

Commenced Publication in 1973

Founding and Former Series Editors:

Gerhard Goos, Juris Hartmanis, and Jan van Leeuwen

Editorial Board

David Hutchison

Lancaster University, UK

Takeo Kanade

Carnegie Mellon University, Pittsburgh, PA, USA

Josef Kittler

University of Surrey, Guildford, UK

Jon M. Kleinberg

Cornell University, Ithaca, NY, USA

Friedemann Mattern

ETH Zurich, Switzerland

John C. Mitchell

Stanford University, CA, USA

Moni Naor

Weizmann Institute of Science, Rehovot, Israel

Oscar Nierstrasz

University of Bern, Switzerland

C. Pandu Rangan

Indian Institute of Technology, Madras, India

Bernhard Steffen

University of Dortmund, Germany

Madhu Sudan

Massachusetts Institute of Technology, MA, USA

Demetri Terzopoulos

New York University, NY, USA

Doug Tygar

University of California, Berkeley, CA, USA

Moshe Y. Vardi

Rice University, Houston, TX, USA

Gerhard Weikum

Max-Planck Institute of Computer Science, Saarbruecken, Germany

James F. Peters Andrzej Skowron
Didier Dubois Jerzy W. Grzymała-Busse
Masahiro Inuiguchi Lech Polkowski (Eds.)

Transactions on Rough Sets II

Rough Sets and Fuzzy Sets

Editors-in-Chief

James F. Peters

University of Manitoba, Department of Electrical and Computer Engineering
Winnipeg, Manitoba R3T 5V6, Canada

E-mail: jfpeters@ee.umanitoba.ca

Andrzej Skowron

University of Warsaw, Institute of Mathematics
Banacha 2, 02-097 Warsaw, Poland

E-mail: skowron@mimuw.edu.pl

Volume Editors

Didier Dubois

Université Paul Sabatier, CNRS, IRIT
118, route de Narbonne, 31062 Toulouse Cedex 4, France

E-mail: dubois@irit.fr

Jerzy W. Grzymała-Busse

University of Kansas, Department of Electrical Engineering and Computer Science
3014 Eaton Hall, 1520 W. 15th St., #2001, Lawrence, KS 66045-7621, USA

E-mail: jerzy@ku.edu

Masahiro Inuiguchi

Osaka University, Department of Mathematical Science for Social Systems
Department of Systems Innovation, Graduate School of Engineering Science
Machikaneyama-cho 1-3, Toyonaka, Osaka 560-8531, Japan

E-mail: inuiguti@sys.es.osaka-u.ac.jp

Lech Polkowski

Polish-Japanese Institute of Information Technology
and Department of Mathematics and Computer Science
University of Warmia and Mazury

Koszykowa 86, 02-008 Warsaw, Poland

E-mail: polkow@pjawst.edu.pl

Library of Congress Control Number: 2004115993

CR Subject Classification (1998): F.4.1, F.1, I.2, H.2.8, I.5.1, I.4

ISSN 0302-9743

ISBN 3-540-23990-1 Springer Berlin Heidelberg New York

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable to prosecution under the German Copyright Law.

Springer is a part of Springer Science+Business Media

springeronline.com

© Springer-Verlag Berlin Heidelberg 2004

Printed in Germany

Typesetting: Camera-ready by author, data conversion by Olgun Computergrafik
Printed on acid-free paper

SPIN: 11322016 06/3142 5 4 3 2 1 0

LNCS Transactions on Rough Sets

This journal subline has as its principal aim the fostering of professional exchanges between scientists and practitioners who are interested in the foundations and applications of rough sets. Topics include foundations and applications of rough sets as well as foundations and applications of hybrid methods combining rough sets with other approaches important for the development of intelligent systems.

The journal includes high-quality research articles accepted for publication on the basis of thorough peer reviews. Dissertations and monographs up to 250 pages that include new research results can also be considered as regular papers. Extended and revised versions of selected papers from conferences can also be included in regular or special issues of the journal.

