The Complexity of Shortest Path and Dilation Bounded Interval Routing*

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Abstract: Interval routing is an attractive space-efficient routing method for point-to-point communication networks which found industrial applications in novel transputer routing technology.
Recently much effort is devoted to relate the efficiency (measured by dilation or stretch factor) to space requirements (measured by compactness or total memory bits) in a variety of compact routing methods [1, 5, 9, 10, 11, 15]. We add new results in this direction for interval routing.
For the shortest path interval routing we give a technique for obtaining lower bounds on compactness. We apply this technique to shuffle exchange graph of order n and get improved lower bound on compactness in the form $\Omega(n^{1/2-\epsilon})$, where ϵ is arbitary positive constant. In [8] we applied this technique also to other interconnection networks, obtaining new lower bounds $\Omega(\sqrt{n/\log n})$ for cube connected cycles and butterfly, and $\Omega(n(\log\log n/\log n)^5)$ for star graph. Previous lower bounds for these networks were only constant [4].
For the dilation bounded interval routing we give a routing algorithm with the dilation $\lceil 1.5D \rceil$ and the compactness $O(\sqrt{n \log n})$ on n-node networks with the diameter D. It is the first nontrivial upper bound on the dilation bounded interval routing on general networks. Moreover, we construct a network on which each interval routing with dilation $1.5D - 3$ needs compactness at least $\Omega(\sqrt{n})$. It is an asymptotical improvement over the previous lower bounds in [15] and it is also better than independently obtained lower bounds in [16].

1 Introduction

Interval routing is an attractive compact routing method for point-to-point communication networks. Interval routing was introduced in [13] and generalized in [17]. It has found industrial applications in INMOS T9000 transputer design.

Interval routing is based on compact routing tables, where the set of nodes reachable via outgoing links is represented by interval labels. By compactness we measure the maximum number of interval labels per link. By dilation we measure the length of the longest routing path in the network.

Most of the previous work was oriented towards optimal (shortest path) interval routing. Several classes of networks have optimal 1-IRS (i.e., routing schemes using up to 1 interval label per link). But there are also networks without optimal 1-IRS [4, 12, 14]. To overcome this inefficiency, a multi-label interval routing schemes were introduced. General n-node networks can be optimaly routed using $\lceil \frac{n}{2} \rceil$ intervals. When no specific assumption about the network topology is made, the number of required intervals does not significantly reduce. In [2], a technique for proving lower bounds on compactness was developed and

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it has been used in [6] to construct n-node networks for which each optimal k-IRS requires $k = \theta(n)$. A similar result for random networks was obtained in [2].

For certain symmetric and regular networks (such as hypercubes or tori), optimal k-IRS exists for small constant k. Natural question arises whether there are also optimal k-IRS for small k for the well-known interconnection networks, such as shuffle exchange (SE), cube connected cycles (CCC), butterfly (BF) and star networks (S). In [4], it was proved that these networks have no optimal 1-IRS. We introduce a technique for obtaining lower bounds on compactness for the optimal IRS on arbitrary networks. Using this technique we give a lower bound $\Omega(n^{\frac{1}{2}-\epsilon})$, $\epsilon > 0$, for SE of order n. In the full version of the paper [8] we applied this technique also to other networks, obtaining lower bounds on compactness in the form $\Omega(\sqrt{n/\log n})$ for CCC and BF, and $\Omega\left(n(\log\log n/\log n)^5\right)$ for S.

Recently, much effort is devoted to relate the efficiency (measured by dilation) to space requirements (measured by compactness). Each network has 1-IRS with dilation $2D$, where D is the diameter of the network [13]. However, there are also networks having long dilation for each 1-IRS. For n-node networks the lower bound for k-IRS with dilation $1.75D - O(1)$ was $k \geq 2$ [14], with dilation $1.25D - O(1)$ it was $k \geq 3$ [15] and with dilation $\frac{2k+1}{2k}D - 1$ and $\frac{6k+1}{6k}D - 1$ it was $k = \Omega(\sqrt[3]{n})$ and $k = \Omega(\sqrt{n})$, respectively [15]. The basic question is whether there are interval routing schemes for arbitrary networks attaining short dilation with reasonable small compactness. We answer this question in the negative way[2] by constructing an n-node network with the diameter D for which each routing scheme with dilation $1.5D - 3$ needs compactness $\Omega(\sqrt{n})$. Moreover, we give a routing algorithm with dilation $\lceil 1.5D \rceil$ and compactness $O(\sqrt{n\log n})$. It is the first nontrivial upper bound for the dilation bounded interval routing on general networks.

