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Research paper

Iterative multiuser detection for high spectral efficiency reverse-link of multibeam satellite via expectation propagation

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Abstract: Multibeam satellite communications employing full frequency reuse have the potential to increase spectral efficiency. However, they suffer from severe inter-beam interference. An expectation propagation based message passing algorithm is proposed for decoding multi-user transmissions in the reverse link of multi-beam satellite communications with full frequency reuse. Compared with an iterative MMSE (Minimum Mean Square Error) interference cancellation algorithm, the proposed algorithm reduces the cubic complexity to square complexity in the number of interfering beams. Numerical results show that the proposed algorithm outperforms the iterative MMSE algorithm slightly in terms of bit error rate when the energy per bit to noise power spectral density ratio is low. The performance of both algorithms is the same for other cases.

Key words: expectation propagation, full frequency reuse, factor graph, iterative multiuser detection, multibeam satellite communication, message passing.

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1 Introduction

Achieving Terabit/s throughput is important in next generation satellite communications[1]. To achieve this goal, a large number of spot-beams and full frequency reuse can be employed to increase spectral efficiency. Consequently, system performance deteriorates severely owing to inter-beam interference. The capacity of a multibeam is analyzed in Ref.[2]; it shows that the theoretical capacity can be improved if multibeam signals are processed jointly. However, an interference cancellation technique for improving system capacity is not developed in Ref.[2]. Different turbo interference cancellation schemes were

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proposed in Refs.[1-5], using soft-input-soft-output modules with an iterative structure. MMSE filter based algorithms estimate an input signal by filtering the original input signal, which is cleaned from interference, which is estimated using the extrinsic information. The complexity of the algorithm is high because the optimal MMSE filter is different for each symbol and iteration, since the covariance matrix of extrinsic information is different for each symbol and iteration. The BP (Belief Propagation) algorithm has been successfully employed in a variety of applications[6,7], e.g., decoding of turbo codes and LDPC (Low Density Parity Check) codes[8], an detection in terrestrial CDMA (Code Division Multiple Access) and MIMO (Multiple-Input Multiple-Output) systems[9-12]. The exact BP algorithm is the optimal approach when a factor graph is cycle free. Nevertheless, the underlying factor graph for the transmission in the multibeam reverse link is dense, and the complexity of the exact BP algorithm is exponential in the number of interfering beams and the modulation alphabet. Hence, the above approaches are impractical for the satellite communications that have a large number of beams.

In this study, it is shown that an EP (Expectation Propagation) based AMP (Approximate Message Passing) algorithm can be used effectively to achieve excellent decoding performance in a multibeam satellite system at low complexities. Firstly, we derive a factor graph representation for multibeam joint processing, and then formulate a joint detection and decoding algorithm by employing the EP algorithm over the resulting factor graph. The performance of the algorithm, which has square complexity in the number of interfering beams, is similar to that of the iterative MMSE algorithm. Although we have focused on a block fading channel and multiuser signal detection in this study, the EP based AMP algorithm can be extended to the sparse frequency selective channels estimation in massive MIMO[13]. As the pilot aided training overhead for channel estimation increases proportionally with the scale of antennas, the proposed low complexity and low overhead algorithm is suitable for massive MIMO systems. In future work, we will extend the algorithm for a time varying channel.

This paper is organized as follows: In Section 2, a model is developed for the fading channel of a multibeam satellite. The joint interference cancellation and decoding method is investigated in Section 3. In Section 4, BER (Bit Error Rate) performance is evaluated through numerical simulations, and compared to that of the conventional method. Conclusions are given in Section 5.

Notations: Lower case letters (e.g.,x) denote scalars, bold lower case letters (e.g.,$\boldsymbol{x}$) denote column vectors, and bold upper case letters (e.g.$\boldsymbol{X}$) denote matrices. The superscript , * and H denote the transpose operation, conjugate operation and conjugate transpose operation, respectively. Symbol $\boldsymbol{I}$ denotes an identity matrix; ln(·) denotes the natural logarithm; and $\mathcal{N}_{\mathbb{C}}(x,\hat{x},\hat{\tau})=(\pi\hat{\tau})^{-1}\exp(-|x-\hat{x}|^2/\hat{\tau})$ denotes a complex Gaussian function. Furthermore, $\boldsymbol{x}\backslash x_i$ denotes the symbols in $\boldsymbol{x}$ with x_i excluded; and $\propto$ denotes equality up to a scale. $\mathrm{E}_{p(x)}[\cdot]$ denotes the statistical expectation operation with respect to the distribution $p(x)$.

