

Theorem 2. The EDGE BLOCKING PROBLEM is NP-hard, even when all edges have weight 1.

Proof. The proof builds on a reduction from the DENSEST SUBGRAPH PROB-LEM. Assume that there is a polynomial-time algorithm \mathcal{A} for the EDGE BLOCK-ING PROBLEM (when all edge weights are 1). Let $H = (V_H, E_H)$, for $V_H :=$



Fig. 1. An example graph H and the obtained graph G after applying Transformer (from Definition 1).

 $\{v_1, \dots, v_{n_H}\}$ and $E_H := \{e_1, \dots, e_{m_H}\}$, and k be the input of the DENSEST SUBGRAPH PROBLEM. If k is smaller than the girth of H, then we can solve the problem in polynomial time according to Lemma 1. Otherwise, we use Transformer from Definition 1 to build graph G from H. Furthermore, consider the coloring \mathcal{C}_0 where node z is colored red and the rest of nodes are white, see Fig. 1 for an example. We define the random distribution to pick this coloring with probability 1. Let $OPT_{DS}(H, k)$ be the optimal solution to the DENSEST SUBGRAPH PROBLEM for the input H and k and $OPT_{EB}(G, k, \mathcal{C}_0)$ be the optimal solution of the EDGE BLOCKING PROBLEM for the input G, k, and \mathcal{C}_0 . We prove that

$$OPT_{DS}(H,k) = OPT_{EB}(G,k,\mathcal{C}_0) - k.$$
(2)

Note that the Transformer generates G from H in polynomial time. Thus, there is an algorithm \mathcal{A}' for the DENSEST SUBGRAPH PROBLEM which first executes Transformer. Then, it runs the algorithm \mathcal{A} to compute $OPT_{EB}(G, k, \mathcal{C}_0)$ and subtracts it by k to obtain $OPT_{DS}(H, k)$ (using Equation (2)). This algorithm clearly runs in polynomial time. This implies that the EDGE BLOCKING PROBLEM is NP-hard based on Theorem 1.

It remains to prove that Equation (2) holds. Let a set $S \subset V_H$ of size k induce a subgraph with $OPT_{DS}(H, k)$ edges. Define $X_S := \{x_i : v_i \in S\}$, which is of size k. If we block the edges between nodes in X_S and z (i.e., $\{(x_i, z) : x_i \in X_S\}$), then all nodes in $Y_S := \{y_j : e_j = \{v_{i_1}, v_{i_2}\}$ for $v_{i_1}, v_{i_2} \in S\}$ remain white because they become disconnected from node z (the only node which is red in C_0). Since $|Y_S| = OPT_{DS}(H, k)$, we have $OPT_{DS}(H, k) + k \leq OPT_{EB}(G, k, C_0)$. (We added k since nodes in X_S also remain white.)

Now, we prove that $OPT_{DS}(H,k) \geq OPT_{EB}(G,k,\mathcal{C}_0) - k$. Since in G all edges have weight 1, all nodes, except the ones which cannot reach z, become red and then orange after at most three rounds. Let E_{XZ} be the set of edges from nodes in X to z and E_{YX} be the set of edges from Y to X. We claim that there is an optimal solution which only blocks edges in E_{XZ} . Let set S_1

be an optimal solution (i.e., blocking edges in S_1 makes $OPT_{EB}(G, k, C_0)$ nodes remain white forever) and $e_i \in S_1$ for some $e_i \in E_{YX}$.

Note that k is at least as large as the girth of H (since we already excluded the other case). This implies that $OPT_{DS}(H,k) \ge k$ since a connected subgraph including the smallest cycle induces at least k edges. Since we already proved that $OPT_{DS}(H,k) \le OPT_{EB}(G,k,\mathcal{C}_0)-k$, we get $2k \le OPT_{EB}(G,k,\mathcal{C}_0)$. Then, there exists a maximal subset $D \subset S_1 \cap E_{XZ}$ which covers at least |D| nodes in Y. We say a node y_j is covered if both its neighbors in X are disconnected from z by blocking edges in D. This is true because otherwise $S_1 \cap E_{XZ}$ covers at most $|S_1 \cap E_{XZ}| - 1$ nodes in Y. In addition to these nodes, the only nodes which could remain white are the nodes in $\{w : (w, w') \in S_1 \text{ for some } w'\}$ whose size is trivially at most $|S_1|$. Thus, at most $|S_1| + |S_1 \cap E_{XZ}| - 1 \le 2|S_1| = 2k - 1$ nodes remain white, which is a contradiction since we argued that the optimal solution is at least 2k. This implies that there is such a node set D.