1.1 Definitions

We assume a point-to-point asynchronous communication network. The network topology is modeled by a simple connected graph $G = (V, E)$, where V is a set of vertices (or processors) and E is a set of edges (or bidirectional links). Assume $|V| = n$. The diameter of G is denoted as $D(G)$. Given a vertex $v \in V$, by $I(v)$ we denote the set of arcs outgoing from v. By $deg(v)$ we denote the degree of v.

In *k-interval routing scheme* (shortly k-IRS), each vertex is labeled by unique element from the set $\{1, \ldots, n\}$ and each arc is labeled by up to k cyclic intervals. The routing is performed in the following way. Let a message destinated to a vertex w currently reach some vertex u, $u \neq w$. Determine the *unique* arc $e \in I(u)$ such that the label of w belongs to an interval assigned to e and transmit a message along e. The scheme should be correct, i.e. it is possible to send a message between any two vertices. The label of a vertex v in routing ρ is denoted $\rho(v)$.

Given a graph G and a k-IRS ρ on G, a *routing path system* (for ρ on G) is the set of routing paths between all pairs of vertices in V. The *dilation*, denoted as $dil(G, \rho)$, is the length of the longest path in the routing path system for ρ on

[2] The same conclusion, independently of [8], was obtained by Tse and Lau [16]. However, they proved weaker results of compactness $\Omega(\log n)$ for dilation $1.5D - O(1)$ and of compactness $\Omega(\sqrt{n})$ for dilation $1.25D - O(1)$.

G. k-IRS is called *optimal*, if all paths in the routing path system are the shortest ones. k-IRS is called *α-bounded* (shortly (k,α)-IRS) if the dilation $dil(G,\rho)$ is limited to α. For optimal routing the *compactness* of G is the minimum k such that there is k-IRS on G. For α-bounded routing the compactness of G denotes the minimum k such that there is (k,α)-IRS on G.

2 Shortest Path Interval Routing

This section is devoted to the shortest path interval routing for some interconnection networks. We present a technique for obtaining a lower bound on compactness for the shortest path routing on arbitrary graphs. A similar technique is given in [2] and also used in [6]. Then, we apply this technique to shuffle exchange graphs and get asymptotical improvement over the previous constant lower bound [4]. Further results concerning cube connected cycles, butterfly and star graphs are given in the full version of the paper [8].

2.1 A Lower Bound Technique for General Graphs

Let $G=(V,E)$ be a simple connected graph. Let $Q=\{q_0,\ldots,q_{l-1}\}$ and $W=\{w_0,\ldots,w_{m-1}\}$ be disjoint subsets of V. We say that W and Q satisfy the *wq-property* iff for any distinct vertices $w_i, w_j \in W$ there exists a vertex $v\in Q$ such that in arbitrary optimal routing scheme the messages from v to w_i and w_j are routed along different outgoing arcs (i.e., for any arc e outgoing from v there don't exist shortest paths to vertices w_i and w_j, both starting with arc e.)

Theorem 1 *Let ρ be an optimal k-IRS of a given graph $G=(V,E)$. Let W and Q be sets satisfying wq-property. Then it holds*

$$k \geq \frac{|W|}{\sum_{v\in Q} deg(v)} \tag{1}$$

Proof: W.l.o.g. assume that $\rho(w_0) < \rho(w_1) < \ldots < \rho(w_{m-1})$. For any v and $e\in I(v)$ denote $R(v,e)$ the set of vertices such that the messages from v destinated to them are routed along arc e in routing ρ. There are at most k intervals on any arc, therefore for any pair $v\in Q$ and $e\in I(v)$ it holds [3] $\sum_{w_j\in W}(w_j\in R(v,e)\wedge w_{j\oplus 1}\notin R(v,e))\leq k$ and consequently

$$\sum_{v\in Q}\sum_{e\in I(v)}\sum_{w_j\in W}(w_j\in R(v,e)\wedge w_{j\oplus 1}\notin R(v,e))\leq k\cdot\sum_{v\in Q} deg(v) \tag{2}$$

On the other hand, for any $w_j, w_{j\oplus 1}$ take the v from the wq-property. Let $e\in I(v)$ be an arc along which messages from v to w_j are routed. From the wq-property $w_{j\oplus 1}\notin R(v,e)$ and therefore $\sum_{v\in Q}\sum_{e\in I(v)}(w_j\in R(v,e)\wedge w_{j\oplus 1}\notin R(v,e))\geq 1$. Hence

$$\sum_{w_j\in W}\sum_{v\in Q}\sum_{e\in I(v)}(w_j\in R(v,e)\wedge w_{j\oplus 1}\notin R(v,e))\geq |W|. \tag{3}$$

Combining inequalities (2) and (3) we get (1). □

[3] We use $\oplus$ and $\ominus$ for the addition and subtraction modulo m.