2 System model

Let us consider a multibeam satellite communications system with N spot beams covering N users, as shown in Fig.1. One user per beam is equipped with one antenna and is scheduled to transmit during a specific time interval. Thus there are at most N active users in a time interval. Perfect symbol and frame synchronization are assumed, and the received signal $\boldsymbol{y}[t]\in\mathbb{C}^{N\times 1}$ in the tth symbol interval is written as

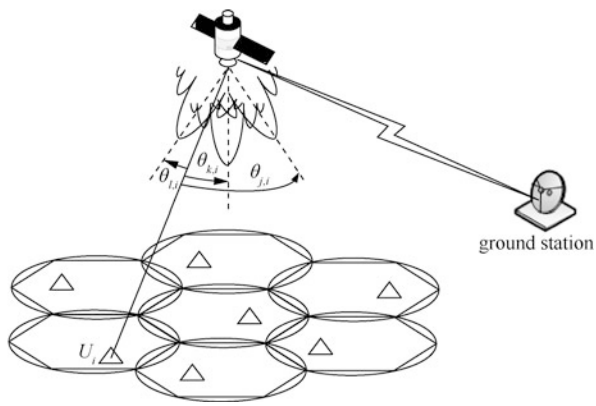


Figure 1 System model for a multibeam satellite system

$$y[t]=\boldsymbol{H}[t]\boldsymbol{x}[t]+\boldsymbol{n}[t], t=1,2,\cdots,T, \quad (1)$$

where $\boldsymbol{H}[t]\in\mathbb{C}^{N\times N}$ is the channel coefficients matrix, $\boldsymbol{n}[t]\in\mathbb{C}^{N\times 1}$ is the complex circularly symmetric Gaussian noise with zero mean, and the covariance is $\mathrm{E}\{\boldsymbol{n}[t]\boldsymbol{n}^{\mathrm{H}}[t]\}=\sigma_n^2\boldsymbol{I}$, and $\boldsymbol{x}[t]=[x_1[t],x_2[t],\cdots,x_N[t]]^{\mathrm{T}}$ represents the transmitted symbol from N users in the tth time interval, where $x_i[t]$ denotes the transmitted symbol from the N user. $x_i[t]$ is chosen from the 2^Q-ary symbol alphabet $\boldsymbol{\mathcal{A}}=\{\alpha_1,\alpha_2,\ldots,\alpha_{2^Q}\}$, where $\alpha_i\in\mathbb{C}$ corresponds to a bit pattern. Each user, e.g., the user, encodes an information bit sequence, $\boldsymbol{b}_i$, to a coded bit sequence, $\boldsymbol{c}_i$, which is mapped to a symbol sequence $\boldsymbol{x}_i=[x_i[1],x_i[2],\cdots,x_i[T]]^{\mathrm{T}}$ with length of T.

Beam gain, Rician fading, and antenna correlation are considered for the channel model of the multibeam satellite reverse link. Let the beam gain matrix be denoted by $\boldsymbol{G}[t]\in\mathbb{C}^{N\times N}$, which models the effects of inter-beam interference. The element $g_{j,i}[t]\in\mathbb{C}$ in $\boldsymbol{G}$ denotes the beam gain between the jth beam and the ith user located in the ith beam, which depends on the radiation pattern of the jth beam and the position of the user in the ith beam, and it is typically approximated as

$$g_{j,i}[t]=b_{\max}\left(\frac{J_1(u_{j,i}[t])}{2u_{j,i}[t]}+36\frac{J_3(u_{j,i}[t])}{u_{j,i}^3[t]}\right)^2, \quad (2)$$

where $b_{\max}$ represents the gain at the beam center, and $u_{j,i}[t]$ is defined as $u_{j,i}[t]=2.071\,23\sin(\theta_{j,i}[t])/\sin(\theta_{3\,\mathrm{dB}})$, J_1 and J_3 are the Bessel functions of the first kind of orders one and three, respectively. $\theta[t]$ is the off-axis angle between the ith beam center and the jth user location, and $\theta_{3\,\mathrm{dB}}$ is to the half power angle. We assume that the distance between each user is large enough for fading coefficients to be independent of each other. Moreover, the fading coefficients between a user and every spot beam are the same, owing to the line of sight environment, and equal distances between the user and the spot-beams. Let $\tilde{h}_i[t]$ denote the Rician fading coefficient for the ith user, which can be modeled by

$$\tilde{h}_i[t]=\sqrt{\frac{K}{K+1}}+\sqrt{\frac{K}{K+1}}\breve{h}_i[t], \quad (3)$$

where K is the Rician factor, and $\breve{h}_i[t]$ is an independent and identically distributed random process with zero mean and unit variance. Under the above assumptions, the channel coefficients matrix $\boldsymbol{H}[t]$ is modeled by

$$\boldsymbol{H}[t]=[\boldsymbol{G}[t]]^{\frac{1}{2}}\boldsymbol{H}[t], \quad (4)$$

where $H=\mathrm{diag}([\tilde{h}_1[t],\tilde{h}_2[t],\cdots,\tilde{h}_N[t]])$ is a diagonal matrix.

3 Multiuser decoding for multibeam reverse link

In this section, the EP algorithm that processes all the interfering spot beams jointly is proposed, and its complexity is discussed.

