Learning Adaptable Risk-Sensitive Policies to Coordinate in Multi-Agent General-Sum Games

Ziyi Liu¹, Yongchun Fang²

NanKai University

¹liuzy@mail.nankai.edu.cn, ²fangyc@nankai.edu.cn

Abstract

In general-sum games, the interaction of selfinterested learning agents commonly leads to socially worse outcomes, such as defect-defect in the iterated stag hunt (ISH). Previous works address this challenge by sharing rewards or shaping their opponents' learning process, which require too strong assumptions. In this paper, we demonstrate that agents trained to optimize expected returns are more likely to choose a safe action that leads to guaranteed but lower rewards. However, there typically exists a risky action that leads to higher rewards in the long run only if agents cooperate, e.g., cooperate-cooperate in ISH. To overcome this, we propose using action value distribution to characterize the decision's risk and corresponding potential payoffs. Specifically, we present Adaptable Risk-Sensitive Policy (ARSP). ARSP learns the distributions over agent's return and estimates a dynamic risk-seeking bonus to discover risky coordination strategies. Furthermore, to avoid overfitting training opponents, ARSP learns an auxiliary opponent modeling task to infer opponents' types and dynamically alter corresponding strategies during execution. Empirically, agents trained via ARSP can achieve stable coordination during training without accessing opponent's rewards or learning process, and can adapt to noncooperative opponents during execution. To the best of our knowledge, it is the first method to learn coordination strategies between agents both in iterated prisoner's dilemma (IPD) and iterated stag hunt (ISH) without shaping opponents or rewards, and can adapt to opponents with distinct strategies during execution. Furthermore, we show that ARSP can be scaled to high-dimensional settings.

1 Introduction

Most existing works in multi-agent reinforcement learning (MARL) focus on the fully cooperative [Foerster *et al.*, 2018a; Rashid *et al.*, 2018; Son *et al.*, 2019; Wang *et al.*, 2020; Qiu *et al.*, 2021] and competitive [Silver *et al.*, 2017; Vinyals *et al.*, 2019] settings. However, these settings only

represent a fraction of potential real-world multi-agent environments. General-sum games, in which multiple self-interested learning agents optimize their own rewards independently and win-win outcomes are only possible through coordination [Matignon *et al.*, 2012], describe many domains such as self-driving cars [Toghi *et al.*, 2021] and human-robot interactions [Shirado and Christakis, 2017].

Coordination is often coupled with risk. There are many scenarios where there exists a safe action that leads to guaranteed but socially and individually lower rewards, and a risky action that leads to higher rewards only if agents cooperate, such as alleviating traffic congestion [Lazar et al., 2021]. This kind of multi-agent coordination problem presents unique challenges that are not presented in single-agent learning [Foerster et al., 2017; Wang et al., 2021], and simply applying MARL algorithms to train self-interested agents typically converge on unconditional mutual defection, which is the globally worst outcome [Tang et al., 2021; Lu et al., 2022].

To avoid such catastrophic outcomes, one set of approaches use explicit reward shaping to force agents to be prosocial, such as by making agents care about rewards of their partners [Peysakhovich and Lerer, 2017b; Tang et al., 2021], which can be viewed as shaping the risk degree of coordination strategies. However, this requires the strong assumption that agents involved are altruistic and can access other agents' reward function. Other works either treat partners as stationary [Papoudakis et al., 2021; Wang et al., 2018] or take into account their learning step in order to shape their policy [Foerster et al., 2017]. By contrast, we focus on general settings where multiple decentralized, separately-controlled, and partially-observable agents interact in the environment and only care about maximizing their own rewards - while the objective is still to increase the probability of coordination. Furthermore, policies learned during training should be adaptable so that the agent is able to dynamically alter its strategies between different modes, e.g., either cooperate or defect, w.r.t. its test-time opponent's behavior.

In this paper, one key insight is that learning from opponent's history behaviors allows the agent to adapt to different opponents during execution. Moreover, given that the other learning agents are non-stationary, decision-making over the agent's return distributions enables the agent to model uncertainties resulting from other agents' behaviors and alter

its risk preference, i.e., from risk-neutral to risk-seeking, to discover coordination strategies. Motivated by the analysis above, we propose ARSP, an Adaptable Risk-Sensitive MARL algorithm and our contributions are summarized as follows:

Leading to stable coordination in decentralized general-sum games. We estimate a dynamic risk-seeking bonus to encourage agents to discover risky coordination strategies. Specifically, the risk-seeking bonus is estimated using a complete distortion risk measure Wang's Transform (WT) [Wang, 2000] and only affects the action selection procedure instead of shaping environment rewards, and decreases throughout training which leading to an unbiased policy.

Adaptable to different opponents during execution. Policies learned independently can overfit other agents' policies in the training phase, failing to adapt to different opponents during execution [Lanctot et al., 2017]. We further propose to train each learning agent with two objectives: a standard Quantile Regression objective [Koenker and Bassett Jr, 1978; Dabney et al., 2018b] and a supervised agent modeling objective which models the behaviors of opponents and affects the intermediate representation learning of the value network. The auxiliary opponent modeling task allows the policy to be influenced by opponent's past behaviors, forcing the intermediate representation to adapt to new opponents.

Evaluating in multi-agent settings. We evaluate ARSP in four different Markov games: Iterated Stag Hunt (ISH) [Wang et al., 2021; Tang et al., 2021], Iterated Prisoners' Dilemma (IPD) [Lu et al., 2022; Foerster et al., 2017; Wang et al., 2018], Monster-Hunt [Tang et al., 2021; Zhou et al., 2021] and Escalation [Tang et al., 2021; Peysakhovich and Lerer, 2017b]. Compared with baseline methods, ARSP agents learn substantially faster, achieves stable coordination during training and can adapt to non-cooperative opponents during execution. Furthermore, we show that our method can be scaled to high-dimensional settings.

2 Related Work

Risk-sensitive and distributional RL. Risk-sensitive policies, which depend upon more than mean of the outcomes, enable agents to handle the intrinsic uncertainties arising from the stochasticity of the environment. In MARL, the intrinsic uncertainties are amplified due to the non-stationarity and partial observability created by other agents that change their policies during the learning procedure [Wang et al., 2022; Papoudakis et al., 2019; Hernandez-Leal et al., 2017]. Distributional RL [Bellemare et al., 2017; Dabney et al., 2018b] provides a new perspective for optimizing policy under different risk preferences within a unified framework [Dabney et al., 2018a; Markowitz et al., 2021]. With distributions of return, it is able to approximate value function under different risk measures, such as Conditional Value at Risk (CVaR) [Rockafellar and Uryasev, 2002; Chow et al., 2015] and WT [Wang, 2000], and thus produces risk-averse or risk-seeking policies. Qiu et al. [Qiu et al., 2021] propose RMIX with the CVaR measure as risk-averse policies. Similar ideas are proposed in LH-IQN [Lyu and Amato, 2018] and DFAC [Sun *et al.*, 2021]. But unlike these works, which focus on the fully cooperative settings and leverage distributional RL to alleviate stochasticity or generate risk-averse policies, our method utilizes return distributions to quantify the decision risk in general-sum games and yields a novel risk-seeking exploration bonus to encourage agents to achieve stable coordination.