Define $D' := \{v_i : (x_i, z) \in D\}$ and $S'_1 := \{v_i : (x_i, z) \in S_1\}$. Let v be a node in $V_H \setminus S'_1$, which has an edge e into D'. (Such a node must exist since k < nand we defined D to be maximal.) Assume that x is the node corresponding to v in X and y is the node corresponding to e in Y. If an edge from y to X is in S_1 , remove it, otherwise remove another edge in $S_1 \cap E_{YX}$ which must exist by assumption, and instead add (x, z) to obtain S_2 . Since node y will be covered (and will remain white), the solution of S_2 is at least as large as the one from S_1 . Thus, there exists an optimal solution S of size k such that $S \cap E_{YX} = \emptyset$. This means that there are $OPT_{EB}(G, k, C_0) - k$ nodes in Y which remain white (i.e., the edges between z and both their neighbors are in S). Note that we subtracted by k since the k nodes in X whose edge to z is removed remain white too.

Let us define $S_H := \{v_i : (x_i, z) \in S\}$. Then, S_H induces a subgraph with $OPT_{EB}(G, k, \mathcal{C}_0) - k$ edges. This implies that $OPT_{DS}(H, k) \ge OPT_{EB}(G, k, \mathcal{C}_0) - k$.

Remark. Note that when all edges have weight 1, the problem of finding the final expected number of orange nodes in the IC model for a given graph G and coloring C_0 is equivalent to a reachability problem, which can be solved in polynomial time, while when any edge weight is allowed, the problem is known to be #P-hard [19]. We proved that the EDGE BLOCKING PROBLEM is NP-hard even when all edge weights are 1. Thus, the hardness comes from the choice of k edges rather than the IC model.

3 Proposed Algorithm

A community refers to a subset of nodes within a graph that exhibits a higher degree of interconnectedness than the rest of the network. Communities are often characterized by a greater density of edges between nodes within the community compared to edges connecting nodes between different communities. Identifying communities within social networks can provide valuable insights into the structure and dynamics of the network.

There are several community detection algorithms in the literature. Some of the most popular ones are the Louvain [2], Leiden [42], Surprise [1], and Walktrap [5] algorithm. We rely on Louvain algorithm for community detection, which works by iteratively optimizing the modularity of a network, which is a measure of how well the nodes in a network are grouped into communities. The Louvain algorithm is fast and scalable, and it has been shown to be effective in detecting communities in a variety of networks. Our algorithm uses this algorithm to first find a set of communities such that the number of inter-community edges is at most k, the budget for the number of edges to be blocked. Then, we simply block all these edges.

The Louvain algorithm receives a graph G and a resolution parameter r. The value of r controls the number of communities (and consequently, the number of inter-community edges) the algorithm will output. Our goal is to generate a set of communities such that the number of inter-community edges is smaller than k but as close as possible to it.

To achieve this, we employ a multi-step process, which is described in Algorithm 1. This essentially follows a hit-and-trial process by updating the resolution parameter and re-running the Louvain algorithm. In addition to graph G and budget k, it also receives an initial resolution parameter r, two repetition parameters h_1 and h_2 , and an increasing factor f > 1. It initially sets $S = \emptyset$ and count = 0. Then, it runs in a while loop until count is larger than the number of repetitions h_1 . Inside this, it first runs a **for** loop for h_2 times. Each time, it runs the Louvain algorithm and finds the inter-community edges. Then, for each of these edge sets \mathcal{E} , if the size of \mathcal{E} is smaller than k, but larger than current S, then we update $S = \mathcal{E}$. This way, the size of S gets closer to the budget k, but it does not exceed it. Note that we run the **for** loop h_2 times, since the Louvain algorithm is nondeterministic. Once the **for** loop is over, we update the resolution factor to r = r * f, where f is the increasing factor. Furthermore, if $|\mathcal{E}| > k$, we increment *count*. Note that at the beginning, *count* might remain zero until r is large enough such that \mathcal{E} (the number of inter-community edges) becomes large. Then, *count* will increase until it exceeds h_1 and then the **while** loop is over. We then return the set S.

4 Evaluation

4.1 Experimental Setup

Social Networks. For our experiments, we use three subgraphs of Facebook, namely Facebook from SNAP dataset [27] and Facebook-Politician and Facebook-Govt from Network Repository [36]. Some graph properties of these networks are listed in the table below.