3.1 Factor graph representation

The joint posterior distribution of the transmitted symbols from N users in the time interval t can be factorized as

$$p(\boldsymbol{x}[t],\boldsymbol{y}[t]|\boldsymbol{H}[t])=p(\boldsymbol{x}[t])p(\boldsymbol{y}[t]|\boldsymbol{x}[t],\boldsymbol{H}[t])$$
$$\propto\prod_{1\leqslant i\leqslant N}p(x_i[t])\prod_{1\leqslant j\leqslant N}\exp\left\{-\frac{1}{\sigma_n^2}\left|y_j[t]-\sum_{i=1}^{N}h_{j,i}[t]x_i[t]\right|^2\right\}, \quad (5)$$

where $h_{i,j}[t]$ is an element in the channel matrix $\boldsymbol{H}[t]$. According to the above mentioned factorization, the factor graph for the decoding of the transmissions in the multibeam system is shown in Fig.2. There are four kinds of nodes in this factor graph. The top node, labeled as code constraints, represents the relationship between the information bits and the coded bits transmitted by each user. The middle node, $\phi(x_i[t], \boldsymbol{c}_i[t]) = \delta(x_i[t], \varphi(\boldsymbol{c}_i[t])), i = 1, 2, \cdots, N$, represents the relationship between the coded bits and the transmitted symbols, where φ is the function mapping the coded bits $\boldsymbol{c}_i[t]$ to the symbols $x_i[t]$, and $\delta(\cdot)$ is the Dirac function. The node, labeled as =, represents the equality constraint, and the node, labeled with f_j, represents the following observation equation.

$$f_j(x[t]) = p(y_j[t] | x_1[t], \cdots, x_N[t]) \propto \exp\left\{ -\frac{1}{\sigma_n^2} \left| y_i[t] - \sum_{j=1}^{N} h_{i,j}[t] x_j[t] \right|^2 \right\}. \quad (6)$$

For notational simplicity, the index of time interval t is henceforth omitted.

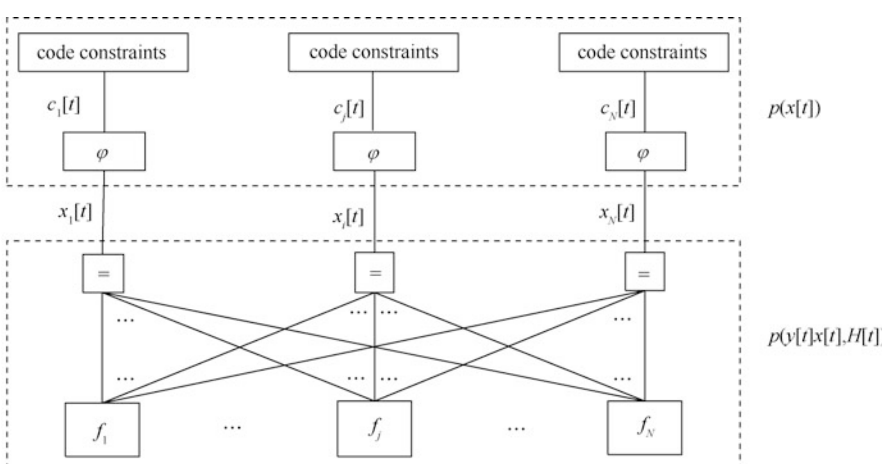
3.2 Expectation propagation based approximate message passing algorithm

The BP algorithms for de-mapper/mapper nodes and decoder nodes have been described well in various literature[8,14,15]. However, direct application of the BP algorithm to the bottom-most detection node leads to a computational explosion. Let $\mu^{(m)}_{x_i \to f_j}(x_i)$ denote the message sent from the variable node x_i to the observation node f_j in the mth iteration, and $\mu^{(m)}_{f_j \to x_i}(x_i)$ denote the message in the opposite direction; Let $\mu^{(m)}_{dec \to x_i}(x_i) = \phi(x_i[t], \boldsymbol{c}_i[t]) \prod_{q=1}^{Q} p^{(m)}(c_{i,q})$ denote the message from the decoder node to the variable node x_i in the mth iteration, where $\{p^{(m)}(c_{i,q})\}$ is the extrinsic information of the coded bits $\{c_{i,q}\}$ fed from the decoder. Let $\mathcal{N}(v)$ denote the set of neighbors of a given node v. The message computations performed at the variable and observation nodes are as follows[6].

$$\mu^{(m)}_{x_i \to f_j}(x_i) = \mu^{(m)}_{dec \to x_i}(x_i) \prod\nolimits_{k \in \mathcal{N}(x_i) \backslash j} \mu^{(m-1)}_{f_k \to x_i}(x_i), \quad (7)$$

$$\mu^{(m)}_{f_j \to x_i}(x_i) = \sum_{\boldsymbol{x} \backslash x_i} f_j(\boldsymbol{x}) \prod\nolimits_{k \in \mathcal{N}(x_j) \backslash i} \mu^{(m)}_{x_k \to f_j}(x_k). \quad (8)$$

As the sum operation in Eq.(8) is exponential in N, it seems that performing BP algorithm exactly is hopeless for large scale multibeam satellite system. If we consider the message $\mu^{(m)}_{x_i \to f_j}(x_i)$ as a complex Gaussian PDF (Probability Density Function) $\hat{\mu}^{(m)}_{x_k \to f_j}(x_k) = \mathcal{N}_{\mathbb{C}}\left(x_k; \hat{x}^{(m)}_{x_k \to f_j}, \hat{\tau}^{(m)}_{x_k \to f_j}\right)$, then $\mu^{(m)}_{f_j \to x_i}(x_i)$ can be calculated by using simple linear operations as



