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Semirings for Soft Constraint Solving and Programming

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Foreword

Constraint satisfaction and constraint programming have shown themselves to be very simple but powerful ideas. A constraint is just a restriction on the allowed combinations of values for a set of variables. If we can state our problem in terms of a set of constraints, and have a way to satisfy such constraints, then we have a solution. The idea is general because it can be applied to several classes of constraints, and to several solving algorithms. Moreover, it is powerful because of its unifying nature, its generality, its declarative aspects and its application possibilities. In fact, many research and application areas have taken advantage of constraints to generalize and improve their results and application scenarios.

In the last 10 years, however, this simple notion of constraint has shown some deficiencies concerning both theory and practice, typically in the way over-constrained problems and preferences are treated. When a problem has no solution, classical constraint satisfaction does not help. Also, classical constraints are not able to model conveniently problems which have preferences, for example over the selection of the most relevant constraints, or about the choice of the best among several solutions which satisfy all the constraints.

Not being able to handle non-crisp constraints is not just a theoretical problem, but it is also particularly negative for applications. In fact, over-constrained and preference-based problems are present in many application areas. Without formal techniques to handle them, it is much more difficult to define a procedure which can easily be repeated to single out an acceptable solution, and sometimes it is not even possible.

For this reason, many researchers in constraint programming have proposed and studied several extensions of the classical concepts in order to address these needs. This has led to the notion of *soft constraints*. After several efforts to define specific classes of soft constraints, like fuzzy, partial and hierarchical, the need for a general treatment of soft constraints became evident, a treatment that could model many different classes altogether and to prove properties for all of them. Two of the main general frameworks for soft constraints were *semiring-based soft constraints* and *valued constraints*.

This book is a revised, extended version of the Ph.D. thesis of Stefano Bistarelli, whom we had the pleasure to supervise at the University of Pisa. It focuses mainly on the semiring-based soft constraint formalism, also comparing it with many of the specific classes and also with valued constraints. Semiring-based soft constraints are so called because they are based on an underlying semiring structure, which defines the set of preferences, the way they are ordered, and how they can be combined. This concept is very general and can be instantiated to obtain many of the classes of soft constraints that have already been proposed, including their solution algorithms, and also some new ones.

The book includes formal definitions and properties of semiring-based soft constraints, as well as their use within constraint logic programming and concurrent constraint programming. Moreover, it shows how to adapt to soft constraints some existing notions and techniques, such as abstraction and interchangeability, and it shows how soft constraints can be used in some application areas, such as security.

This book is a great starting point for anyone interested in understanding the basics of semiring-based soft constraints, including the notion of soft constraint propagation, and also in getting a hint abut the applicability potential of soft constraints. In fact, it is the first book that summarizes most of the work on semiring-based soft constraints. Although most of its content also appears in published papers, this is the only place where this material is gathered in a coherent way.

This book is the result of several threads of collaborative work, as can be seen from the many publications that are cited in the bibliography and whose content is reflected in the book. Therefore many authors have contributed to the material presented here. However, Stefano Bistarelli succeeded in providing a single line of discourse, as well as a unifying theme that can be found in all the chapters. This uniform approach makes the material of this book easily readable and useful for both novice and experienced researchers, who can follow the various chapters and find both informal descriptions and technical parts, as well as application scenarios.

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Preface

The *Soft Constraints* idea is able to capture many real-life situations that cannot be represented and solved with the classical crisp constraint framework. In this book we first describe a general framework for representing many soft constraint systems, and then we investigate the related theoretic and application-oriented issues.

Our framework is based on a semiring structure, where the carrier of the semiring specifies the values to be associated with each tuple of values of the variable domain, and the two semiring operations, $+$ and $\times$, model constraint projection and combination, respectively. The semiring carrier and operations can be instantiated in order to capture all the *non-crisp* constraints representing fuzziness, optimization, probability, hierarchies, and others. The solution of each instance of the soft Constraint Satisfaction Problem (CSP) is computed by using the appropriate $\times$ and $+$ semiring operation.