Test-time adaptation across different opponents. Many real world scenarios require agents to adapt to different opponents during execution. However, most of existing works focus on learning a fixed and team-dependent policy in fully cooperative settings [Sunehag et al., 2017; Rashid et al., 2018; Son et al., 2019; Qiu et al., 2021; Wang et al., 2020], which can not adapt to slightly altered environments or different opponents during execution. Other works either use the population-based training method to train an adaptable agent [Tang et al., 2021; Lupu et al., 2021; Strouse et al., 2021], or adapt to different opponents under the Tit-for-Tat principle [Wang et al., 2018; Peysakhovich and Lerer, 2017a] in the IPD. Our work is closely related to test-time training methods [Sun et al., 2020; Hansen et al., 2020]. However, they focus on image recognition or single agent policy adaption. Ad hoc teamwork [Stone et al., 2010; Zhang et al., 2020] also requires agents to adapt to new teams, but they focus on cooperative games and has different concerns with us.

Opponent modeling. Our approach to learning adaptable policies can be viewed as a kind of opponent modeling method [Albrecht and Stone, 2018]. Most existing works either focus on modeling opponent's intention [Wang et al., 2013; Raileanu et al., 2018] or exploit opponent learning dynamics [Foerster et al., 2017; Zhang and Lesser, 2010]. We consider a more general setting where agents can not only model opponents during training but can transfer their learned knowledge to different opponents at test-time. Policy reconstruction methods [Raileanu et al., 2018] which make explicit predictions about opponents' actions are similar with our approach. However, instead of predicting the opponent's future actions and converging to a fixed policy, ARSP learns from opponents' past behaviors to infer their strategies and can transfer its learned knowledge to different opponents during execution.

3 Preliminaries

Stochastic games. In this work, we consider multiple self-interested learning agents interact with each other. We model the problem as a Partially-Observable Stochastic Game (POSG) [Shapley, 1953; Hansen $et\ al.$, 2004], which consists of N agents, a state space $\mathcal S$ describing the possible configurations of all agents, a set of actions $\mathcal A^1,\ldots,\mathcal A^N$ and a set of observations $\mathcal O^1,\ldots,\mathcal O^N$ for each agent. At each time step, each agent i receives its own observation $o^i\in\mathcal O^i$, and selects an action $a^i\in\mathcal A^i$ based on a stochastic policy $\pi^i:\mathcal O^i\times\mathcal A^i\mapsto [0,1]$, which results in a joint action vector a. The environment then produces a new state s' based on the transition function P(s'|s,a). Each agent i obtains rewards as a function of the state and its action $R^i:\mathcal S\times\mathcal A^i\mapsto\mathbb R$. The initial states are determined by the

distribution $\rho: \mathcal{S} \mapsto [0,1]$. We treat the reward "function" R^i of each agent as a random variable to emphasize its stochasticity, and use $Z^{\pi^i}(s,a^i) = \sum_{t=0}^T \gamma^t R^i(s_t,a^i_t)$ to denote the random variable of the cumulative discounted rewards where $S_0 = s$, $A^i_0 = a^i, \gamma$ is a discount factor and T is the time horizon.

Distorted expectation. Distorted expectation is a risk weighted expectation of value distribution under a specific distortion function [Wirch and Hardy, 2001]. A function $g:[0,1]\mapsto [0,1]$ is a distortion function if it is non-decreasing and satisfies g(0)=0 and g(1)=1 [Balbás et al., 2009]. The distorted expectation of Z under g is defined as $\Psi(Z)=\int_0^1 F_Z^{-1}(\tau)dg(\tau)=\int_0^1 g'(\tau)F_Z^{-1}(\tau)d\tau$, where F_Z^{-1} is the quantile function at $\tau\in[0,1]$ for the random variable Z. We introduce two common distortion functions as follows:

- CVaR is the expectation of the lower or upper tail of the value distribution, corresponding to risk-averse or risk-seeking policy respectively. Its distortion function is $g(\tau) = \min(\tau/\alpha, 1)$ (risk-averse) or $\max(0, 1 (1 \tau)/\alpha)$ (risk-seeking), $\alpha \in (0, 1)$ denotes confidence level.
- WT is proposed by Wang [Wang, 2000]: $g_{\lambda}(\tau) = \Phi\left(\Phi^{-1}(\tau) + \lambda\right)$, where Φ is the distribution of a standard normal. The parameter λ is called the market price of risk and reflects systematic risk. $\lambda > 0$ for risk-averse and $\lambda < 0$ for risk-seeking.

 CVaR_α assigns a 0-value to all percentiles below the α or above $1-\alpha$ significance level which leads to erroneous decisions in some cases [Balbás $et~al.,\,2009$]. Instead, WT is a complete distortion risk measure and ensures using all the information in the original loss distribution which makes training much more stable, and we will empirically demonstrate it in sec. 5.

4 Methods

In this section, we describe our proposed Adaptable Risk-Sensitive Policy (ARSP) method. We first introduce the motivation behind our risk-seeking bonus in sec. 4.1 and then clarify the mathematical formulation of the bonus in sec. 4.2. Furthermore, we propose the auxiliary opponent modeling task to learn adaptable policies in sec. 4.3. The details of test-time policy adaptation under different opponents are present in sec. 4.4.

4.1 Motivation

Let's consider a classical matrix game in the game theory: Stag Hunt (SH). Two players choose either Stag action or Hare action at the same time. If both agents choose to hunt stag, they receive the highest payoff a. If they both hunt hare, each of them will receive d reward. However, if one player choose to hunt stag and the other hunt hare, the player who hunt stag will receive the lowest payoff c or even be punished, and the player who hunt hare will receive a guaranteed reward b. Table 1 models this situation. There exists two pure strategy Nash Equilibrium (NE): (Stag, Stag) and (Hare,

Hare). The Stag NE is the only pareto optimal NE, but it is risky since if the other agent defects, the agent will receive a big loss c, e.g., c=-10. The following theorem similar to [Tang *et al.*, 2021] shows if agents compute the expected payoff function's gradient to update its policy parameters , then the probability the strategies converge to the Stag NE via policy gradient is very low.

Table 1: The stag-hunt game, $a > b \ge d > c$.

	Stag	Hare
Stag	a, a	c, b
Hare	b, c	d, d

Theorem 1. Suppose $a-b=\epsilon(d-c)$ for some $0<\epsilon<1$ and each play has its own policy $\pi_i(\theta_i)$. $P[\pi_i(\theta_i)=S]=\theta_i$ and $P[\pi_i(\theta_i)=H]=1-\theta_i$. If initialize $\theta_1,\theta_2\sim Unif[0,1]$, then the probability that agents discover the pareto optimal NE via policy gradient is $\frac{\epsilon^2}{\epsilon^2+2\epsilon+1}$

We provide the proof in the appendix. Theorem 1 shows that if the risk is high, i.e., c is low, then the ϵ is low and the probability of converging to Stag NE via policy gradient is very low. It is noteworthy that the expectation ignores the complete information of agent's payoff distribution, e.g., the upper and lower tail information when the return distribution is asymmetric.