2.2 A Lower Bound for Shuffle Exchange

Denote the left cyclic shift and the right cyclic shift operations on binary strings as L and R respectively and the shuffle operation corresponding to altering least significant bit as S.

Shuffle exchange graph of degree d (denoted as $SE(d)$) is a graph whose vertices are all binary strings of length d and two vertices u, v are connected by an edge if v can be obtained from u using L,R or S operation. The arc (u, v) is called L-arc, R-arc or S-arc depending on whether $v = L(u)$, $v = R(u)$ or $v = S(u)$. To each path $C \equiv v_0, \ldots, v_p$ in $SE(d)$ assign *the characteristic sequence* $C' \equiv e_0, \ldots, e_{p-1}$, where $e_i \in \{L, R, S\}$ is the name of the arc (v_i, v_{i+1}).

Claim 1 *Let C be a path in $SE(d)$ from v_0 to v_p, and C' be its characteristic sequence. Then $\#_S C' \geq |\#_1 v_0 - \#_1 v_p|$.*

It is convenient to represent vertices of $SE(d)$ as binary strings with cursors denoting the least significant bit, cyclicly. For example, $111\underline{1}0101$ denotes the string 10111110. To move to neighbouring vertex it's enough to move the cursor to the left, to the right or change the bit pointed by cursor. If $C \equiv v_0, \ldots, v_p$ is a path with characteristic sequence $C' \equiv e_0, \ldots, e_{p-1}$, then the cursor positions $k^{(i)}, 0 \leq i \leq p$ are as follows: $k^{(0)} = 0$, $k^{(i+1)} = k^{(i)} \ominus 1$ if $e_i = L$, $k^{(i+1)} = k^{(i)} \oplus 1$ if $e_i = R$, and $k^{(i+1)} = k^{(i)}$ if $e_i = S$. The string with cursor which represents v_i is $a^{(i)} = a^{(i)}_{d-1} \ldots \underline{a^{(i)}_{k^{(i)}}} \ldots a^{(i)}_0$.

In [8] we have proved the following lemma [4] used in Theorem 2.

Lemma 1 *Let $C \equiv v_0, \ldots, v_p$ be a path in the graph $SE(d)$ with associated characteristic sequence $C' \equiv e_0, \ldots, e_{p-1}$ and cursor positions $k^{(0)}, \ldots, k^{(p)}$. Let $x_1 < \ldots < x_{t-1}$ be the positions at which $v_0 = a^{(0)}$ and $L^{k^{(p)}}(v_p) = a^{(p)}$ differ and let $x_0 = 0$. It holds $\#_{L,R} C' \geq d - \max_{i \in \{0,\ldots,t-1\}}(x_{i \oplus 1} \ominus x_i)$. Moreover, if the equality holds, then there are either only L's or only R's in $C'/_{L,R}$.*

Theorem 2 *For arbitrary constant $\epsilon > 0$ each optimal k-IRS of the shuffle exchange graph $SE(d)$ requires $k = \Omega\left(|V|^{\frac{1}{2}-\epsilon}\right)$ intervals.*

Proof: Let $d = 2(m+1)^2 + p - 1$, where $p = O(\sqrt{d})$. Consider the following sets W and Q:

$$W = 1^p(\{0,1\}^m 1)^m 0^m \underline{1} 0^m (1\{0,1\}^m)^m$$

$$Q = \bigcup\{0^{p+m(m+1)-|a|-1} \underline{0} a 0^m 0 0^{m(m+2)}\} \cup \bigcup\{0^{p+m(m+2)} 0 0^m b \underline{0} 0^{m(m+1)-|b|-1}\}$$

where the first union in Q is taken over all suffixes a of all strings from $(\{0,1\}^m 1)^m$ with the length different from $(m+1)i + 1$ for all $i \in \{0, \ldots, m-1\}$ and the second union is taken over all prefixes b of all strings from $(1\{0,1\}^m)^m$ with the length different from $(m+1)i + 1$ for all $i \in \{0, \ldots, m-1\}$.