Figuer 2 Factor graph for the multiuser decoding of transmission in the multibeam reverse-link

follows.

$$\begin{aligned}&\mu_{f_j\to x_i}^{(m)}(x_i)\\&\cong\int_{x\backslash x_i} f_j(x)\prod_{k\neq i}\mathcal{N}_{\mathbb{C}}\left(x_k;\hat{x}_{x_k\to f_j}^{(m)},\hat{\tau}_{x_k\to f_j}^{(m)}\right)\\&\propto\mathcal{N}_{\mathbb{C}}\left(h_{j,i}x_i;z_{f_j\to x_i}^{(m)},\tau_{f_j\to x_i}^{(m)}\right),\end{aligned}\tag{9}$$

where $z_{f_j\to x_i}^{(m)}=y_j-\sum_{k\neq i}h_{j,k}\hat{x}_{x_k\to f_j}^{(m)}$ and $\tau_{f_j\to x_i}^{(m)}=\sigma_n^2+\sum_{k\neq i}\left|h_{j,k}\right|^2\hat{\tau}_{x_k\to f_j}^{(m)}$.

By substituting $\mu_{f_j\to x_i}^{(m-1)}(x_i)=\mathcal{N}_{\mathbb{C}}\left(h_{j,i}x_i;z_{f_j\to x_i}^{(m-1)},\tau_{f_j\to x_i}^{(m-1)}\right)$ into Eq.(7), we obtain

$$\begin{aligned}&\mu_{x_i\to f_j}^{(m)}(x_i)\\&=\mu_{dec\to x_i}^{(m)}(x_i)\mathcal{N}_{\mathbb{C}}\left(x_i;\zeta_{x_i\to f_j}^{(m-1)},\gamma_{x_i\to f_j}^{(m-1)}\right)\\&\cong\mu_{dec\to x_i}^{(m)}(x_i)\mathcal{N}_{\mathbb{C}}\left(x_i;\zeta_{x_i\to f_j}^{(m-1)},\gamma_{x_i}^{(m-1)}\right)\\&\propto\frac{\mu_{dec\to x_i}^{(m)}(x_i)\mathcal{N}_{\mathbb{C}}\left(x_i;\zeta_{x_i\to f_j}^{(m-1)},\gamma_{x_i}^{(m-1)}\right)}{\sum\mu_{dec\to x_i}^{(m)}(x_i)\mathcal{N}_{\mathbb{C}}\left(x_i;\zeta_{x_i\to f_j}^{(m-1)},\gamma_{x_i}^{(m-1)}\right)},\end{aligned}\tag{10}$$

where $\gamma_{x_i\to f_j}^{(m-1)}=\sum_{k\neq j}1\Big/\left(\left|h_{k,i}\right|^2\Big/\tau_{f_j\to x_i}^{(m-1)}\right)$ and $\zeta_{x_i\to f_j}^{(m-1)}=\gamma_{x_i}^{(m-1)}\sum_{k\neq j}h_{k,i}^{*}z_{f_k\to x_i}^{(m-1)}\Big/\tau_{f_j\to x_i}^{(m-1)}$. However, $\mu_{x_i\to f_j}^{(m)}(x_i)$ in Eq.(10) is not a Gaussian PDF, since the normalized message $\mu_{dec\to x_i}^{(m)}(x_i)$ from the decoder is a discrete distribution. To obtain a Gaussian PDF $\hat{\mu}_{x_i\to f_j}^{(m)}(x_i)$ to replace $\mu_{x_i\to f_j}^{(m)}(x_i)$, a natural approach is to minimize the KL (Kull-back-Leibler) divergence, $KL\left(\mu_{x_i\to f_j}^{(m)}(x_i)\,||\,\hat{\mu}_{x_i\to f_j}^{(m)}(x_i)\right)$. Then, the following equations can be obtained[16]

$$\hat{x}_{x_i\to f_j}^{(m)}=\sum_{\alpha_i\in\mathcal{A}}\alpha_i\mu_{x_i\to f_j}^{(m)}(x_i=\alpha_i),\tag{11}$$

$$v_{x_i\to f_j}^{(m)}=\sum_{\alpha_i\in\mathcal{A}}\left|\alpha_i-\hat{x}_{x_i\to f_j}^{(m)}\right|^2\mu_{x_i\to f_j}^{(m)}(x_i=\alpha_i).\tag{12}$$