Edge Weights. Most real-world networks are unweighted, and one needs to introduce a meaningful procedure for weight assignment. Using the communication information of individuals on various real-world networks, the authors in [32,13]

Algorithm 1 Pseudocode for our proposed algorithm

Input: $G(V, E, \omega)$, Resolution r, Increasing Factor f, Repetitions h_1 and h_2 , and Budget k

Output: Set of edges S of size at most k to be blocked.

1:	procedure Algorithm (G, r, f, h_1, h_2, k)
2:	$\mathrm{S}=\emptyset$
3:	$\mathrm{count}=0$
4:	while $(count \le h_1)$ do
5:	for i from 1 to $h_2 \ {f do}$
6:	$\mathbf{C}=$ set of communities returned by the Louvain algorithm for G,r
7:	$\mathcal{E} = ext{set} ext{ of inter-community edges for } C$
8:	$\mathbf{if} \ \mathcal{E} > S \ \mathrm{and} \ \mathcal{E} <= k \ \mathbf{then}$
9:	$S = \mathcal{E}$
10:	end if
11:	end for
12:	update $r = r * f$
13:	$\mathbf{if} \ \mathcal{E} > k \ \mathbf{then}$
14:	$\operatorname{count}++$
15:	end if
16:	end while
17:	return S
18:	end procedure

Table 1. Some characteristics of the networks used in the experiments, including average degree d_{avg} , maximum degree d_{max} , diameter D, average clustering coefficient K_{avg} , and number of triangles T.

Network	n	m	d_{avg}	d_{max}	D	K_{avg}	Т
Facebook	4039	88234	43.691	1045	8	0.6055	1612010
Facebook-Govt	7057	89429	25.344	697	10	0.410	523854
Facebook-Politician	5908	41706	14.118	323	14	0.6055	174632

observed that there is a strong correlation between the number of shared friends of two individuals and their level of communication. Consequently, they proposed the usage of similarity measures, such as Jaccard-like parameters, to approximate the weights of connections between nodes. This is also aligned with the well-studied strength of weak ties hypothesis [14]. Therefore, we assign the edge weights according to the Jaccard index [18] in our set-up. More precisely, for each edge $(v, u) \in E$, we set $\omega((v, u)) = \frac{|\hat{N}(v) \cap \hat{N}(u)|}{N(v) \cup N(v)}$. We use $|\hat{N}(v) \cap \hat{N}(u)|$ instead of $|N(v) \cap N(u)|$ in the numerator to ensure that the weight of an edge is never equal to zero.

Some of the prior algorithms that we discuss in Section 4.2 rely on a measure of distance between two nodes. Since the edge weights represent the strength of the relations, it is conventional to use their "opposite" form when calculating distance. More precisely, for an edge (v, u), we use $1 - \omega((v, u))$.

Algorithm Parameters. For our algorithm, as discussed in Section 3, we need to set the initial resolution parameter r, the repetitions h_1 and h_2 , and increasing factor f > 1. In our experiments, we set r = 0.01 for Facebook and Facebook-Politician and r = 0.05 for Facebook-Govt, f = 1.05, and $h_1 = h_2 = 5$. Note that the closer f is to 1 and the larger h_1 and h_2 are, the more precise our algorithm would be. There is nothing specifically unique about these choices. They are just some reasonable choices that allow our algorithm to perform well on the datasets used, as will be discussed in Section 4.2.

Containment Factor. As mentioned, the Independent Cascade model serves as a simulation tool to emulate the process of misinformation spreading. Initially, a set R of nodes is red and the rest is white. To measure the effectiveness of an edge blocking algorithm that blocks edges in a set S, we rely on *containment factor*

$$cf = 100 \cdot \frac{\phi(G(V, E, \omega), R) - \phi(G(V, E \setminus S, \omega), R)}{\phi(G(V, E, \omega), R)}.$$
(3)

Here $\phi(G(V, E, \omega), R)$ and $\phi(G(V, E \setminus S, \omega), R)$ denote the expected final number of orange nodes (when initially nodes in R are red) before and after blocking edges in S. (Note that we focus on orange nodes, since all red nodes eventually become orange.) Thus, $\phi(G(V, E, \omega), R)$ is the number of nodes that become orange before blocking any edges, and cf measures what percentage of them will remain white once edges in S are blocked.

Note that maximizing cf is the same as maximizing the final number of white nodes, used in the EDGE BLOCKING PROBLEM. To be consistent with prior work, cf. [49], we use cf in our evaluations to compare the algorithms.