Clearly, $|W| = 2^{2m^2}$ and $|Q| = 2.(2^{m^2+1} - 1)$. We need to show that W, Q satisfy the wq-property of Theorem 1. Consider w_1, w_2 from W, $w_1 \neq w_2$. W.l.o.g. suppose that w_1 and w_2 differ at some position to the left of the cursor. Then

[4] We use $\#_L C'$ for the number of occurences of L in C' and $C'/L, R$ for the maximal subsequence of C' consisting of L, R.

$w_1 = 1^p r_1 0 q 0^m \underline{1} 0^m s_1$, $w_2 = 1^p r_2 1 q 0^m \underline{1} 0^m s_2$. Choose the following v from Q: $v = 0^p 0^{|r_1|} \underline{0} q 0^m 0 0^m 0^{|s_1|}$. Take the following path from v to w_i: move the cursor to the left until it reaches the same position as the cursor in w_i, and along the way change all bits in which w_i and v differ. We have obtained a path of the length $\#_1 w_i - \#_1 v + d - |q| - m - 1$. Due to the Claim 1, for any shortest path from v to w_i with associated characteristic sequence C', we obtain $\#_S C' \geq \#_1 w_i - \#_1 v$. Combining this bound with the previous upper bound for the length of the path we obtain $\#_{L,R} C' \leq d - |q| - m - 1$. Observe that $L^{k^{(p)}}(w_i)$ doesn't contain $m+1$ consecutive 0's for any $k^{(p)}$. If $x_1 < \ldots < x_{t-1}$ are positions at which v and $L^{k^{(p)}}(w_i)$ differ and $x_0 = 0$, then

- If $x_i < x_{i \oplus 1} \leq d - 1 - |q|$, we have $x_{i\oplus 1} \ominus x_i \leq m + 1$, because of previous observation and also due to the fact that bits $0, \ldots, d-1-|q|$ are 0's in v.
- If $x_i \leq d - 1 - |q|$ and either $x_{i\oplus 1} = 0$ or $x_{i \oplus 1} > d - 1 - |q|$, then $x_{i\oplus 1} \ominus x_i \leq m + 1 + |q|$ due to the same reason.
- If $x_i > d - 1 - |q|$, then simply $x_{i \oplus 1} \ominus x_i \leq |q| - 1$.

So we have $\max_{i\in\{0,\ldots,t-1\}}(x_{i\oplus 1} \ominus x_i) \leq m + 1 + |q|$ and using Lemma 1 we get $\#_{L,R} C' \geq d - |q| - m - 1$. Therefore, for the shortest path it holds $\#_{L,R} C' = d - |q| - m - 1$ and from the second part of Lemma 1 it follows that there are only R's or only L's in $C'/_{L,R}$. The case that there are only L's does not work, because we will need more than d cursor moves to the right. It follows, that there is exactly one shortest path from v to w_1, which starts with R-edge and there is exactly one shortest path from v to w_2, which starts with S-edge, therefore wq-property from Theorem 1 is satisfied and the following bound on k necessary for any optimal k-IRS of $SE(d)$ holds:

$$k \geq \frac{|W|}{\sum_{v \in Q} deg(v)} = \frac{2^{2\cdot m^2}}{3 \cdot 2 \cdot (2^{m^2+1} - 1)} > 2^{m^2-4}.$$

It holds $m = \lfloor \sqrt{(d - O(\sqrt{d}))/2} \rfloor - 1$. Hence $2^{m^2-4} = 2^{d(\frac{1}{2} - O(d^{-1/2}))}$ and therefore for any positive constant ϵ it holds $k = \Omega(|V|^{\frac{1}{2}-\epsilon})$. □

3 Interval Routing with Bounded Dilation

Dilation bounded interval routing was studied in [12, 13, 14, 15]. Each graph has $(1, 2D)$-IRS [13] and can be optimaly routed with compactness $|V|/2$. Moreover, there are graphs for which $(1.75D - 1)$-bounded routing requires compactness at least 2 [14] and $(1.25D - 1)$-bounded routing compactness at least 3 [15]. The basic question is whether one can hope to find interval routing scheme for an arbitrary graph with short dilation and simultaneously with reasonably small compactness. The main result of this section is a negative answer to this question, stating that there are graphs for which routing with dilation $1.5D - 3$ needs compactness $\Omega(\sqrt{|V|})$. We also show that $O(\sqrt{|V| \log |V|})$ compactness is sufficient for routing arbitrary graphs with dilation $\lceil 1.5D \rceil$.