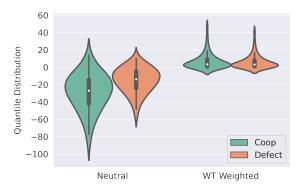
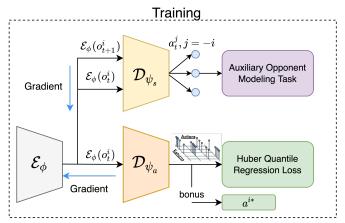


Figure 1: Quantile value distribution of cooperation and defection in Iterated Stag Hunt weighted by WT compared with risk-neutral policy.

Fig.1 left part shows the quantile value distribution of hunting stag (cooperation, Coop) and hunting hare (defection, Defect) in the Iterated Stag Hunt (ISH) learned by a risk-neutral policy. We repeat the stag hunt matrix game ten times in the ISH. The return expectation of Defect is higher than that of Coop, but the Coop distribution has a longer upper tail which means that it has a higher potential payoff. However, the expectation unable to express this information. Fig.1 right part shows the WT weighted quantile distribution learned by a risk-seeking policy. The risk-seeking policy gives the upper tail higher weight when computing distorted expectations, so the agent is more tolerant of the risk and pays more attention to the potential payoff.



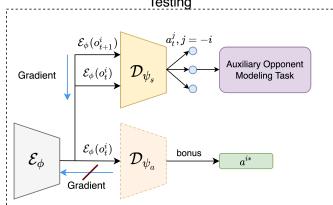


Figure 2: **Left:** Diagram of ARSP architecture during training. Outputs of \mathcal{E}_{ϕ} are fed into \mathcal{D}_{ψ_a} and \mathcal{D}_{ψ_s} , so features are shared between policy and auxiliary opponent modeling. The prediction head \mathcal{D}_{ψ_s} outputs other agents' actions. **Right:** Test-Time policy adaptation. The agent can not receive environment rewards during testing, so we only optimize the auxiliary opponent modeling objective.

4.2 Risk-Seeking Bonus

Based on analysis above, we propose to use WT distortion function to reweight the expectation of quantile distribution. By following [Dabney *et al.*, 2018b], we first represent the return distribution of each agent i with policy π^i by a uniform mix of M supporting quantiles:

$$Z_{\theta}^{\pi^{i}}(o^{i}, a^{i}) \doteq \frac{1}{M} \sum_{k=1}^{M} \delta_{\theta_{k}^{\pi^{i}}(o^{i}, a^{i})}$$
 (1)

where δ_x denotes a Dirac Delta function at $x \in \mathbb{R}$, and each $\theta_k^{\pi^i}$ is an estimation of the quantile corresponding to the quantile fractions $\hat{\tau}_k \doteq \frac{\tau_{k-1} + \tau_k}{2}$ with $\tau_k \doteq \frac{k}{M}$ for $0 \leq k \leq M$. The state-action value $Q^{\pi^i}(o^i, a^i)$ can then be approximated by $\frac{1}{M} \sum_{k=1}^M \theta_k^{\pi^i}(o^i, a^i)$.

Furthermore, we propose the risk-seeking bonus Ψ defined as:

$$\Psi(Z_{\theta}^{\pi^{i}}) = \int_{0}^{1} g_{\lambda}'(\tau) F_{Z_{\theta}^{\pi^{i}}}^{-1}(\tau) d\tau \approx \frac{1}{M} \sum_{k=1}^{M} g_{\lambda}'(\hat{\tau}_{k}) \theta_{k}^{i}, \quad (2)$$

where $g'_{\lambda}(\tau)$ is the derivatives of WT distortion function at $\tau \in [0,1]$, and λ controls the risk-seeking level. Fig.1 right part shows the WT weighted quantile distribution in which the upper quantile values are multiplied by bigger weights and lower quantile values are multiplied by smaller weights to encourage agents to discover risky coordination strategies.

A naive approach to exploration would be to use the variance of the estimated distribution as a bonus. [Mavrin et al., 2019] shows that the exploration bonus from truncated variance outperforms bonus from the variance. Specifically, the Right Truncated Variance tells about lower tail variability and the Left Truncated Variance tells about upper tail variability. For instantiating optimism in the face of uncertainty, the upper tail variability is more relevant than the lower tail, especially if the estimated distribution is asymmetric. So we adopt the Left Truncated Variance of quantile distribution to

further leverage the intrinsic uncertainty for efficient exploration. The left truncated variance is defined as

$$\sigma_{+}^{2} = \frac{1}{2M} \sum_{j=\frac{M}{2}}^{M} \left(\theta_{\frac{M}{2}} - \theta_{j}\right)^{2}, \tag{3}$$

and analysed in [Mavrin et~al., 2019]. The index starts from the median, i.e., M/2, rather than the mean due to its well-known statistical robustness [Huber, 2011; Rousseeuw et~al., 2011]. We anneal the two exploration bonuses dynamically so that in the end we produce unbiased policies. The anneal coefficients are defined as $c_{tj} = c_j \sqrt{\frac{\log t}{t}}, j = 1, 2$ which is the parametric uncertainty decay rate [Koenker and Hallock, 2001], and c_j is a constant factor. This approach leads to choosing the action such that

$$a^{i*} = \arg\max_{a^i \in \mathcal{A}^i} \left(Q^{\pi^i}(o^i, a^i) + c_{t1} \Psi(Z^{\pi^i}(o^i, a^i)) + c_{t2} \sqrt{\sigma_+^2(o^i, a^i)} \right)$$
(4)

These quantile estimates are trained using the Huber [Huber, 1992] quantile regression loss. The loss of the quantile value network of each agent i at time step t is then given by

$$\mathcal{J}\left(o_{t}^{i}, a_{t}^{i}, r_{t}^{i}, o_{t+1}^{i}; \theta^{i}\right) = \frac{1}{M} \sum_{k=0}^{M-1} \sum_{i=0}^{M-1} \rho_{\hat{\tau}_{k}}^{\kappa} \left(\delta_{kj}^{ti}\right) \quad (5)$$

where $\delta_{kj}^{ti} \doteq r_t^i + \gamma \theta_j^i \left(o_{t+1}^i, \pi^i \left(o_{t+1}^i\right)\right) - \theta_k^i (o_t^i, a_t^i)$, and $\rho_{\hat{\tau}_k}^{\kappa}(x) \doteq |\hat{\tau}_k - \mathbb{I}\left\{x < 0\right\}| \frac{\mathcal{L}_{\kappa}(x)}{\kappa}$ where \mathbb{I} is the indicator function and $\mathcal{L}_{\kappa}(x)$ is the Huber loss:

$$\mathcal{L}_{\kappa}(x) \doteq \begin{cases} \frac{1}{2}x^2 & \text{if } x \leq \kappa \\ \kappa \left(|x| - \frac{1}{2}\kappa\right) & \text{otherwise} \end{cases}$$
 (6)

4.3 Auxiliary Opponent Modeling Task

In order to alter the agent's strategies under different opponents, we share parameters between policy and auxiliary opponent modeling task. Specifically, we split the Q value network into two parts: feature extractor \mathcal{E}_{ϕ} and decision maker

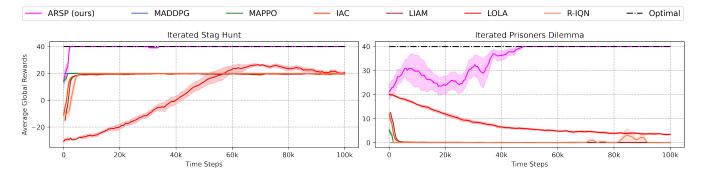


Figure 3: Mean evaluation returns for ARSP, MADDPG, MAPPO, IAC, LIAM and LOLA on two repeated matrix games. The average global rewards equal to 40 means that all agents have learned coordination strategy, i.e., cooperating at each time step.