This uniform representation can be used to give sufficient conditions for the correctness and applicability of local consistency and dynamic programming algorithms. In particular:

- We show that using an idempotent $\times$ operator the classical local consistency (and also dynamic programming) techniques can be used to reduce the complexity of the problem without modifying its solution.
- We adapt to the soft framework partial local consistency and labeling techniques, which require fewer pruning steps of the domain. This means that, although they are able to remove fewer non-optimal solutions than classical algorithms can, partial local consistency algorithms can be beneficial because they are faster and easier implemented.
- We extend general local consistency algorithms that use several pruning rules until the fix-point is reached.

Solving a soft CSP is generally harder than solving the corresponding crisp CSP. For this reason we introduce an abstraction/concretization mapping over soft CSPs in order to solve a problem in an easier environment and then use the abstract results to speed up the search of solutions in the concrete one. Several mappings between existing soft frameworks are given. These mappings will especially be useful for applying soft local consistency techniques in a safe, easy, and faster way. Also useful, when looking for optimal solutions, are the notions of *substitutability* and *interchangeability.* In crisp CSPs they have been used as a basis for search heuristics, solution adaptation, and abstraction techniques.

The next part of the book involves some programming features: as classical constraint solving can be embedded into Constraint Logic Programming (CLP) systems, so too can our more general notion of constraint solving be handled within a logic language, thus giving rise to new instances of the CLP scheme. This not only gives new meanings to constraint solving in CLP, but it also allows one to treat in a uniform way optimization problem solving within CLP, without the need to resort to ad hoc methods. In fact, we show that it is possible to generalize the semantics of CLP programs to consider the chosen semiring and thus solve problems according to the semiring operations. This is done by associating each ground atom with an element of the semiring and by using the two semiring operations to combine goals. This allows us to perform in the same language both constraint solving and optimization. We then provide this class of languages with three equivalent semantics, model-theoretic, fix-point, and proof-theoretic, in the style of CLP programs. The language is then used to show how the soft CLP semantics can solve shortest-path problems. In a way similar to the soft CLP language we also extend the semantics of the Concurrent Constraints (cc) language. The extended cc language uses soft constraints to prune and direct the search for a solution.

The last part of the book aims to describe how soft constraints can be used to solve some security-related problems. In the framework, the crucial goals of *confidentiality* and *authentication* can be achieved with different levels of security. In fact, different messages can enjoy different levels of confidentiality, or a principal can achieve different levels of authentication with different principals.

Acknowledgement. This monograph is a revised and extended version of my doctoral dissertation which was submitted to the University of Pisa Computer Science Department and accepted in March 2001.

The results presented here have been influenced by many people and I would like to take this opportunity to thank them all.

I wish to thank first of all the supervisor of my Ph.D. thesis Ugo Montanari; the main part of this book came from the research performed under his significant guidance.

I want also to thank Francesca Rossi, my unofficial supervisor; she shared with me the passion for constraints. Many of the ideas collected in this book came from ideas we developed together.

A special thanks is also due to all the friends who shared with me some of the results I collected in this book: Giampaolo Bella, Boi Faltings, Rosella Gennari, Hélène Fargier, Yan Georget, Nicoleta Neagu, Elvinia Riccobene, Thomas Schiex, and Gérard Verfaillie.

Many thanks are due to the external reviewers of my Ph.D. thesis, Philippe Codognet and Pascal Van Hentenryck; they carefully read a preliminary version of the thesis and provided many useful comments and suggestions.

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Finally, I am also grateful both to the anonymous referees of this book and to Jan van Leeuwen; they gave me a number of hints on a draft version of the book which helped to improve the final presentation.

Some thanks also to Anna Kramer and Alfred Hofmann who helped me with the last details needed for the publication of this book.

December 2003 Stefano Bistarelli

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