3.1 A Lower Bound on Dilation Bounded Interval Routing

Assume $B \subseteq \{1, \ldots, n\}$. A set A is called *k-interval representable* (shortly k-I) in the set B if there exist k cyclic intervals $I_1, \ldots, I_k$ such that $(\bigcup_{i=1}^{k} I_i) \cap B = A$.

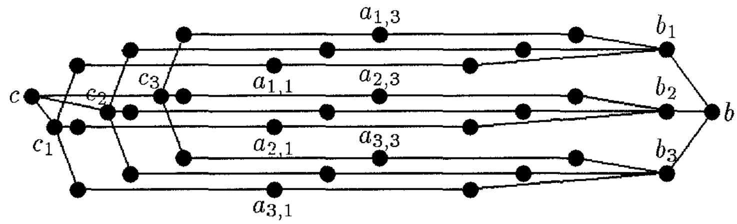
The elements of the set B are cyclicly ordered, therefore define *successor* of $b \in B$ as the next element in this cyclic ordering. An element a of $A \subseteq B$ is called an *isolated* element in A w.r.t. B, if its successor in B is not in A, otherwise a is called an *inner* element in A w.r.t. B. It is obvious, that if A is k-I w.r.t. B then the number of isolated elements in A is at most k and that there are at least $|A| - k$ inner elements in A.

Lemma 2 *Assume $M = \{a_{i,j} \mid 1 \leq i \leq s, 1 \leq j \leq v\}$ is $s \times v$ matrix of distinct elements from $\{1, \ldots, n\}$ such that every column $C_j = \{a_{i,j} \mid 1 \leq i \leq s\}$ and every row $R_i = \{a_{i,j} \mid 1 \leq j \leq v\}$ is k-I in M. Then $k \geq \frac{sv}{s+v}$.*

Proof: Let P be the number of isolated elements in sets $R_1, \ldots, R_s$ w.r.t. M. In every k-I set there are at most k isolated elements, so we have $P \leq sk$. Similarly, there are at least $v(s-k)$ inner elements in sets $C_1, \ldots, C_v$ and one can observe that each of them is isolated in some R_i. It follows $P \geq v(s-k)$. Combining both inequalities we get $k \geq \frac{sv}{s+v}$. □

Further, we construct a graph $F(s,v,r)$ such that due to the Lemma 2 each interval routing scheme on F with the dilation bounded by $1.5D - 3$ requires compacity at least $\frac{sv}{s+v}$.

Graph $F(s,v,r)$ is defined as follows. There are $s \times v$ middle vertices $\{a_{i,j}\}$ which form $s \times v$ rectangle, v column vertices $\{c_i\}$, s row vertices $\{b_j\}$ and two special vertices b, c. A column vertex c_i (row vertex b_j) is connected with every vertex from the i-th column (j-th row) of the rectangle via unique path of the length r. The vertex c is connected with all column vertices c_i and the vertex b with all row vertices b_j. Graph $F(s,v,r)$ has $(2r-1)sv + s + v + 2$ vertices, $2svr + s + v$ edges and its diameter is $2r + 2$. We give an example of $F(3,3,2)$.



Theorem 3 *For arbitrary k, there is a graph F of the size $\Theta(k^2)$ such that there is no $(k, 1.5D-3)$-IRS of the graph F.*

Proof: Let ρ be some $(k, 1.5D-3)$-IRS of the graph $F(s,v,r)$. As ρ is $(1.5D-3)$-bounded, for all i, j, messages from c (from b) must be routed along arc (c, c_j) (along arc (b, b_i)), otherwise the length of some routing path would be at least $3r+1$, thus longer than $1.5D-3$. Now, take $s \times v$ matrix M consisting of labels of vertices $a_{i,j}$, $1 \leq i \leq s$, $1 \leq j \leq v$. Columns and rows of this matrix must be k-I in M and therefore applying Lemma 2, we get $k \geq \frac{sv}{s+v}$. Choosing $s = v = 2(k+1)$ we get a contradiction, hence there does not exist $(1.5D-3)$-bounded k-IRS of the graph $F(2k+1, 2k+1, r)$. □

Corollary 1 *There are graphs $F = (V, E)$ such that each $(k, 1.5D-3)$-IRS of F needs $k = \Omega(\sqrt{|V|})$.*

3.2 An Upper Bound on Dilation Bounded Interval Routing

In this subsection we show that every graph has interval routing with dilation $\lceil 1.5D \rceil$ and compactness $O(\sqrt{|V| \log |V|})$. We need the following lemma.