 \mathcal{D}_{ψ_a} . The parameters of the Q value network Q_{θ^i} for agent i are sequentially divided into ϕ^i and ψ_a^i , i.e., $\theta^i = (\phi^i, \psi_a^i)$. The auxiliary opponent modeling task shares a common feature extractor \mathcal{E}_{ϕ^i} with the value network. We can update the parameters of \mathcal{E}_{ϕ^i} during execution using gradients from the auxiliary opponent modeling task, such that π_{θ^i} can generalize to different opponents. The supervised prediction head and its specific parameters are $\mathcal{D}_{\psi_s^i}$ with ψ_s^i . The details of our network architecture are shown in Fig. 2.

During training, the agent i can collect a set of trajectory transitions $\{(o_t^i,o_{t+1}^i,\mathbf{a}_t^{-i})\}_{t=0}^T$ where \mathbf{a}_t^{-i} indicates the joint actions of other agents except i at time step t. We use the embeddings of agent i's observations o_t^i and o_{t+1}^i to predict the joint actions \mathbf{a}_t^{-i} , i.e., the $\mathcal{D}_{\psi_s^i}$ is a multi-head neural network whose outputs are multiple soft-max distributions over the discrete action space or predicted continuous actions of each other agent, and the objective function of the auxiliary opponent modeling task can be formulated as

$$\mathcal{L}\left(o_{t}^{i}, o_{t+1}^{i}, \mathbf{a}_{t}^{-i}; \phi^{i}, \psi_{s}^{i}\right)$$

$$= \frac{1}{N-1} \sum_{j=1, j \neq i}^{N} \ell\left(a_{t}^{j}, \mathcal{D}_{\psi_{s}^{i}}\left(\mathcal{E}_{\phi^{i}}\left(o_{t}^{i}\right), \mathcal{E}_{\phi^{i}}\left(o_{t+1}^{i}\right)\right)^{j}\right),$$

$$(8)$$

where $\ell(\cdot)$ is the cross-entropy loss function for discrete actions or mean squared error for continuous actions. The strategies of opponents will change constantly during the procedure of multi-agent exploration and thus various strategies will emerge. The agent can leverage them to gain some experience about how to make the best response by jointly optimizing the auxiliary opponent modeling task and quantile value distribution. The joint training problem is therefore

$$\min_{\phi^{i}, \psi_{s}^{i}, \psi_{a}^{i}} \mathcal{J}\left(o_{t}^{i}, a_{t}^{i}, r_{t}^{i}, o_{t+1}^{i}; \phi^{i}, \psi_{a}^{i}\right) + \mathcal{L}\left(o_{t}^{i}, o_{t+1}^{i}, \mathbf{a}_{t}^{-i}; \phi^{i}, \psi_{s}^{i}\right)$$
(9)

4.4 Test-Time Policy Adaptation under Different Opponents

During testing time, we can not optimize \mathcal{J} anymore since the reward is unavailable, but we assume the agent can observe actions made by its opponents during execution, then

we can continue optimizing \mathcal{L} to update the parameters of shared feature extractor \mathcal{E}_{ϕ} . Learning from opponents' past behaviors at test time makes the agent adapt to different opponents efficiently. This can be formulated as

$$\min_{\phi^i, \psi^i_s} \mathcal{L}\left(o^i_t, o^i_{t+1}, \mathbf{a}^{-i}_t; \phi^i, \psi^i_s\right) \tag{10}$$

5 Experimental Setup

5.1 Environments

s Iterated Prisoner's Dilemma. The prisoner's dilemma is one of the most widely-studied and important general-sum games, with applications in evolutionary biology, economics, politics, sociology, and other fields. In the single-shot prisoner's dilemma, agents can choose to cooperate (C) or defect (D) against each other, and there is only one Nash equilibrium [Fudenberg and Tirole, 1991], where both agents defect. The payoff of the result is presented in Table 2. A common extension of the prisoner's dilemma is the IPD, in which the prisoner's dilemma is played repeatedly, with players able to observe their opponent's past decisions. In infinitely IPD, the folk theorem [Myerson, 1997] shows that there are infinitely many Nash equilibria. For example, grim-trigger, in which the agent starts out cooperating and keeps cooperating as long as the other agent cooperates. But as soon as the other agent defects, the agent defect every play after that, and tit for tat (TFT), in which the agent copies the other agent's last move. It is noteworthy that cooperative strategy generally form equilibria with each other: if both agents are playing grim-trigger, then they will keep cooperating and none of them can do any better with a different strategy.

Table 2: Payoff Matrix for the Prisoner's Dilemma.

	C	D	
С	(2, 2)	(-1, 3)	
D	(3, -1)	(0,0)	

Iterated Stag Hunt. Iterated Stag Hunt (ISH) is an extension of Stag Hunt matrix game similar with IPD. The one-step payoff matrix used in our experiments is shown in Table 3. There exist two NEs in ISH - keep cooperation and keep defect. Agents in both IPD and ISH can condition their

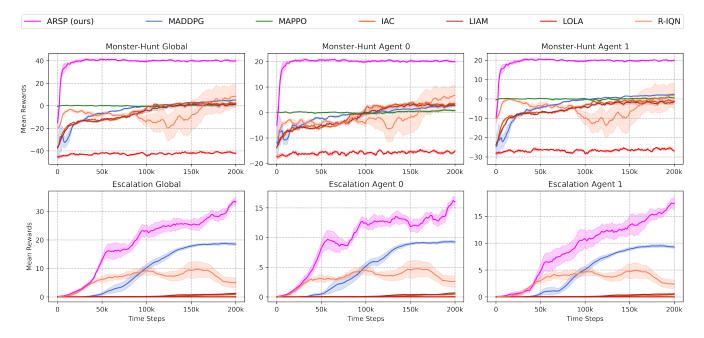


Figure 4: Mean evaluation returns for ARSP, MADDPG, MAPPO, IAC, LIAM and LOLA on Monster-Hunt and Escalation. Global rewards are summation of both agents rewards.

