OPTIMAL FILTER FOR EDGE DETECTION METHODS AND RESULTS

Serge CASTAN, Jian ZHAO
Lab. IRIT-CERFIA, UPS
118, route de Narbonne
31062 Toulouse FRANCE

Jun SHEN
Southeast University
Nanjing, CHINA

Abstract

In this paper, we give a new demonstration in which it is proved that the symmetric exponential filter is the optimal edge detection filter in the criteria of the signal to noise ratio, localization precision and unique maximum. Then we deduce the first and the second directional derivative operators for symmetric Exponential Filter and realize them by first order recursive algorithm, and propose to detect the edges by maxima of Gradient (GEF), or by the zeros crossing of Second directional Derivative along the gradient direction (SDEF).

I. Introduction

Edge detection is one of the most important subjects in image processing, which finds wide applications in the pattern recognition, the scene analysis and the 3-D vision, because the edges correspond in general to the important changes of physical or geometrical properties of objects in the scene and they are widely used as primitives in the pattern recognition, the image matching etc.

The edges coincide, generally speaking, grey level transition, they can be detected by maxima of gradient or the zero-crossing of the second derivatives calculated by some differential operators. Because the differential operators are sensitive to noise, a preprocessing such as smoothing is often necessary to eliminate the noise. A well-known soomthing filter is Gaussian filter and the edges can therefore be detected by a Laplacien-Gaussian filter. But there is an essential difficulty of the Laplacien-Gaussian filter which is the contradiction between the smoothing effect and the precision of edge localization. To overcome this difficulty, We proposed the optimal linear filter based on one step model (a step edge and the white noise) and the multi-edge model [9][10][11]. This optimal smoothing filter is a symmetric exponential filter of an infinitely large window size and can be realized by very simple recursive algorithm. It is proved that the band limited Laplacien of an input image filtered by this filter can be calculated from the Difference between the input and the output of this Recursive Filter (DRF). The edges detected by DRF method are less noisy and with a much better precision of localization.

The maxima of gradient or zeros of the second directional derivative along the gradient are a natural definition of intensity edges. Zeros of the Laplacian are only extensively used for their computational convenience. However, we must stress here that the zeros crossing of the Laplacien are not always coincided with the maxima of gradient, for example, the zeros of the Laplacien are farther apart than the maxima of gradient for circularly symmetric patterns, this lack of localization by the Laplacien can also be seen in the fact that zeros of Laplacien "swing wide" of corners. Therefore, it had better to detect the edges by maxima of gradient or zeros of the second directional derivative along the gradient .

In this paper, we give a new demonstration in which it is proved that the symmetric exponential filter is the optimal edge detection filter in the criteria of the signal to noise ratio, localization precision and unique maximum. then we deduce the first and the second directional derivative operators for symmetric Exponential Filter, and realize them by the first order recursive algorithm. Using these operators, we propose two methods for edges detection, one uses maxima of Gradient (GEF), another uses the zeros crossing of Second directional Derivative along the gradient (SDEF). It is done for the theoretical analysis on comparing the performance of the filters. The performances of the GEF, SDEF methods and DRF method for edge detection are compared experimentally, the results show that the new methods are less sensitive to noise and have much better precision of edge localization.

II. Optimal Filter for Edges Detection

Let f(x) be the low-pass smoothing filter function that we want to find which gives the best results for step edge detection. The input signal is a step edge plus the white noise.[9,10]

We define a measure SRN of the signal to noise performance of the operator which is independent of the input signal:

$$SRN = f^2(0)/(\int_{-\infty}^{\infty} f^2(x)dx) \qquad (1)$$

Because the maxima of the first derivative correspond to the edge points, an optimal filter f'(x) should maximaze the ratio of the output in response to step input to the output in response to the noise, i.e. maximaze the criterion SRN (the CANNY's criterion Σ). Besides, the edge position is localized by the zero-crossing of the second derivative, therefore we should diminish noise energy in the second derivative of output signal, i.e. maximaze at the same time the other criterion as follows:

$$C = 1/\int_{-\infty}^{\infty} f''^2(x)dx \qquad (2)$$

To maximaze at the same time the criteria (1) and (2), we can maximaze the criterion Cr which is the product of the two criteria as follows:

$$Cr = f^2(0)/(\int_{-\infty}^{\infty} f^2(x)dx \cdot \int_{-\infty}^{\infty} f''^2(x)dx) \qquad (3)$$