The message $\mu_{x_i\to dec}^{(m)}(x_i)$ is the multiplication of all the incoming messages $\{\mu_{f_j\to x_i}^{m}(x_i)\}$. The message towards the decoder can be obtained as

$$\begin{aligned}\mu_{x_i\to dec}^{(m)}(x_i)&=\prod_j\mu_{f_j\to x_i}^{(m)}(x_i)\\&\propto\mathcal{N}_{\mathbb{C}}(x_i;\zeta_{x_i}^{(m)},\gamma_{x_i}^{(m)}),\end{aligned}\tag{13}$$

where $\gamma_{x_i}^{(m)}=1\Big/(\sum_j\left|h_{j,i}\right|^2\Big/\tau_{f_j\to x_i}^{(m)})$ is the variance and $\zeta_{x_i}^{(m)}=\gamma_{x_i}^{(m)}\sum_j h_{j,i}^{*}z_{f_j\to x_i}^{(m)}\Big/\tau_{f_j\to x_i}^{(m)}$ is the mean. For the channel decoding, the message $\mu_{x_i\to dec}^{(m)}(x_i)$ is finally mapped into the LLR (Log Likelihood Ratio) of the coded bits that are corresponding to the symbol x_i

$$\mathcal{L}^{(m)}(c_{i,q})=\ln\frac{\sum_{\mathcal{A}_{i,q}^{1}}\mathcal{N}_{\mathbb{C}}\left(x_i;\zeta_{x_i}^{(m)},\gamma_{x_i}^{(m)}\right)\prod_{k\neq q}p^{(m)}(c_{i,k})}{\sum_{\mathcal{A}_{i,q}^{0}}\mathcal{N}_{\mathbb{C}}\left(x_i;\zeta_{x_i}^{(m)},\gamma_{x_i}^{(m)}\right)\prod_{k\neq q}p^{(m)}(c_{i,k})},\tag{14}$$

where $\mathcal{A}_{i,q}^{1}$ and $\mathcal{A}_{i,q}^{0}$ denote the subsets of all the symbols, in which the qth bit has a value of 1 and 0, respectively, and $p^{(m)}(c_{i,k})$ is the extrinsic probability of the coded bit $c_{i,k}$ fed from the decoder. The decoders decode the information bits and generate the probability information about the coded bits, which is used in the next iteration.

The computation of $\{\mu_{x_i\to f_j}^{(m)}(x_i)\}$ shown by Eq.(10) is time-consuming. In addition, the number of messages $\{\mu_{x_i\to f_j}^{(m)}(x_i)\}$ that need to be tracked is as large as N^2. Using EP, we can reduce the complexity in the computation of $\{\mu_{x_i\to f_j}^{(m)}(x_i)\}$. If we define a symbol belief as

$$\beta_{x_i}^{(m)}(x_i)=\frac{\mu_{dec\to x_i}^{(m)}(x_i)\prod_j\mu_{f_j\to x_i}^{(m-1)}(x_i)}{\sum_{x_i\in\mathcal{A}}\mu_{dec\to x_i}^{(m)}(x_i)\prod_j\mu_{f_j\to x_i}^{(m-1)}(x_i)},\tag{15}$$

and consider it as a complex Gaussian PDF $\hat{\beta}_{x_i}^{(m)}(x_i)=\mathcal{N}_{\mathbb{C}}(x_i;\hat{x}_{x_i}^{(m)},v_{x_i}^{(m)})$, Then, by minimizing the KL divergence $KL(\mu_{x_i}^{(m)}(x_i)\|\hat{\beta}_{x_i}^{(m)}(x_i))$, we can obtain $\hat{x}_{x_i}^{(m)}=\mathrm{E}_{\hat{\beta}_{x_i}^{(m)}(x_i)}[x_i]$ and $v_{x_i}^{(m)}=\mathrm{E}_{\hat{\beta}_{x_i}^{(m)}(x_i)}\left\{\left|x_i\right|^2\right\}-\left|\hat{x}_{x_i}^{m}\right|^2$. Then, the approximate message $\hat{\mu}_{x_i\to f_j}^{(m)}(x_i)$ is computed from the approximate symbol belief $\hat{\beta}_{x_i}^{(m)}(x_i)$. According to the semantics of the factor graph, we can obtain

$$\hat{\mu}^{(m)}_{x_i\to f_j}(x_i) \propto \frac{\beta^{(m)}_{x_i}(x_i)}{\mu^{(m-1)}_{f_j\to x_i}(x_i)}, \tag{16}$$

$\beta^{(m)}_{x_i}(x_i)$ is approximated by using the Gaussian PDF $\hat{\beta}^{(m)}_{x_i}(x_i) = \mathcal{N}_{\mathbb{C}}\left(x_i;\hat{x}^{(m)}_{x_i},\nu^{(m)}_{x_i}\right)$ and $\mu^{(m-1)}_{f_j\to x_i}(x_i) = \mathcal{N}_{\mathbb{C}}\left(h_{j,i}x_i;z^{(m-1)}_{f_j\to x_i},\tau^{(m-1)}_{f_j\to x_i}\right)$ is a Gaussian PDF. Thus, $\tilde{\mu}^{(m)}_{x_i\to f_j}(x_i)$ is also a Gaussian PDF, given by

$$\begin{aligned}\tilde{\mu}^{(m)}_{x_i\to f_j}(x_i) &\propto \frac{\beta^{(m)}_{x_i}(x_i)}{\mu^{(m-1)}_{f_j\to x_i}(x_i)} \\ &\approx \frac{\hat{\beta}^{(m)}_{x_i}(x_i)}{\mu^{(m-1)}_{f_j\to x_i}(x_i)} \\ &= \mathcal{N}_{\mathbb{C}}\left(x_i;\hat{x}^{(m)}_{x_i\to f_j},\nu^{(m)}_{x_i\to f_j}\right),\end{aligned} \tag{17}$$

where $\hat{x}^{(m)}_{x_i\to f_j}$ and $\nu^{(m)}_{x_i\to f_j}$ can be derived using the canonical form of the Gaussian PDF as follows.