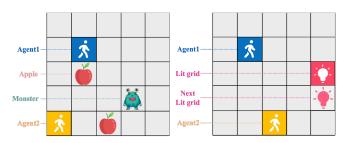


Figure 5: Monster-Hunt and Escalation

actions on past history. Similar like [Foerster *et al.*, 2017; Lu *et al.*, 2022], We consider memory-1 players, i.e., the agents act based on the results of last one rounds.

Table 3: Payoff Matrix for the Stag Hunt.

	C	D	
C	(2, 2)	(-10, 1)	
D	(1, -10)	(1, 1)	

Monster-Hunt (MH). The Monster-Hunt [Tang *et al.*, 2021] is a multi-agent grid-world environment with high dimensional state space. It is a 5×5 grid and consists of two agents, two apples and one monster. The apples are static while the monster keeps moving towards its closest agent. When a single agent meets the monster, it gets a penalty of -10. If two agents catch the monster together, they both get a bonus of 5. If one agent meets an apple, it gets a bonus of 2. Whenever an apple is eaten or the monster meets an agent, the entity will respawn randomly in the grid. The cooperative strategy, i.e., both agents catch the monster together, is risky

since the agent will suffer a big loss if the other agent defect. The episode length is 20 in our experiment.

Escalation. Escalation is a 5×5 grid-world with sparse rewards, consisting of two agents and a static light. If both agents step on the light simultaneously, they receive a bonus of 1, and then the light moves to a random adjacent grid. If only one agent steps on the light, he gets a penalty of $1.5 \times L$, where L denotes the latest consecutive cooperation steps, and the light will respawn randomly. To maximize their individual payoffs, agents must coordinate to stay together and step on the light grid simultaneously. For each integer L, there is a corresponding coordination strategy where each agent follows the light for L steps then simultaneously stop coordination. However, the agent will suffer more losses once the other agent defect with the increase of cooperation steps. The episode length is 30 in our experiment.

5.2 Baseline Comparisons

- Independent Actor-Critic (IAC): IAC is a naive decentralized policy gradient method. Each agent learns policy and value networks based on its local observations and treat other agents as part of the environment.
- Local Information Agent Modelling (LIAM): LIAM [Papoudakis *et al.*, 2021] learns latent representations of the opponent from the ego agent's local information using encoder-decoder architecture. The opponent's observations and actions are utilized as reconstruction targets for the decoder, and the learned latent representation conditions the policy of the ego agent in addition to its local observation. The policy and model are optimized based on A2C algorithm.
- Learning with Opponent Learning Awareness

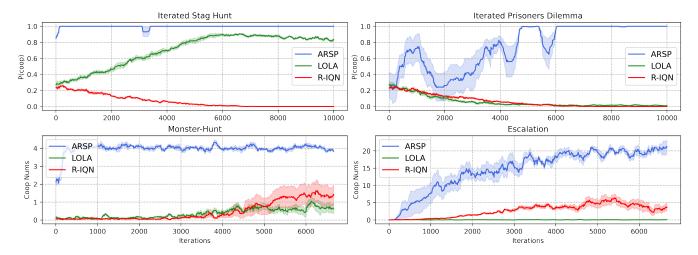


Figure 6: The probability of agents cooperate with each other in ISH and IPD (first row) during training and the number of mutual cooperation steps in one episode in Monster-Hunt and Escalation (second row) during training.

(LOLA): LOLA [Foerster *et al.*, 2017] assumes that other agents are naive learners and considers the learning processes of other agents. LOLA takes a gradient through the opponent's update function to shape the opponent. In the self-play setting, LOLA is one of the first methods to discover the tit-for-tat (TFT) strategy in the IPD.

- Multi-Agent Deep Deterministic Policy Gradient (MADDPG): MADDPG [Lowe *et al.*, 2017] is an extension of DDPG [Lillicrap *et al.*, 2015] methods in MARL where the critic is augmented with extra information about the policies of other agents, and is applicable to mixed cooperative-competitive environments.
- Multi-Agent Proximal Policy Optimization (MAPPO): MAPPO [Yu et al., 2021] is a variant of PPO which is specialized for multi-agent settings. MAPPO learns a centralized critic and adopts a collections of tricks, e.g., input normalization and layer normalization to improve agent's performance.
- Risk-Sensitive Implicit Quantile Networks (R-IQN):
 R-IQN [Dabney et al., 2018a] is a dencentralized risk-sensitive distributional reinforcement learning method. It learns the quantile values for sampled quantile fractions with an implicit quantile value network (IQN) that maps from quantile fractions to quantile values. By sampling quantile fractions τ from different distortion risk measures instead of uniform distribution, agents can be risk-seeking or risk-averse. We use WT as the distortion risk measure to construct risk-seeking agents and compare them with our ARSP agents.

6 Results

In this subsection, we evaluate all methods on four multiagent environments and use 5 different random seeds to evaluate each method. We pause training every 50 episodes and run 30 independent episodes to evaluate the average performance of each method.

6.1 Iterated Games

Fig. 3 shows the average global rewards, i.e., the summation of all agents' average returns, of all methods evaluated during training in ISH and IPD. The shadowed part represents a 95% confidence interval. The average global rewards equal to 40 means that both agents' have discovered coordination strategy, i.e., cooperating at each time step. ARSP vastly outperforms all other learning methods in the IPD and ISH. Notably, it is the only algorithm to achieve the optimal coordination strategy stably in a sample efficient way. LOLA outperforms other baseline methods except ARSP in ISH and IPD. However, LOLA agents are unable to achieve coordination stably. R-IQN is a risk-seeking method similar to ARSP but failed to discover coordination strategies. We suspect that this happens because R-IQN implicitly achieves risk-seeking by sampling quantile fractions from the distorted distribution instead of uniform, whereas ARSP explicitly constructs riskseeking exploration bonus to encourage agents to discover coordination strategies more efficiently. The first row of Fig. 6 shows the probability of agents cooperate with each other in ISH and IPD during training. ARSP agents can converge to coordination strategies efficiently in probability one. To our best knowledge, it is the first method to achieve this result.

6.2 Grid-Worlds

We further show the effectiveness of ARSP in two high dimensional grid-world games - MH and Escalation [Tang *et al.*, 2021]. Both of them have multiple NEs with different payoff.

The first row of Fig. 4 is the evaluation results of ARSP and other baselines in MH. Global rewards are the summation of all agents' returns in the environment. In MH, ARSP agents can rapidly discover the high payoff NE where two agents stay together and wait for the monster. Notably, ARSP agents can stably converge to the coordination strategy in less than 25k time steps, which shows the superiority of ARSP in sample efficiency. However, most other baseline agents can only converge to some guaranteed but lower payoff NEs, i.e.,

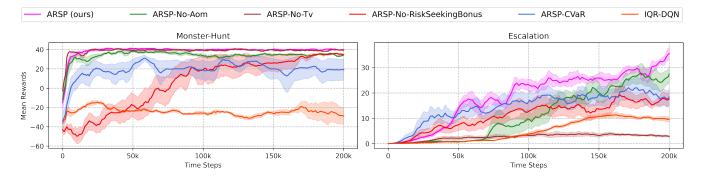


Figure 7: Mean evaluation return of ARSP compared with other ablation methods in two grid-world multi-agent environments.

avoid stag and eat apples alone. LOLA do not achieve significant results in MH, and similar results also appear in [Foerster *et al.*, 2018b]. We suspect that LOLA can not scale to high dimensional settings. R-IQN performs better than other baselines in MH and the second row of Fig 6 shows that R-IQN agents can achieve one or two co-operations in one episode, demonstrating that the risk-seeking policy plays an important role in discovering coordination strategies. Similar results are also achieved in the Escalation environment, where ARSP significantly outperforms other baselines both in asymptotic performance and sample efficiency. Although with the increase of cooperation times, the punishment suffered by agents for betrayal is also increasing in the Escalation, agents can still achieve stable cooperation shown in Fig 6.