In order to avoid the introduction of additional maxima at the positions other than the edge position, this maximum at x=0 should be unique, i.e. f(x) should satisfy[10]:

$$\max_{x \in (-\infty, \infty)} f(x) = f(0) > 0 \qquad (4)$$

and : $f(x_1) > f(x_2)$ when $|x_1| < |x_2|$

and in order to maximaze Cr, f(x) should be an even function [10]. Eq.(3) can be written as :

$$Cr = f^2(0)/(2 \cdot \int_0^{\infty} f^2(x)dx \cdot \int_0^{\infty} f''^2(x)dx) \qquad (5)$$

Except signal to noise ratio and unique maximum, the other important performance of the filter for edges detection is the localization precision. For the localization criterion we proceed as follows.

Because the zero-crossing of the second derivative correspond to the edge position, we can find edge position x1 by the inequality as follows:

$$\begin{cases} S_o''(x_1^-) \geq 0 \\ S_o''(x_1^+) \leq 0 \end{cases} \quad \text{or :} \quad \begin{cases} S_o''(x_1^-) \leq 0 \\ S_o''(x_1^+) \geq 0 \end{cases} \qquad (6)$$

where, $S_o(x)$ is the output signal, $x_1^- = \lim_{\varepsilon \to 0}(x_1 - \varepsilon)$ et $x_1^+ = \lim_{\varepsilon \to 0}(x_1 + \varepsilon)$ $\quad \varepsilon > 0$

$$S_o''(x) = (S(x)*f(x))'' + (N(x)*f(x))'' = S_V(x) + S_N(x) \qquad (7)$$

If f'(x) and f''(x) are continuous at x=0, we have:

$$x_1 \approx -S_N(0)/ A \cdot f''(0)$$

We can notice that this ratio means CANNY's localization criterion [15], obtained from a more general approach.

From this formula, if the noise exist ($S_N(0) \neq 0$), the localization error ($x_1 \neq 0$) always exist. So, with this CANNY's criterion, it is theoretically impossible to find a filter without any localization error ($x_1 = 0$).

But if f'(x) is not continuous at x=0, and f(x) is an even function which satisfy the formula (4), we have :

$|f'(0^-)| = |f'(0^+)|$, $f'(0^-) > 0$ and $f'(0^+) < 0$, f'(x) is odd function.

Provided the absolute value of the noise is less than that of the signal, i.e. $S_N(0) < |f'(0^+)|$, we can satisfy the inequality (6) at $x_1=0$ (no error).

So, we should put in the condition as follows to the criterion (5) for obtaining the best localization precision ($x_1 = 0$).

$$f'(0^+) < 0 \qquad (8)$$

Besides, a stable infinite window size filter must satisfy the boundary conditions :

$$f(\infty) = 0,\ f'(\infty) = 0,\ f''(\infty) = 0 \qquad (9)$$

To sum up, the optimal edge detection filter has now been defined implicitly by maximazing the criterion (5) (signal to noise ratio) , and this filter must satisfy the conditions (4) (unique maximum), (8) (localization precision) and (9) (stability).

We consider at first a filter f(x) with a limited window size 2W, the equations (5) become:

$$Cr = f^2(0)/(2 \cdot \int_0^W f^2(x)dx \cdot \int_0^W f''^2(x)dx) \qquad (10)$$

In order to find the optimal function which maximazes the criterion (10), we can form a composite function $\psi(x,f',f'')$, and finding a solution for this unconstrained problem as follows is equivalent to finding the solution to the constrained problem.

$$C_N = \int_0^W \psi(x,f',f'')dx \qquad (11)$$

where: $\psi(x,f',f'') = f'^2(x) + \lambda \cdot f''^2(x)$

This is an example of what is known as reciprocity in variational problems. The optimal solution should satisfy the Euler equation :

$$2 \cdot \lambda \cdot f''''(x) - 2 \cdot f'' = 0 \qquad (12)$$

The general solution f'(x) of this differential equation is :

$$f'(x) = C_1 e^{-\alpha x} + C_2 \cdot e^{\alpha x} + C_3 \qquad (13)$$

where $\alpha = 1/(\lambda)^{1/2}$ and C_1, C_2, C_3 are the constants.

For obtaining an infinite window size filter, we should consider the case where $w \to \infty$.

From Eq. (13), the constants C_1, C_2, C_3 can be determined as follows by the conditions (8) and

(9).

