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Transactions on Rough Sets II

Rough Sets and Fuzzy Sets

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LNCS Transactions on Rough Sets

This journal subline has as its principal aim the fostering of professional exchanges between scientists and practitioners who are interested in the foundations and applications of rough sets. Topics include foundations and applications of rough sets as well as foundations and applications of hybrid methods combining rough sets with other approaches important for the development of intelligent systems.

The journal includes high-quality research articles accepted for publication on the basis of thorough peer reviews. Dissertations and monographs up to 250 pages that include new research results can also be considered as regular papers. Extended and revised versions of selected papers from conferences can also be included in regular or special issues of the journal.

Preface

This collection of articles is devoted to fuzzy as well as rough set theories. Both theories are based on rigorous ideas, methods and techniques in logic, mathematics, and computer science for treating problems for which approximate solutions are possible only, due to their inherent ambiguity, vagueness, incompleteness, etc. Vast areas of decision making, data mining, knowledge discovery in data, approximate reasoning, etc., are successfully explored using methods worked out within fuzzy and rough paradigms.

By the very nature of fuzzy and rough paradigms, outlined above, they are related to distinct logical schemes: it is well-known that rough sets are related to modal logics $S5$ and $S4$ (Orłowska, E., Modal logics in the theory of information systems, *Z. Math. Logik Grund. Math.* 30, 1984, pp. 213 ff.; Vakarelov, D., Modal logics for knowledge representation systems, *LNCS* 363, 1989, pp. 257 ff.) and to finitely-valued logics (Pagliani, P., Rough set theory and logic-algebraic structures. In *Incomplete Information: Rough Set Analysis*, Orłowska, E., ed., Physica/Springer, 1998, pp. 109 ff.; Polkowski, L. A note on 3-valued rough logic accepting decision rules, *Fundamenta Informaticae* 61, to appear).

Fuzzy sets are related to infinitely-valued logics (fuzzy membership to degree $r \in [0, 1]$ expressing truth degree r) (Goguen, J.A., The logic of inexact concepts, *Synthese* 18/19, 1968–9, pp. 325 ff.; Pavelka, J., On fuzzy logic I, II, III, *Z. Math. Logik Grund. Math.* 25, 1979, pp. 45 ff., pp. 119 ff., pp. 454 ff.; Dubois, D., Prade, H., *Possibility Theory*, Plenum Press, 1988; Hájek, P., *Metamathematics of Fuzzy Logic*, Kluwer, 1998).

Algebraic as well as topological features of roughness and fuzziness are distinct. Topologically, rough sets may be described by means of topologies on families of sets (Polkowski, L., *Rough Sets. Mathematical Foundations*, Physica/Springer, 2002) whereas fuzzy sets by their nature fall into the province of topologies on function spaces (Ying-Ming Liu, Mao Kang Luo, *Fuzzy Topology*, World Scientific, 1998). Algebraically, rough sets form structures known as Łukasiewicz algebras, Heyting algebras, Post algebras, etc. (Pagliani, op. cit., Polkowski, op. cit.), whereas fuzzy set algebra involves point-wise operations on fuzzy membership functions suggested by various logical interpretations of fuzzy union, intersection, etc. (Novák, V., Perfilieva, I., Močkoř, J., *Mathematical Principles of Fuzzy Logic*, Kluwer, 1999).

Despite some differences, there have been attempts to reconcile the two theories and to form a hybrid paradigm, rough-fuzzy, or fuzzy-rough, depending on whether rough constructs are introduced in the fuzzy set framework, or conversely, fuzzy constructs are defined in the rough set framework (Dubois, D., Prade, H., Putting rough sets and fuzzy sets together. In *Intelligent Decision Systems. Handbook of Applications and Advances of Rough Sets Theory*, Słowiński, R., ed., Kluwer, 1992, pp. 203 ff.; Dubois, D., Prade, H., Similarity

versus preference in fuzzy-set based logics. In *Incomplete Information: Rough Set Analysis*, Orłowska, E., ed., Physica/Springer, 1998, pp. 441 ff.; Nakamura, A., Fuzzy rough sets, *Notes on Multiple-Valued Logic in Japan*, 9, 1988, pp. 1 ff.; Cattaneo, G., Generalized rough sets. Preclusivity fuzzy-intuitionistic (BZ) lattices, *Studia Logica*, 58, 1997, pp. 47 ff.; Pedrycz, W., Shadowed sets: bridging fuzzy and rough sets. In *Rough Fuzzy Hybridization*, Pal, S.K., Skowron, A., eds., Springer, Singapore, 1999, pp. 179 ff.; Inuiguchi, M., Tanino, T., A new class of necessity measures and fuzzy rough sets based on certainty qualifications, *LNAI* 2005, 2000, pp. 261 ff.).

The volume presented to the reader contains papers devoted to rough set theory, to fuzzy set theory, and to both theories. These papers highlight important aspects of those theories from theoretical as well as application points of view.

It is our pleasure that this volume appears in the Lecture Notes in Computer Science series of Springer-Verlag in the newly initiated sub-series of Transactions on Rough Sets. We are indebted to the editors of the subseries, Profs. Peters and Skowron for their invitation to publish the volume in this subseries. Our thanks go also to Prof. Janusz Kacprzyk who suggested that we prepare a collection of papers devoted simultaneously to rough and fuzzy theories. We would like to thank the authors, whose chapters are included in this volume, for making this possible. Our thanks go to the editors of Springer-Verlag, for their dedicated work toward giving the volume its final shape as well as to Dr. Piotr Synak who helped us with LaTeX.

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