Honorary Editor:

Zdzisław Pawlak

Editors-in-Chief:

James F. Peters, Andrzej Skowron

Editorial Board

M. Beynon

G. Cattaneo

A. Czyżewski

J.S. Deogun

D. Dubois

I. Duentzsch

S. Greco

J.W. Grzymała-Busse

M. Inuiguchi

J. Järvinen

D. Kim

J. Komorowski

C.J. Liao

T.Y. Lin

E. Menasalvas

M. Moshkov

T. Murai

M. do C. Nicoletti

H.S. Nguyen

S.K. Pal

L. Polkowski

H. Prade

S. Ramanna

R. Słowiński

J. Stepaniuk

R. Świniarski

Z. Suraj

M. Szczuka

S. Tsumoto

G. Wang

Y. Yao

N. Zhong

W. Ziarko

Preface

This collection of articles is devoted to fuzzy as well as rough set theories. Both theories are based on rigorous ideas, methods and techniques in logic, mathematics, and computer science for treating problems for which approximate solutions are possible only, due to their inherent ambiguity, vagueness, incompleteness, etc. Vast areas of decision making, data mining, knowledge discovery in data, approximate reasoning, etc., are successfully explored using methods worked out within fuzzy and rough paradigms.

By the very nature of fuzzy and rough paradigms, outlined above, they are related to distinct logical schemes: it is well-known that rough sets are related to modal logics *S5* and *S4* (Orłowska, E., Modal logics in the theory of information systems, *Z. Math. Logik Grund. Math.* 30, 1984, pp. 213 ff.; Vakarelov, D., Modal logics for knowledge representation systems, *LNC3* 363, 1989, pp. 257 ff.) and to finitely-valued logics (Pagliani, P., Rough set theory and logic-algebraic structures. In *Incomplete Information: Rough Set Analysis*, Orłowska, E., ed., Physica/Springer, 1998, pp. 109 ff.; Polkowski, L. A note on 3-valued rough logic accepting decision rules, *Fundamenta Informaticae* 61, to appear).

Fuzzy sets are related to infinitely-valued logics (fuzzy membership to degree $r \in [0, 1]$ expressing truth degree r) (Goguen, J.A., The logic of inexact concepts, *Synthese* 18/19, 1968–9, pp. 325 ff.; Pavelka, J., On fuzzy logic I, II, III, *Z. Math. Logik Grund. Math.* 25, 1979, pp. 45 ff., pp. 119 ff., pp. 454 ff.; Dubois, D., Prade, H., *Possibility Theory*, Plenum Press, 1988; Hájek, P., *Metamathematics of Fuzzy Logic*, Kluwer, 1998).

Algebraic as well as topological features of roughness and fuzziness are distinct. Topologically, rough sets may be described by means of topologies on families of sets (Polkowski, L., *Rough Sets. Mathematical Foundations*, Physica/Springer, 2002) whereas fuzzy sets by their nature fall into the province of topologies on function spaces (Ying-Ming Liu, Mao Kang Luo, *Fuzzy Topology*, World Scientific, 1998). Algebraically, rough sets form structures known as Łukasiewicz algebras, Heyting algebras, Post algebras, etc. (Pagliani, op. cit., Polkowski, op. cit.), whereas fuzzy set algebra involves point-wise operations on fuzzy membership functions suggested by various logical interpretations of fuzzy union, intersection, etc. (Novák, V., Perfilieva, I., Močkoř, J., *Mathematical Principles of Fuzzy Logic*, Kluwer, 1999).

Despite some differences, there have been attempts to reconcile the two theories and to form a hybrid paradigm, rough-fuzzy, or fuzzy-rough, depending on whether rough constructs are introduced in the fuzzy set framework, or conversely, fuzzy constructs are defined in the rough set framework (Dubois, D., Prade, H., Putting rough sets and fuzzy sets together. In *Intelligent Decision Systems. Handbook of Applications and Advances of Rough Sets Theory*, Słowiński, R., ed., Kluwer, 1992, pp. 203 ff.; Dubois, D., Prade, H., Similarity