6.3 Adaptation Study

This subsection investigates the ability of the pre-trained ARSP agent to adapt to different opponents during execution. The cooperative opponents are trained by ARSP method because ARSP is the only method to produce cooperative strategies, and the non-cooperative opponents are MADDPG agents. During evaluation, random seeds of four environments are different from that during training, and hyperparameters of the ARSP, e.g., risk-seeking level λ , are same and fixed between different opponents. Furthermore, the pretrained agent can no longer use environment rewards to update its policy and it must utilize the auxiliary opponent modeling task to adapt to different opponents. The network details and hyperparameters can be found in the Appendix.

Table 4: Mean evaluation return of ARSP with and without auxiliary opponent modeling task on four multi-agent environments.

	ARSP-No-Aom		ARSP	
	CC	CD	CC	CD
ISH	20	-100	20	0.65
IPD	20	-5	20	-1.08
M-H	20.62	-15.03	21.36	-12.07
Escalation	9.45	-0.545	11.3	0.175

Table 4 shows the mean evaluation return of the ARSP agent with and without auxiliary opponent modeling (Aom) task on four multi-agent environments when interacting

with different opponents. All returns are averaged on 100 episodes. ARSP-No-Aom means that the agent is trained without Aom task. CC indicates the opponent is a cooperative agent while CD means the opponent will defect. In ISH and IPD, the ARSP agent chooses to cooperate at the first time step. After observing its opponent's decision, it will choose to keep cooperating or defecting, depending on its opponent's type. So the ARSP agent can avoid being exploited by its opponent and gets guaranteed rewards, while ARSP-No-Aom can not, as shown in Table 4. Similar results also appear in M-H and Escalation. Furthermore, when interacting with cooperative opponents, ARSP agent can adapt to their policies and receive higher individual rewards than ARSP-No-Aom, e.g., 21.36 in M-H and 11.3 in Escalation. The experiment results also demonstrate that policies learned independently can overfit other agents' policies during training, and our Aom method provides a way to tackle this problem.

6.4 Ablations

In this subsection, we perform an ablation study to examine the components of ARSP to better understand our method. ARSP is based on QR-DQN and has three components: the risk-seeking exploration bonus, the left truncated variance (Tv) and the auxiliary opponent modeling task (Aom). We design and evaluate six different ablations of ARSP in two grid-world environments, as show in Fig. 7. The evaluation return of ARSP-No-Aom which we ablate the Aom module and retain all other features of our method is a little lower than that of ARSP but has a much higher variance, indicating that learning from opponent's behaviors can stable training and improve performance. Moreover, the ARSP-No-Aom is a completely decentralized method whose training without any opponent information, and the ablation results of ARSP-No-Aom indicate that our risk-seeking bonus is the determining factor for agents to achieve coordination strategies in our experiments. We observe that ablating the left truncated variance module leads to a lower return than ARSP in the Escalation but no difference in the Monster-Hunt. Furthermore, ablating the risk-seeking bonus increases the training variance, leads to slower convergence and worse performance. It is noteworthy that the Escalation is a sparse reward and hard-exploration multi-agent environment because two decentralized agents can not get any reward until they navigate to and step on the light simultaneously and constantly. These two ablations indicate that the exploration ability of left truncated variance is important for our method and the risk-seeking bonus can encourage agents to coordinate with each other stably and converge to high-risky cooperation strategies efficiently. We also implement our risk-seeking bonus with CVaR instead of WT, and the results are shown as ARSP-CVaR. The ARSP-CVaR performs worse than our method and has a higher training variance. Finally, we ablate all components of the ARSP and use ϵ -greedy policy for exploration which leads to the IQR-DQN algorithm. As shown in Fig. 7, IQR-DQN can not learn effective policies both in the Monster-Hunt and the Escalation.

7 Conclusion & Future Work

In this paper, we presented Adaptable Risk-Sensitive Policy (ARSP), a novel adaptable risk-sensitive reinforcement learning method for multi-agent general-sum games where winwin outcomes are only achieved through coordination. By estimating the risk-seeking exploration bonus, ARSP agents can efficiently discover risky coordination strategies and converge to high payoff NEs stably. To avoid overfitting training opponents and learn adaptable policies, we propose the auxiliary opponent modeling task, which leverages the opponent's history behaviors to infer its strategies and alter the ego agent's policy dynamically during execution, thus adapting to different opponents.

More specifically, ARSP agents can discover the optimal coordination strategy in both ISH and IPD and converge to it in probability one. To the best of our knowledge, it is the first method to achieve this result. Furthermore, ARSP can also scale to more complex, high-dimensional multi-agent games and achieve similar results in Monster-Hunt and Escalation. We also show that the ARSP agent can efficiently adapt to its opponent's policy, avoid being exploited by non-cooperative opponent and further improve its coordination performance for cooperative opponent.

The risk-seeking bonus in ARSP is estimated using WT distorted expectation and its risk-sensitive level is a hyperparameter. Developing the method that can adjust agent' risk-sensitive level dynamically by utilizing its observation, rewards, or opponents' information is the direction of our future work.

References

- [Albrecht and Stone, 2018] Stefano V Albrecht and Peter Stone. Autonomous agents modelling other agents: A comprehensive survey and open problems. *Artificial Intelligence*, 258:66–95, 2018.
- [Balbás *et al.*, 2009] Alejandro Balbás, José Garrido, and Silvia Mayoral. Properties of distortion risk measures. *Methodology and Computing in Applied Probability*, 11(3):385–399, 2009.
- [Bellemare *et al.*, 2017] Marc G Bellemare, Will Dabney, and Rémi Munos. A distributional perspective on reinforcement learning. In *International Conference on Machine Learning*, pages 449–458. PMLR, 2017.