$$\nu^{(m)}_{x_i\to f_j} = \left(\frac{1}{\nu^{(m)}_{x_i}} - \frac{\left|h_{j,i}\right|^2}{\tau^{(m-1)}_{f_j\to x_i}}\right)^{-1}, \tag{18}$$

$$\hat{x}^{(m)}_{x_i\to f_j} = \nu^{(m)}_{x_i\to f_j}\left(\frac{\hat{x}^{(m)}_{x_i}}{\nu^{(m)}_{x_i}} - \frac{h^*_{j,i}z^{(m-1)}_{f_j\to x_i}}{\tau^{(m-1)}_{f_j\to x_i}}\right). \tag{19}$$

The computation of $\hat{x}^{(m)}_{x_i} = \mathrm{E}_{\hat{\beta}^{(m)}_{x_i}(x_i)}[x_i]$ and $\nu^{(m)}_{x_i} = \mathrm{E}_{\hat{\beta}^{(m)}_{x_i}(x_i)}\left\{\left|x_i\right|^2\right\} - \left|\hat{x}^{m}_{x_i}\right|^2$ has a slightly high complexity. Note that according to the semantics of the factor graph, $\beta^{(m)}_{x_i}(x_i)$ is the posteriori probability of x_i. To reduce the complexity of in computing $\hat{x}^{(m)}_{x_i}$ and $\nu^{(m)}_{x_i}$, we use $\tilde{p}^{(m)}(x_i) = \prod_{q=1}^{Q}\tilde{p}^{(m)}(c_{i,q})$ to replace $\beta^{(m)}_{x_i}(x_i)$, where $\tilde{p}^{(m)}(x_i)$ is the posteriori probability of the coded bit $c_{i,q}$ fed back from the decoders. Then the mean $\hat{x}^{(m)}_{x_i}$ and the variance $\nu^{(m)}_{x_i}$ are given by

$$\hat{x}^{(m)}_{x_i} = \sum_{\alpha_S\in\mathcal{A}}\alpha_S\tilde{p}^{(m)}(x_i=\alpha_S), \tag{20}$$

$$\nu^{(m)}_{x_i} = \sum_{\alpha_S\in\mathcal{A}}\left|\alpha_S\right|^2\tilde{p}^{(m)}(x_i=\alpha_S) - \left|\hat{x}^{(m)}_{x_i}\right|^2. \tag{21}$$

Summarizing the above discussion, the EP algorithm is shown below (Algorithm 1).

Algorithm 1 The EP algorithm

Initialization: set $z^{(0)}_{f_j\to x_i}=0, \tau^{(0)}_{f_j\to x_i}=\infty$.
  and $\left\{\tilde{p}^{(m)}(c_{i,q})=\frac{1}{2}, p^{(m)}(c_{i,q})=\frac{1}{2}\right\}$.
for i=1→N **do**
  $\tilde{p}^{(m)}(x_i)=\prod_{q=1}^{Q}\tilde{p}^{(m)}(c_{i,q})$
  $\hat{x}^{(m)}_{x_i}=\sum_{\alpha_S\in\mathcal{A}}\alpha_S\tilde{p}^{(m)}(x_i=\alpha_S)$
  $\nu^{(m)}_{x_i}=\sum_{\alpha_S\in\mathcal{A}}\left|\alpha_S\right|^2\tilde{p}^{(m)}(x_i=\alpha_S)-\left|\hat{x}^{(m)}_{x_i}\right|^2$
  for i=1→M **do**
    $\nu^{(m)}_{x_i\to f_j}=\left(\frac{1}{\nu^{(m)}_{x_i}}-\frac{\left|h_{j,i}\right|^2}{\tau^{(m-1)}_{f_j\to x_i}}\right)^{-1}$
    $\hat{x}^{(m)}_{x_i\to f_j}=\nu^{(m)}_{x_i\to f_j}\left(\frac{\hat{x}^{(m)}_{x_i}}{\nu^{(m)}_{x_i}}-\frac{h^*_{j,i}z^{(m-1)}_{f_j\to x_i}}{\tau^{(m-1)}_{f_j\to x_i}}\right)$
  end for
end for
for j=1→M **do**
  $\tau^{(m)}_{f_j}=\sigma^2_n+\sum_i\left|h_{j,i}\right|^2\nu^{(m)}_{x_i\to f_j}$
  $z^{(m)}_{f_j}=y_j-\sum_i h_{j,i}\hat{x}^{(m)}_{x_i\to f_j}$
  for i=1→N **do**
    $\tau^{(m)}_{f_j\to x_i}=\tau^{(m)}_{f_j}-\left|h_{j,i}\right|^2\nu^{(m)}_{x_i\to f_j}$
    $z^{(m)}_{f_j\to x_i}=z^{(m)}_{f_j}+h_{j,i}\hat{x}^{(m)}_{x_i\to f_j}$
  end for
end for
for i=1→N **do**
  $\gamma^{(m)}_{x_i}=\left(\sum_j\frac{\left|h_{j,i}\right|^2}{\tau^{(m)}_{f_j\to x_i}}\right)^{-1}, \zeta^{(m)}_{x_i}=\gamma^{(m)}_{x_i}\sum_j\frac{h^*_{j,i}z^{(m)}_{f_j\to x_i}}{\tau^{(m)}_{f_j\to x_i}},$
  $\tilde{p}^{(m)}_{eq}(x_i)=\mu^{(m)}_{dec\to x_i}(x_i)\mathcal{N}_{\mathbb{C}}\left(x_i;\zeta^{(m)}_{x_i},\gamma^{(m)}_{x_i}\right)$
  for q=1→Q **do**
    $\mathcal{L}^{(m)}_e(c_{i,q})=\ln\frac{\sum_{\mathcal{A}^1_{i,q}}\tilde{p}^{(m)}_{eq}(x_i)}{\sum_{\mathcal{A}^0_{i,q}}\tilde{p}^{(m)}_{eq}(x_i)}-\mathcal{L}^{(m)}(c_{i,q})$
  end for
end for

