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# Automated Deduction in Geometry

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Revised Papers



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## Preface

Automated deduction in geometry has several roots that go back to developments in the 20th century. These fundamental methods in ADG are the quantifier elimination method of Tarski and Collins, the method of characteristic sets of Ritt and Wu, and the Gröbner basis method of Buchberger. Based on these algorithmic techniques various geometric theorem provers have been developed in recent decades. Applications of ADG theorem provers range from computer-aided geometric design to robotics and education.

In 1992 H. Hong, D.M. Wang, and F. Winkler organized the workshop “*Algebraic Approaches to Geometric Reasoning*” in Schloss Weinberg near Linz, Austria. The idea of this workshop was to bring together mathematicians, computer scientists and people working in applications, to discuss the newest results in ADG. After this first workshop D.M. Wang took over the responsibility of organizing a series of workshops in ADG. He organized and chaired ADG 1996 in Toulouse. The following workshops were ADG 1998 in Beijing organized and chaired by X.S. Gao, and ADG 2000 in Zürich organized and chaired by J. Richter-Gebert. The proceedings of ADG 1996, ADG 1998, and ADG 2000 have all appeared in the series Lecture Notes in Artificial Intelligence.

I was asked by D.M. Wang to organize and chair ADG 2002. I gladly accepted this offer, and ADG 2002 took place on September 4–6, 2002, in Schloss Hagenberg, the home of the *Research Institute for Symbolic Computation (RISC-Linz)* of the Johannes Kepler University of Linz, Austria. The speakers at the workshop were invited to revise their work and submit papers to the proceedings. These submissions were reviewed by the program committee and 13 papers were selected to be included in the proceedings of ADG 2002. Let me just briefly review the various contributions to these proceedings.

Several of the papers are concerned with new theoretical developments in ADG: G. Bodnár investigates an important subproblem in the resolution of singularities, namely the question of whether two (or several) varieties have the normal crossing property. H. Li presents an innovative new representation of objects in affine and projective algebraic geometry using Cayley algebra and bracket algebra. J.C. Owen and S.C. Power deal with a configuration problem of points in a plane. Such configuration problems can be phrased in terms of graphs. They show that certain related graph problems cannot be solved by radicals. I.J. Tchoupaeva proposes a coordinate-free representation of varieties in Grassmann algebra along with Gröbner basis techniques for computing in such an environment. X. Chen and D.K. Wang use a projection algorithm for quasi varieties in geometry theorem proving and show that their method provides optimal nondegeneracy conditions for geometric theorems.

Another group of papers is concerned with software and computer systems for geometric theorem proving: X.S. Gao and Q. Lin describe their software package MMP/Geometer, implementing Wu’s method for Euclidean and differential

geometry. H.G. Gräbe presents the *SymbolicData* GEO records, a proposed standard for proof schemes. A. Pasko and V. Adzhiev design a specialized language for shape modeling. D.M. Wang has further developed his system GEOTHER, and shows how it can be applied for solving geometric proving problems.

Finally, there is a group of papers investigating applications of ADG to problems in computer-aided geometric design and robotics: C. Jermann, B. Neveu, and G. Trombettoni consider the problem of structural rigidity in geometric constraint satisfaction problems. They propose a new approach to this concept, which takes into account geometric aspects of rigidity. M. Shalaby, B. Jüttler, and J. Schicho propose a new method for constructing a low degree  $C^1$  implicit spline representation of a given parametric planar curve. L. Yang in his paper introduces an invariant method based on distance geometry to construct the constraint equations for spatial constraint solving. S. Corvez and F. Rouillier use computer algebra tools to give a classification of 3-revolute-jointed manipulators based on the cuspidal behavior.

I want to thank the RISC-Linz institute for its hospitality during the workshop, and the Government of Upper Austria, the University of Linz, the Linzer Hochschulfonds, and the Wirtschaftskammer Österreich for their financial support. Last but not least I want to thank Manasi Athale for her work in collecting all these contributions and preparing them for printing.

The series of workshops in ADG will continue, with ADG 2004 being organized by Neil White at the University of Florida.

October 2003

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