- [Chow et al., 2015] Yinlam Chow, Aviv Tamar, Shie Mannor, and Marco Pavone. Risk-sensitive and robust decision-making: a cvar optimization approach. Advances in neural information processing systems, 28, 2015.
- [Dabney et al., 2018a] Will Dabney, Georg Ostrovski, David Silver, and Rémi Munos. Implicit quantile networks for distributional reinforcement learning. In *International conference on machine learning*, pages 1096–1105. PMLR, 2018.
- [Dabney et al., 2018b] Will Dabney, Mark Rowland, Marc Bellemare, and Rémi Munos. Distributional reinforcement learning with quantile regression. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 32, 2018.
- [Foerster *et al.*, 2017] Jakob N Foerster, Richard Y Chen, Maruan Al-Shedivat, Shimon Whiteson, Pieter Abbeel, and Igor Mordatch. Learning with opponent-learning awareness. *arXiv preprint arXiv:1709.04326*, 2017.
- [Foerster et al., 2018a] Jakob Foerster, Gregory Farquhar, Triantafyllos Afouras, Nantas Nardelli, and Shimon Whiteson. Counterfactual multi-agent policy gradients. In *Proceedings of the AAAI conference on artificial intelligence*, volume 32, 2018.
- [Foerster et al., 2018b] Jakob Foerster, Gregory Farquhar, Maruan Al-Shedivat, Tim Rocktäschel, Eric Xing, and Shimon Whiteson. Dice: The infinitely differentiable monte carlo estimator. In *International Conference on Machine Learning*, pages 1529–1538. PMLR, 2018.
- [Fudenberg and Tirole, 1991] Drew Fudenberg and Jean Tirole. *Game theory*. MIT press, 1991.
- [Hansen *et al.*, 2004] Eric A Hansen, Daniel S Bernstein, and Shlomo Zilberstein. Dynamic programming for partially observable stochastic games. In *AAAI*, volume 4, pages 709–715, 2004.
- [Hansen *et al.*, 2020] Nicklas Hansen, Rishabh Jangir, Yu Sun, Guillem Alenyà, Pieter Abbeel, Alexei A Efros, Lerrel Pinto, and Xiaolong Wang. Self-supervised policy adaptation during deployment. *arXiv* preprint *arXiv*:2007.04309, 2020.
- [Hernandez-Leal *et al.*, 2017] Pablo Hernandez-Leal, Michael Kaisers, Tim Baarslag, and Enrique Munoz de Cote. A survey of learning in multiagent environments: Dealing with non-stationarity. *arXiv preprint arXiv:1707.09183*, 2017.
- [Huber, 1992] Peter J Huber. Robust estimation of a location parameter. In *Breakthroughs in statistics*, pages 492–518. Springer, 1992.
- [Huber, 2011] Peter J Huber. Robust statistics. In *International encyclopedia of statistical science*, pages 1248–1251. Springer, 2011.
- [Koenker and Bassett Jr, 1978] Roger Koenker and Gilbert Bassett Jr. Regression quantiles. *Econometrica: journal of the Econometric Society*, pages 33–50, 1978.

- [Koenker and Hallock, 2001] Roger Koenker and Kevin F Hallock. Quantile regression. *Journal of economic perspectives*, 15(4):143–156, 2001.
- [Lanctot et al., 2017] Marc Lanctot, Vinicius Zambaldi, Audrunas Gruslys, Angeliki Lazaridou, Karl Tuyls, Julien Pérolat, David Silver, and Thore Graepel. A unified gametheoretic approach to multiagent reinforcement learning. Advances in neural information processing systems, 30, 2017.
- [Lazar et al., 2021] Daniel A Lazar, Erdem Bıyık, Dorsa Sadigh, and Ramtin Pedarsani. Learning how to dynamically route autonomous vehicles on shared roads. Transportation research part C: emerging technologies, 130:103258, 2021.
- [Lillicrap *et al.*, 2015] Timothy P Lillicrap, Jonathan J Hunt, Alexander Pritzel, Nicolas Heess, Tom Erez, Yuval Tassa, David Silver, and Daan Wierstra. Continuous control with deep reinforcement learning. *arXiv preprint arXiv:1509.02971*, 2015.
- [Lowe et al., 2017] Ryan Lowe, Yi I Wu, Aviv Tamar, Jean Harb, OpenAI Pieter Abbeel, and Igor Mordatch. Multiagent actor-critic for mixed cooperative-competitive environments. Advances in neural information processing systems, 30, 2017.
- [Lu et al., 2022] Christopher Lu, Timon Willi, Christian A Schroeder De Witt, and Jakob Foerster. Model-free opponent shaping. In *International Conference on Machine Learning*, pages 14398–14411. PMLR, 2022.
- [Lupu et al., 2021] Andrei Lupu, Brandon Cui, Hengyuan Hu, and Jakob Foerster. Trajectory diversity for zero-shot coordination. In *International Conference on Machine Learning*, pages 7204–7213. PMLR, 2021.
- [Lyu and Amato, 2018] Xueguang Lyu and Christopher Amato. Likelihood quantile networks for coordinating multi-agent reinforcement learning. *arXiv preprint arXiv:1812.06319*, 2018.
- [Markowitz *et al.*, 2021] Jared Markowitz, Ryan Gardner, Ashley Llorens, Raman Arora, and I-Jeng Wang. A risk-sensitive policy gradient method. 2021.
- [Matignon *et al.*, 2012] Laetitia Matignon, Guillaume J Laurent, and Nadine Le Fort-Piat. Independent reinforcement learners in cooperative markov games: a survey regarding coordination problems. *The Knowledge Engineering Review*, 27(1):1–31, 2012.
- [Mavrin *et al.*, 2019] Borislav Mavrin, Hengshuai Yao, Linglong Kong, Kaiwen Wu, and Yaoliang Yu. Distributional reinforcement learning for efficient exploration. In *International conference on machine learning*, pages 4424–4434. PMLR, 2019.
- [Myerson, 1997] Roger B Myerson. *Game theory: analysis of conflict.* Harvard university press, 1997.
- [Papoudakis et al., 2019] Georgios Papoudakis, Filippos Christianos, Arrasy Rahman, and Stefano V Albrecht.