versus preference in fuzzy-set based logics. In *Incomplete Information: Rough Set Analysis*, Orłowska, E., ed., Physica/Springer, 1998, pp. 441 ff.; Nakamura, A., Fuzzy rough sets, *Notes on Multiple-Valued Logic in Japan*, 9, 1988, pp. 1 ff.; Cattaneo, G., Generalized rough sets. Preclusivity fuzzy-intuitionistic (BZ) lattices, *Studia Logica*, 58, 1997, pp. 47 ff.; Pedrycz, W., Shadowed sets: bridging fuzzy and rough sets. In *Rough Fuzzy Hybridization*, Pal, S.K., Skowron, A., eds., Springer, Singapore, 1999, pp. 179 ff.; Inuiguchi, M., Tanino, T., A new class of necessity measures and fuzzy rough sets based on certainty qualifications, *LNAI* 2005, 2000, pp. 261 ff.).

The volume presented to the reader contains papers devoted to rough set theory, to fuzzy set theory, and to both theories. These papers highlight important aspects of those theories from theoretical as well as application points of view.

It is our pleasure that this volume appears in the Lecture Notes in Computer Science series of Springer-Verlag in the newly initiated sub-series of Transactions on Rough Sets. We are indebted to the editors of the subseries, Profs. Peters and Skowron for their invitation to publish the volume in this subseries. Our thanks go also to Prof. Janusz Kacprzyk who suggested that we prepare a collection of papers devoted simultaneously to rough and fuzzy theories. We would like to thank the authors, whose chapters are included in this volume, for making this possible. Our thanks go to the editors of Springer-Verlag, for their dedicated work toward giving the volume its final shape as well as to Dr. Piotr Synak who helped us with L^AT_EX.

May 2004

Didier Dubois
Jerzy Grzymała-Busse
Masahiro Inuiguchi
Lech Polkowski

Table of Contents

Feature Selection with Rough Sets for Web Page Classification	1
<i>Aijun An, Yanhui Huang, Xiangji Huang, and Nick Cercone</i>	
On Learning Similarity Relations in Fuzzy Case-Based Reasoning	14
<i>Eva Armengol, Francesc Esteva, Lluís Godo, and Vicenç Torra</i>	
Incremental versus Non-incremental Rule Induction for Multicriteria Classification	33
<i>Salvatore Greco, Roman Słowiński, Jerzy Stefanowski, and Marcin Żurawski</i>	
Three Strategies to Rule Induction from Data with Numerical Attributes .	54
<i>Jerzy W. Grzymala-Busse</i>	
Fuzzy Transforms	63
<i>Irina Perfilieva</i>	
Possible Equivalence Relations and Their Application to Hypothesis Generation in Non-deterministic Information Systems	82
<i>Hiroshi Sakai</i>	
Applications of Fuzzy Logic Functions to Knowledge Discovery in Databases	107
<i>Noboru Takagi, Hiroaki Kikuchi, and Masao Mukaidono</i>	
Fuzzy Integral Based Fuzzy Switching Functions	129
<i>Eiichiro Takahagi</i>	
First Steps Towards Computably-Infinite Information Systems	151
<i>Peter Apostoli, Akira Kanda, and Lech Polkowski</i>	
Data Structure and Operations for Fuzzy Multisets	189
<i>Sadaaki Miyamoto</i>	
A Non-controversial Definition of Fuzzy Sets	201
<i>Jaroslav Ramík and Milan Vlach</i>	
Algebraic Structures for Rough Sets	208
<i>Gianpiero Cattaneo and Davide Ciucci</i>	
Rough Mereology as a Link Between Rough and Fuzzy Set Theories. A Survey	253
<i>Lech Polkowski</i>	

Fuzzy Rough Sets Based on Residuated Lattices	278
<i>Anna Maria Radzikowska and Etienne E. Kerre</i>	
Semantics of Fuzzy Sets in Rough Set Theory	297
<i>Yiyu Yao</i>	
A New Proposal for Fuzzy Rough Approximations and Gradual Decision Rule Representation	319
<i>Salvatore Greco, Masahiro Inuiguchi, and Roman Słowiński</i>	
Emergent Rough Set Data Analysis	343
<i>Yasser Hassan and Eiichiro Tazaki</i>	
Author Index	363