4.2 Comparison of Algorithms

We compare our proposed algorithm against algorithms from prior work.

- **RNDM:** A set of edges is randomly selected to be blocked.
- **HWT:** Edges with the largest weight are blocked.
- **DEG** [19,46]: The edges for which the sum of the degree of their two endpoints are the largest are blocked.
- WDEG: This is the same as DEG, except the weighted degrees (the sum of the weight of adjacent edges for each node) are considered.
- CLO: The edges for which the sum of the closeness of their two endpoints are the largest are blocked.
- WCLO: This is the same as CLO, except the edge weights (their "opposite" actually, as explained in Section 4.1) are considered when calculating closeness.
- **BET** [7]: The edges with the highest betweenness centrality are blocked.
- WBET: The edges with the highest weighted betweenness are blocked.
- PGRK [3,46]: The edges for which the sum of the PageRank centrality of their two endpoints are the largest are blocked.

- IEED [49]: In each iteration, a "critical" edge is determined and blocked from the network. Criticality is determined using nodes' influence and edges' blocking efficiency, weighed using a notion of entropy. (Please refer to [49] for more details on this algorithm.)

For each of our three networks, we select a randomly chosen set R of nodes of size |R| = 0.001n to be red initially (and the rest white). We let the number of blocked edges to range from 0.01m to 0.2m. Then, we compute the containment factor cf for all the algorithms by blocking the corresponding edges and running the Independent Cascade model. For each experiment, we select |R| nodes to be red, and then run the Independent Cascade Model 10 times to obtain the cf for the same set of initial red nodes. We run each of these experiments 10 times for different sets of initial red nodes and report the average value of cf. (The standard deviations are given in Appendix A.)



Fig. 2. The containment factor for different algorithms on Facebook (top-left), Facebook-Govt (top-right), and Facebook-Politician (bottom) networks.

The outcomes of our experiments are provided in Fig. 2. The vertical axis denotes the containment factor of the algorithms, while the horizontal axis is the percentage of edges blocked. As expected, it can be seen that as the percentage of blocked edges increases, the containment factor of the methods increases. We observe that our proposed algorithm consistently outperforms all other algo-

rithms, especially by a significant margin for higher percentages of blocked edges. Our proposed algorithm is followed by BET, WBET, and IEED. The only case where our algorithm does not perform better than the other algorithms is for small percentages of blocked edges on the Facebook-Govt dataset.

5 Conclusion

We studied the problem of mitigating misinformation spreading in social networks using blocking edges. After providing a formal formulation of the problem, we proved that it is NP-hard. Then, we proposed an intuitive community-based algorithm, which first partitions the node set into well-connected communities by leveraging the Louvain algorithm. Then, it blocks the inter-community edges. Through experiments on real-world social networks, we observed that this algorithm, despite its simplicity, consistently and significantly outperforms the prior algorithms.

There are several potential future research avenues. It would be interesting to devise more effective strategies to choose the final resolution parameter in our proposed algorithm such that the number of inter-community edges is as close as possible to the budget k. Furthermore, other community detection algorithms can be explored, rather than the Louvain algorithm. One also might apply a community detection algorithm to devise a *node* blocking strategy. Finally, studying the EDGE BLOCKING PROBLEM where each edge has a given cost would be interesting from both a practical and theoretical perspective.

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A Standard Deviations

The standard deviation obtained for the proposed algorithm in cf for the three datasets is given in Table 2.

Percentage	Facebook	Facebook-Govt	Facebook-Politician
1	13.13838739	0.536237924	13.62691459
2	11.47495844	0.41963079	11.68562978
3	7.922823641	0.406590157	12.87673285
4	7.828210311	3.765703269	11.54790164
5	5.521524447	12.09778772	6.344192445
6	5.794904371	9.049993554	8.917378102
7	4.393543369	10.34596116	6.689110222
8	6.45707192	9.882091322	4.062169098
9	7.01420812	9.639261152	4.078979175
10	4.586620155	11.71557316	4.285124269
11	5.191117306	9.887803037	6.042205631
12	4.300630574	9.524686288	3.913324957
13	3.196539101	9.898218078	2.538393193
14	4.516281164	9.493850407	3.342031983
15	5.215616401	10.76495219	3.927128581
16	5.813759159	4.749856723	4.339572944
17	2.844741933	5.597944464	2.68540086
18	5.075444151	7.42852049	2.974853124
19	3.379384559	5.622314272	3.102571693
20	3.678885097	5.077641732	2.681650404

Table 2. Standard deviation obtained for the proposed algorithm.