$C_2 = C_3 = 0$ and $C_1 < 0$

Because f'(x) is an odd function, f'(x) can be written in the form :

$$f'(x) = \begin{cases} C_1 e^{-\alpha x} & x>0 \\ - C_1 e^{-\alpha x} & x<0 \end{cases} \qquad (14)$$

And we can equally get the low-pass filter f(x) as follows, which satisfy the condition (4).

$$f(x) = C \cdot e^{-|x|} \qquad (15)$$

where the constant $C > 0$.

Thus far we find the optimal edge detection filter which is a symmetric exponential filter, in the criteria of the output signal to noise ratio, localization precision and unique maximum. The means of these criteria are similar to that of CANNY, but we search the filter with an infinite window size and no function continuity constraint is used. So, our result is more general than that of CANNY.

III. The First and the Second Directional Derivative Operators for Symmetric Exponential Filter

A normalized symmetric exponential filter on 1-D can be written :

$$f_L(x) = C \cdot a_0 \cdot (1-a_0)^{|x|} = f_1(x) * f_2(x) = C \cdot (f_1(x) + f_2(x) - a_0 \cdot \delta(x)) \qquad (16)$$

where: $C = 1/(2-a_0)$, $*$ means the convolution.

$$f_1(x) = \begin{cases} a_0 \cdot (1-a_0)^x & x \geq 0 \\ 0 & x<0 \end{cases} \qquad f_2(x) = \begin{cases} 0 & x>0 \\ a_0 \cdot (1-a_0)^{-x} & x \leq 0 \end{cases} \qquad (17)$$

we can write the first derivative operator of exponential filter:

$$f_L'(x) = f_2(x) - f_1(x) \qquad (18)$$

And we can obtain the normalized second derivative operator of exponential filter :

$$f_L''(x) \approx f_1(x) + f_2(x) - 2 \cdot \delta(x) \qquad (19)$$

Because the exponential fonction is separable, we can write out 2-D exponential filter:

$$f(x,y) = f_L(x) \cdot f_L(y) \qquad (20)$$

From the equations (18),(19) and (20), the first and the second directional derivative operators for symmetric exponential filter can be written like this:

$$f_x(x,y) = f_L(y) \cdot (f_2(x) - f_1(x)) \qquad (21)$$
$$f_y(x,y) = f_L(x) \cdot (f_2(y) - f_1(y)) \qquad (22)$$
$$f_{xx}(x,y) = f_L(y) \cdot (f_1(x) + f_2(x) - 2 \cdot \delta(x)) \qquad (23)$$
$$f_{yy}(x,y) = f_L(x) \cdot (f_1(y) + f_2(y) - 2 \cdot \delta(y)) \qquad (24)$$

IV. The Recursive Algorithm for realizing the These Directional Derivative Operators of Symmetric Exponential Filter

The exponential filter is an IIR filter corresponding to an infinite window size, so we should realize the fonctions $f_1(x)$ and $f_2(x)$ (see formula (17)) by a recursive algorithm.

Supposing $I(x,y)$ is the input image, $I_1(x,y) = I(x,y) * f_1(x)$ and $I_2(x,y) = I(x,y) * f_2(x)$, we have the recursive algorithm :

$$I_1(x,y) = I_1(x-1,y) + a_0 \cdot (I(x,y) - I_1(x-1,y)) \qquad (25)$$
$$I_2(x,y) = I_2(x+1,y) + a_0 \cdot (I(x,y) - I_2(x+1,y))$$

From the equations (16),(21),(22),(23) and (24), the band-limited first and second directional derivative of input image can be calculated by the recursive algorithm f_1 and f_2 as follows

$$I_x(x,y) = I(x,y) * f_1(y) * f_2(y) * (f_2(x) - f_1(x)) \qquad (26)$$
$$I_{xx}(x,y) = I(x,y) * f_1(y) * f_2(y) * (f_2(x) + f_1(x)) - 2 \cdot I(x,y) * f_1(y) * f_2(y) \qquad (27)$$
$$I_y(x,y) = I(x,y) * f_1(x) * f_2(x) * (f_2(y) - f_1(y)) \qquad (28)$$
$$I_{yy}(x,y) = I(x,y) * f_1(x) * f_2(x) * (f_2(y) + f_1(y)) - 2 \cdot I(x,y) * f_1(x) * f_2(x) \qquad (29)$$

With this algorithm, we can calculate at the same time the band-limited first and second directional derivative I_x and I_{xx} (or I_y and I_{yy}) of input image.