3.3 Complexity

In this section, we compare the complexity of the proposed EP Algorithm with that of the iterative MMSE detection algorithm. Tab.1 shows the necessary FLOPs (Floating-Point Operations) counted per iteration. Complicated nonlinear functions such as the exponential function are implemented using LUTs (Loop-Up Tables) in DSP or FPGA. It is assumed that $\{\tilde{p}^{(m)}(c_{i,q}), p^{(m)}(c_{i,q})\}$ can be calculated by the decoders. We consider that a complex square matrix inverse needs $2N^3$ FLOPs of multiplication and addition.

For QPSK where Q=2 and $|\mathcal{A}|$=4. The complexity of the two algorithms versus the number of antennas is shown in Fig.3. It's observed that the complexity of the EP algorithm is clearly much lower than that of the MMSE algorithm. We define the total complexity as the total number of FLOPs of multiplications, divisions, additions and subtractions. The total complexity of the MMSE algorithm that is normalized over that of the of EP algorithm is shown in Fig.4. When the number of antennas is for QPSK, the total complexity of the EP algorithm is approximately 80 times less than that of the MMSE algorithm.

4 Numerical results

We consider a TDMA satellite communications system with N=127 spot beams, in which full frequency reuse is employed. Each of the N users, located randomly in the beams, is assumed to employ either QPSK modulation, 8PSK modulation or 16PSK modulation. A rate-1/2 convolutional code that has a length of 1 024 is used. As the interferences are mainly come from the adjacent beams, the number of interfering beams for one beam is set to be M=6.

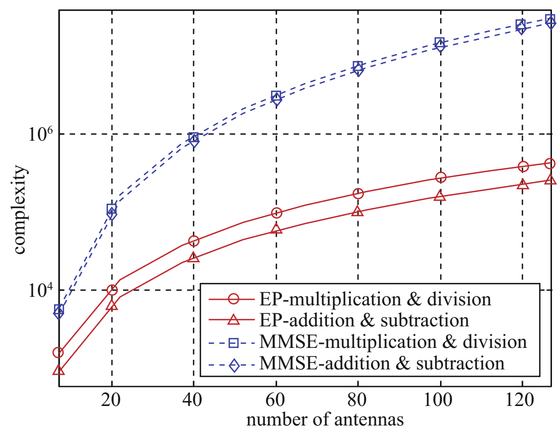


Figure 3 Complexity versus number of antennas N with QPSK for EP and MMSE algorithm

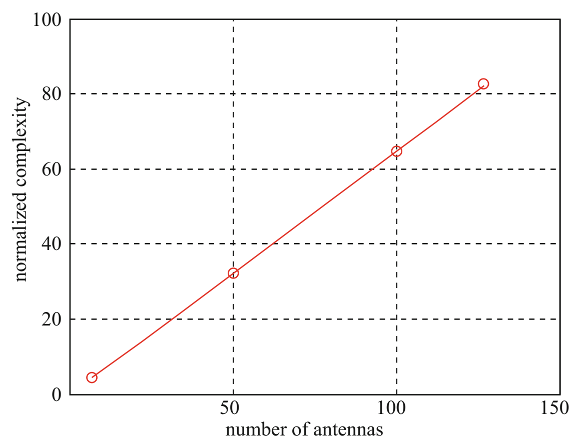


Figure 4 Total complexity of MMSE algorithm normalized over that of the EP algorithm

Fig.5 shows the numerical results of the transmissions in the multibeam reverse-link in terms of BER versus E_b/N_0, in which the performance of