- Dealing with non-stationarity in multi-agent deep reinforcement learning. *arXiv preprint arXiv:1906.04737*, 2019.
- [Papoudakis et al., 2021] Georgios Papoudakis, Filippos Christianos, and Stefano Albrecht. Agent modelling under partial observability for deep reinforcement learning. Advances in Neural Information Processing Systems, 34, 2021.
- [Peysakhovich and Lerer, 2017a] Alexander Peysakhovich and Adam Lerer. Consequentialist conditional cooperation in social dilemmas with imperfect information. *arXiv* preprint arXiv:1710.06975, 2017.
- [Peysakhovich and Lerer, 2017b] Alexander Peysakhovich and Adam Lerer. Prosocial learning agents solve generalized stag hunts better than selfish ones. *arXiv preprint arXiv:1709.02865*, 2017.
- [Qiu et al., 2021] Wei Qiu, Xinrun Wang, Runsheng Yu, Rundong Wang, Xu He, Bo An, Svetlana Obraztsova, and Zinovi Rabinovich. Rmix: Learning risk-sensitive policies forcooperative reinforcement learning agents. Advances in Neural Information Processing Systems, 34, 2021.
- [Raileanu et al., 2018] Roberta Raileanu, Emily Denton, Arthur Szlam, and Rob Fergus. Modeling others using oneself in multi-agent reinforcement learning. In *Interna*tional conference on machine learning, pages 4257–4266. PMLR, 2018.
- [Rashid et al., 2018] Tabish Rashid, Mikayel Samvelyan, Christian Schroeder, Gregory Farquhar, Jakob Foerster, and Shimon Whiteson. Qmix: Monotonic value function factorisation for deep multi-agent reinforcement learning. In *International Conference on Machine Learning*, pages 4295–4304. PMLR, 2018.
- [Rockafellar and Uryasev, 2002] R Tyrrell Rockafellar and Stanislav Uryasev. Conditional value-at-risk for general loss distributions. *Journal of banking & finance*, 26(7):1443–1471, 2002.
- [Rousseeuw et al., 2011] Peter J Rousseeuw, Frank R Hampel, Elvezio M Ronchetti, and Werner A Stahel. Robust statistics: the approach based on influence functions. John Wiley & Sons, 2011.
- [Shapley, 1953] Lloyd S Shapley. Stochastic games. *Proceedings of the national academy of sciences*, 39(10):1095–1100, 1953.
- [Shirado and Christakis, 2017] Hirokazu Shirado and Nicholas A Christakis. Locally noisy autonomous agents improve global human coordination in network experiments. *Nature*, 545(7654):370–374, 2017.
- [Silver *et al.*, 2017] David Silver, Julian Schrittwieser, Karen Simonyan, Ioannis Antonoglou, Aja Huang, Arthur Guez, Thomas Hubert, Lucas Baker, Matthew Lai, Adrian Bolton, et al. Mastering the game of go without human knowledge. *nature*, 550(7676):354–359, 2017.
- [Son *et al.*, 2019] Kyunghwan Son, Daewoo Kim, Wan Ju Kang, David Earl Hostallero, and Yung Yi. Qtran: Learning to factorize with transformation for cooperative multi-

- agent reinforcement learning. In *International Conference* on *Machine Learning*, pages 5887–5896. PMLR, 2019.
- [Stone et al., 2010] Peter Stone, Gal A Kaminka, Sarit Kraus, and Jeffrey S Rosenschein. Ad hoc autonomous agent teams: Collaboration without pre-coordination. In *Twenty-Fourth AAAI Conference on Artificial Intelligence*, 2010.
- [Strouse *et al.*, 2021] DJ Strouse, Kevin McKee, Matt Botvinick, Edward Hughes, and Richard Everett. Collaborating with humans without human data. *Advances in Neural Information Processing Systems*, 34:14502–14515, 2021.
- [Sun *et al.*, 2020] Yu Sun, Xiaolong Wang, Zhuang Liu, John Miller, Alexei Efros, and Moritz Hardt. Test-time training with self-supervision for generalization under distribution shifts. In *International Conference on Machine Learning*, pages 9229–9248. PMLR, 2020.
- [Sun et al., 2021] Wei-Fang Sun, Cheng-Kuang Lee, and Chun-Yi Lee. Dfac framework: Factorizing the value function via quantile mixture for multi-agent distributional q-learning. In *International Conference on Machine Learning*, pages 9945–9954. PMLR, 2021.
- [Sunehag et al., 2017] Peter Sunehag, Guy Lever, Audrunas Gruslys, Wojciech Marian Czarnecki, Vinicius Zambaldi, Max Jaderberg, Marc Lanctot, Nicolas Sonnerat, Joel Z Leibo, Karl Tuyls, et al. Value-decomposition networks for cooperative multi-agent learning. arXiv preprint arXiv:1706.05296, 2017.
- [Tang et al., 2021] Zhenggang Tang, Chao Yu, Boyuan Chen, Huazhe Xu, Xiaolong Wang, Fei Fang, Simon Du, Yu Wang, and Yi Wu. Discovering diverse multiagent strategic behavior via reward randomization. arXiv preprint arXiv:2103.04564, 2021.
- [Toghi *et al.*, 2021] Behrad Toghi, Rodolfo Valiente, Dorsa Sadigh, Ramtin Pedarsani, and Yaser P Fallah. Social coordination and altruism in autonomous driving. *arXiv* preprint arXiv:2107.00200, 2021.
- [Vinyals *et al.*, 2019] Oriol Vinyals, Igor Babuschkin, Wojciech M Czarnecki, Michaël Mathieu, Andrew Dudzik, Junyoung Chung, David H Choi, Richard Powell, Timo Ewalds, Petko Georgiev, et al. Grandmaster level in starcraft ii using multi-agent reinforcement learning. *Nature*, 575(7782):350–354, 2019.
- [Wang et al., 2013] Zhikun Wang, Katharina Mülling, Marc Peter Deisenroth, Heni Ben Amor, David Vogt, Bernhard Schölkopf, and Jan Peters. Probabilistic movement modeling for intention inference in human–robot interaction. *The International Journal of Robotics Research*, 32(7):841–858, 2013.
- [Wang et al., 2018] Weixun Wang, Jianye Hao, Yixi Wang, and Matthew Taylor. Towards cooperation in sequential prisoner's dilemmas: a deep multiagent reinforcement learning approach. arXiv preprint arXiv:1803.00162, 2018.

- [Wang et al., 2020] Jianhao Wang, Zhizhou Ren, Terry Liu, Yang Yu, and Chongjie Zhang. Qplex: Duplex dueling multi-agent q-learning. arXiv preprint arXiv:2008.01062, 2020.
- [Wang et al., 2021] Woodrow Z Wang, Mark Beliaev, Erdem Bıyık, Daniel A Lazar, Ramtin Pedarsani, and Dorsa Sadigh. Emergent prosociality in multi-agent games through gifting. arXiv preprint arXiv:2105.06593, 2021.
- [Wang et al., 2022] Woodrow Zhouyuan Wang, Andy Shih, Annie Xie, and Dorsa Sadigh. Influencing towards stable multi-agent interactions. In Conference on Robot Learning, pages 1132–1143. PMLR, 2022.
- [Wang, 2000] Shaun S Wang. A class of distortion operators for pricing financial and insurance risks. *Journal of risk and insurance*, pages 15–36, 2000.
- [Wirch and Hardy, 2001] Julia L Wirch and Mary R Hardy. Distortion risk measures: Coherence and stochastic dominance. In *International congress on insurance: Mathematics and economics*, pages 15–17, 2001.
- [Yu et al., 2021] Chao Yu, Akash Velu, Eugene Vinitsky, Yu Wang, Alexandre Bayen, and Yi Wu. The surprising effectiveness of ppo in cooperative, multi-agent games. arXiv preprint arXiv:2103.01955, 2021.
- [Zhang and Lesser, 2010] Chongjie Zhang and Victor Lesser. Multi-agent learning with policy prediction. In *Twenty-fourth AAAI conference on artificial intelligence*, 2010.
- [Zhang et al., 2020] Tianjun Zhang, Huazhe Xu, Xiaolong Wang, Yi Wu, Kurt Keutzer, Joseph E Gonzalez, and Yuandong Tian. Multi-agent collaboration via reward attribution decomposition. arXiv preprint arXiv:2010.08531, 2020.
- [Zhou et al., 2021] Zihan Zhou, Wei Fu, Bingliang Zhang, and Yi Wu. Continuously discovering novel strategies via reward-switching policy optimization. In *Deep RL Workshop NeurIPS 2021*, 2021.