V. Edges Detection

The band-limited first and second directional derivative of input image can be obtained by the algorithms as stated above. Using them, we can then realize the edge detection for an image.

The maxima of gradient or zeros of the second directional derivative along the gradient are a natural way of characterizing and localizing intensity edges, so we present here to detect the edges from the maxima of gradient or zeros of the second directional derivative along the gradient by using the differential operators of exponential filter.

1. Edges from the maxima of gradient

Using the first directional derivative operator of the exponential filter, the two band-limited first directional derivatives I_x and I_y can be calculated, and the gradient vector can be therefore determined approximatively for every point in image. The gradient magnitude image is then non maxima suppressed in the gradient direction and thresholded with hysterisis, i.e. if the entire segment of the contour lies above a low threshold T1, and at least one of part of which is above a high threshold T2, that contour is output. The non maxima suppression scheme requires three points, one of which will be the current point, and the other two should be estimated of the gradient magnitude at points displaced from the current point by vector normal to the edge direction.

2. Edges from the zero-crossings of the second directional derivative along the gradient direction

Because edges detected from local gradient maxima can not be a pixel width (less good precision of localization), we propose an other method which detect the edges from the zeros crossing of the second directional derivative along the gradient direction.

We can calculate I_x, I_y, I_{xx} and I_{yy} by using the method shown in paragraphe IV, and therefore obtain approximatively the gradient vector and the second derivative in the gradient direction for every point in image. We extract at first the zero crossing of second derivative along the gradient direction on which the gradient magnitude must be above a low threshold, so an edges image is obtained. To this image, the entire segment of the contour will be kept, if the gradient magnitude on at least one part of this contour is above a high threshold.

VI. Comparision of Performance of the Filters

Filtering is a problem of estimation from noisy signal, and edge detection is a problem of estimating the position of maximal local signal change. Up to now, many works are done for edges detection in image, and different filters are proposed, for example, Gaussien filter, Canny filter, exponential filter, Deriche filter etc...

We appreciate the performance of the filters as follows :

(1) Precision of edge localization

According to our analysis [10], we can calculate the average localization error x_e for Gaussian filter, Canny filter [15], Deriche filter [16] and the exponential filter : $x_{eG} = 4\cdot(2\cdot e\cdot\pi)^{1/2}/\alpha$, $x_{ec} \approx 0.81/\alpha$, $x_{eD} = 4\cdot e^{-1}/\alpha = 1.47/\alpha$, $x_{eE} = 0$, i.e. $x_{eG} > x_{eD} > x_{ec} > x_{eE} = 0$.

So, we can see that the exponential filter localizes edge points with the best precision.

(2) Signal/Noise ratio on the edge point detected

Because x_e is the average estimation for the position of the edge point detected, we propose to calculate Signal/Noise ratio (Eq.(7)) at the point x_e.

And the signal/noise ratio for the Gaussien filter, Canny filter, Deriche filter and the exponential filter is : $SNR_G = 2\cdot\sigma\cdot e^{-32\pi\sigma}/\pi^{1/2}$, $SNR_c = 0.39/\alpha$, $SNR_D = 0.64/\alpha$, $SNR_E = 1/\alpha$, i.e. $SNR_E > SNR_D > SNR_c > SNR_G$.

Then, we see that the exponential filter has the best noise eliminating effect among the above four.

(3) Complexity of calculation

For the complexity of calculation, we only tell the difference from exponential filter and Deriche filter [16], because they are implemented by recursive algorithms which have a simpler calculation.

Because ISEF can be realized by first order recursive filter, the ISEF algorithms are much simpler than that of Deriche filter. Besides, the ISEF algorithms can be implemented independently to every line and every column, it can be easily realized by a parallel system.

According to the analysis results above, the ISEF filter is superior to the others at the 3 principal aspects of the performance of the filter.

VII. Experimental Results

Our new algorithms have been tested for different types of images and provided very good results. For comparision purpose, we take two examples : an indoor scenes image (Fig. 1) and a synthetic very noisy image (Fig. 2).

The experimental results show that GEF and SDEF methods are less sensitive to noise than DRF method, because the maxima of gradient or zero-crossing of directional second derivative rather than the Laplacian are used. The edges detected by GEF method are not always one pixel width, so its precision of localization is less good than that of SDEF method.