Table 1 Complexity comparison in terms of FLOPs per iteration

algorithm	multiplication & division	addition & subtraction	LUT
EP	$27N^2+14N+5\vert\mathcal{A}\vert N+2QN$	$16N^2-N+4\vert\mathcal{A}\vert N+(\vert\mathcal{A}\vert-1)QN$	$(Q-1)N$
MMSE	$15N^3+12N^2+14N+5\vert\mathcal{A}\vert N+2QN$	$13N^3+6N^2+3N+4\vert\mathcal{A}\vert N+(\vert\mathcal{A}\vert-1)NQ$	$(Q-1)N$

the EP algorithm (labeled as EP) and the iterative MMSE interference cancellation algorithm (labeled as iteraMMSE) is compared for 10 iterations. We observe that the EP algorithm with low complexity performs well in the system with full frequency reuse, and approaches the performance of iterative MMSE interference cancellation algorithm. Irrespective of the type of modulation, when E_b/N_0 is below a threshold value, which is 3 dB, 5.5 dB and 8 dB for QPSK, 8PSK and 16PSK, respectively, the EP algorithm performs slightly better than the MMSE algorithm. However, when E_b/N_0 greater the value, the performance of both algorithm is the same.

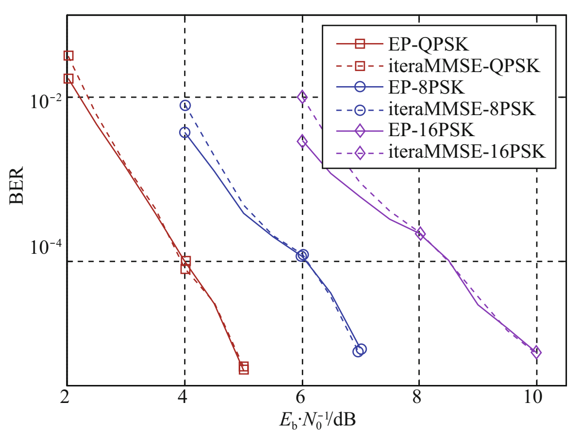


Figure 5 BER performance for the multibeam satellite reverse-link, K=10 dB, full frequency reuse

Fig.6 shows the BER performance versus number of iterations for both algorithms. Four iterations are sufficient to achieve convergence in both case.

Fig.7 shows the BER performance for the EP algorithm for different iterations. The modulation is 8PSK. It is observed that convergence is achieved quickly during the first 3 iterations. In the first iteration, although E_b/N_0 increases, the BER does not decrease, since inter beam interference is not cancelled. The BER performance at the third iteration is higher than that at the second iteration by 2.5 dB at 10^{-2}. It is also observed that the change in the performance after the 5th is negligible.

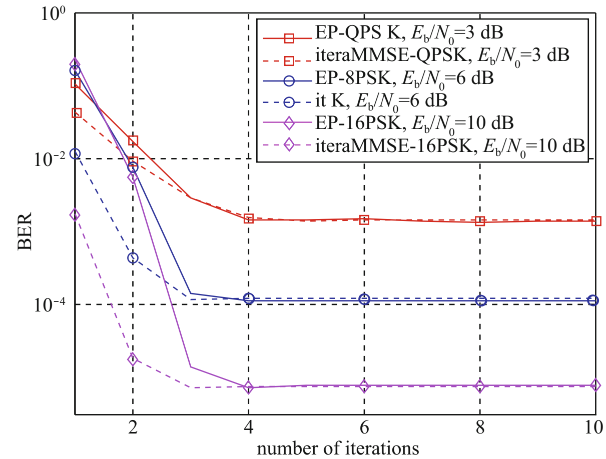


Figure 6 BER performance verse number of iterations for the multibeam satellite reverse link, K=10 dB, full frequency reuse

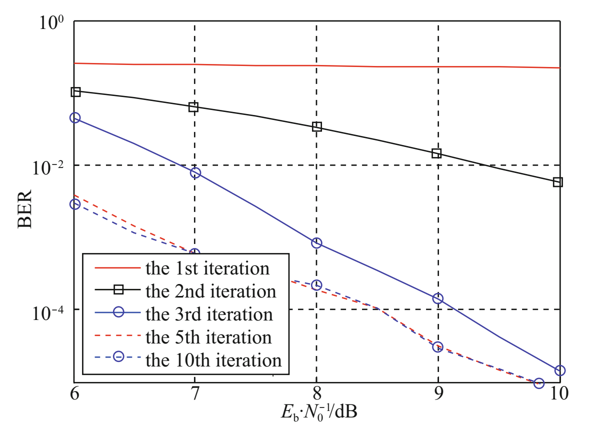


Figure 7 BER performance of the EP algorithm in the multibeam satellite reverse-link, 8PSK, K=10 dB, full frequency reuse

5 Conclusion

In this paper, an EP algorithm is proposed for the multibeam reverse link of satellite communications with full frequency reuse, which scales down the complexity per beam from $\mathcal{O}((N)^3)$ to $\mathcal{O}(N^2)$. Numerical results show that the performance of the proposed algorithm is slightly better than that of the iterative MMSE interference cancellation algorithm when E_b/N_0 is below the threshold value. When E_b/N_0

is above the threshold value, the performance of both algorithm is the same. Based on its good performance and low complexity, the EP algorithm has the potential to realize full frequency reuse in multibeam satellite communications, which can increase the spectral efficiency considerably.

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