Lemma 3 *Let $G=(V,E)$ be a graph. There is a set $C \subseteq V$ such that $|C| = O(\sqrt{|V| \log |V|})$ and for $v \in V$ it holds $d(v, C) \leq \lceil \frac{1}{2} D \rceil$.*

Proof: Let $V = \{1, ..., n\}$ and $m = \lceil \sqrt{n \ln n} \rceil$. For every vertex $v \in V$ define the set $V_v \subseteq V$ as the set of vertices whose distance from v is at most $\lceil \frac{1}{2} D \rceil$. If there exists $v \in V$ such that $|V_v| \leq m$, then it is obvious that we can set $C = V_v$ and the lemma holds. If such v doesn't exist (i.e, for all $v \in V$ it holds $|V_v| > m$), we prove the lemma by contradiction. Suppose that the lemma doesn't hold. Therefore if we take the union of any m sets from $V_1, ..., V_n$, then at least one element from V is not contained in this union. There are $\binom{n}{m}$ possibilities how to choose these m sets and from the pigeon-hole principle follows that there exists $a \in V$ such that a is missing in at least $\binom{n}{m}/n$ choices. On the other hand $|V_a| > m$, therefore a is not contained in at most $n - m$ sets and the number of choices with a missing is at most $\binom{n-m}{m}$. From this we get inequality $\binom{n-m}{m} \geq \binom{n}{m}/n$, which is a contradiction. □

Theorem 4 *Let $G = (V, E)$ be a graph. There is an interval routing scheme of G with the dilation $\lceil 1.5D \rceil$ and compactness $O(\sqrt{|V|. \log |V|})$.*

Proof: Take the set $C = \{c_1, ..., c_m\} \subseteq V$ from the previous lemma. Divide the set V into non-intersecting subsets $R_1, ..., R_m$ such that for any vertex $v \in R_i$ it holds $d(c_i, v) \leq \lceil \frac{1}{2} D \rceil$ and the subgraph of G induced by R_i (denoted as G/R_i) is connected for all $i \in \{1, ..., m\}$. Subgraphs G/R_i are called clusters and vertices c_i cluster centers. Given the set C we can find this division as follows. Set $\forall i \in \{1, ..., m\} : R_i = \{c_i\}$. Then repeat $\lceil \frac{1}{2} D \rceil$ times: for each $i \in \{1, ..., m\}$ set $R_i := R_i \cup \{$ free vertices adjacent to $R_i\}$.

Construct BFS spanning tree T_i from each center $c_i \in C$. First, create tree-labeling scheme on the subtree T_i/R_i from the root c_i following the technique from [13] (two intervals per arc are required). Vertices in R_i will have consecutive labels for all $i \in \{1, ..., m\}$. Then, assign interval corresponding to R_i to each arc of T_i not belonging to the cluster G/R_i and oriented towards the center c_i. Such interval routing scheme has compactness at most $m + 1$ (as each arc belongs to at most m trees, in $m - 1$ trees it is assigned 1 interval and in one tree it is assigned two intervals). The dilation is at most $D + \lceil D/2 \rceil = \lceil 1.5D \rceil$. □

As a consequence of the above techniques for general graphs we can obtain asymptoticaly tight trade-offs between dilation and compactness for some special classes of graphs. In [8] we proved that the compactness $\theta(\sqrt{n})$ can be achieved for dilation up to $1.25D - 1$ and $O(1)$ for dilation $1.25D$ on multiglobe graphs and the compactness $\theta(\sqrt{n})$ can be achieved for dilation D and $O(1)$ for dilation $(1 + \epsilon)D$ on globe graphs.

4 Conclusion

We proved that large compactness is needed for optimal interval routing on certain regular and symmetric topologies used in parallel architectures. The main question remains whether this phenomenon holds also for near-optimal interval routing on these topologies.

We also improved a lower bound on compactness for the dilation bounded interval routing on general n-vertex graphs[5]. An upper bound shows that for interval routing with dilation $\lceil 1.5D \rceil$ the compactness is $O(\sqrt{n \log n})$. Thus the compactness threshold is achieved for dilation $1.5D - O(1)$. The main unresolved problem is to exhibit a tight trade-off between dilation and compactness for general graphs.

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[5] Recently the lower bound was improved by Flammini and Nardelli [3] to compactness $\Omega(n/\log n)$ for dilation $1.5D - 2$ and the same result was independently obtained by Gavoille [7].