VIII. Conclusion

The symmetric exponential filter of an infinite large window size is an optimal linear filter deduced from one step edge model and the multi-edge model, now we further prove that the symmetric exponential filter is the optimal edge detection filter in the criteria of the signal to noise ratio, the localization precision and unique maximum. Obviously, the real images will be still more complicated than these models, however DRF method has already provided good results for different type of images. The results obtained through the two new methods further show the superior performance of this filter. The theoretical analysis for the performance of the filters shows also that the exponential filter is superior to the other current filters.

The first and second directional derivative operators can be realized by recursive algorithm and calculated at the same time. The new algorithms are therefore very simple as well as DRF algorithm, and they are also easy to implemente in a parallel way.

From the experimental results, the new methods show better effect than that of the other methods.

References
[1] W.K. PRATT, Digital Image Processing, New York, 1978.
[2] J. PREWITT, Object Enhancement and Extraction,Picture Processing and Psychopictories, Etd. by B. Lipkin and A. Rosenfeld, New York, pp. 75-149, 1970.
[3] M. HUECKEL, An Operator Which Locates Edges in Digitized Pictures. J.A.C.M., Vol. 18, pp 113-125, 1971.
[4] R.O. DUDA and P.E. HART, Pattern Classification and Scene Analysis. Wiley, New York, 1973.
[5] R. HARALICK,Edge and Region Analysis for Digital Image Data. C.G.I.P., Vol. 12, pp 60-73, 1980.
[6] R.HARALICK and L.WATSON, A Facet Model for Image Data. C.G.I.P., Vol. 15, pp 113-129, 1981.
[8] D. MARR and E.C. HILDRETH, Theory of Edge Detection. Proc. R. Soc. Lond. B, Vol. 207, pp 187-217, 1980.
[9] J. SHEN and S. CASTAN, Un nouvel algorithme de detection de contours, proceedings of 5th Conf. on P.R.&.A.I. (in French), Grenoble, 1985.
[10] J. SHEN and S. CASTAN, An Optimal Linear Operator for Edge Detection. Proc. CVPR'86, Miami.
[11] J. SHEN and S. CASTAN, Edge Detection Based on Multi-Edge Models.Proc. SPIE'87, Cannes, 1987.
[12] J. SHEN and S. CASTAN, Further Results on DRF Method for Edge Detection. 9th I.C.P.R., ROME, 1988.
[13] V.TORRE and T.A.POGGIO, On Edge Detection IEEE Transaction on Pattern Analysis and Machine Intelligence, Vol. Pami-8, N° 2, March 1986.
[14] J.S.CHEN and G.MEDIONI, Detection, Localization, and Estimation of Edges. IEEE Transaction on Pattern Analysis and Machine Intelligence, Vol. 11, N° 2, February 1989.
[15] J.F.CANNY, Finding Edges And Lines in Images. MIT Technical Raport N° 720, 1983.
[16] R. DERICHE, Optimal Edge Detection Using Recursive Filtering. In proc. First International Conference on Computer Vision, London, June 8-12 1987.

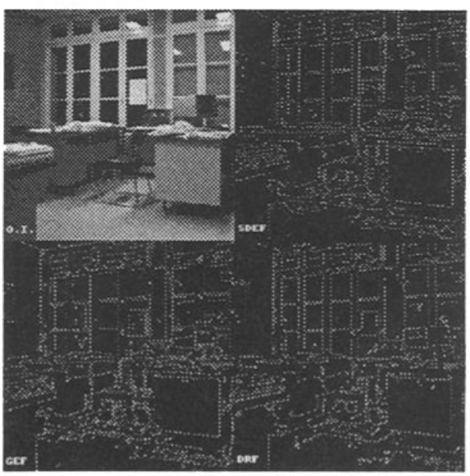

Fig. 1

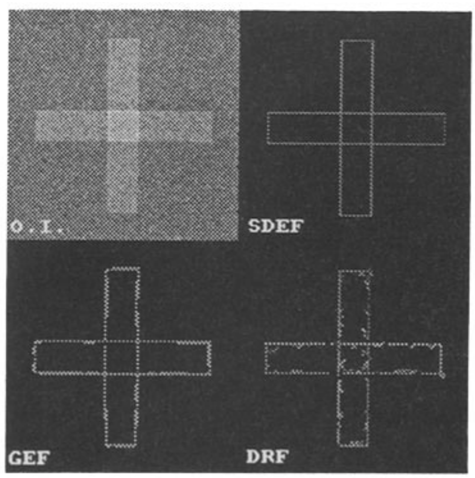

Fig. 2