Edge Contours Using Multiple Scales *

Donna J. Williams† and Mubarak Shah
Computer Science Department
University of Central Florida, Orlando, FL 32816

1 Introduction

The field of computer vision is concerned with extracting information contained in images about scenes they depict. The effectiveness of the early levels of processing is crucial in determining how successful higher level processing will be. Edge point detectors typically return an edge map identifying the location of points where the intensity gradient is high, together with some gradient and direction information. The next step is to group points into edge segments. Several algorithms have been developed for linking edge points, e.g., [5].

The choice of the scale to use in smoothing an image has been much studied. Smaller scales result in too much noise and fine texture while larger scales result in delocalization of edges and gaps. One approach to this problem is to use multiple scales, e.g. [4]. Witkin [9] introduced the concept of *scale space*, where the zero crossings of the second derivative are examined for a continuous spectrum of scales rather than a few discrete values. The properties of scale space have been examined by a number of authors, among them [1,2,6].

This paper presents a method of producing connected edge contours which are suitable for higher processing. The algorithm uses a gradient of Gaussian operator to determine gradient magnitude and direction, followed by non-maxima suppression to identify ridges in the gradient map. Canny[3] has shown this to be a near optimal edge detector. The resulting ridge is often more than 1 pixel wide, and may have small noisy spurs. In this paper the gradient maxima points are thinned to one pixel wide and linked into contours by an algorithm using weights to measure noise, curvature, gradient magnitude, and contour length. The set of points giving the largest average weight is chosen. This algorithm is then extended to one using multiple scales in the edge linking step and to a third algorithm which uses multiple scales during non-maxima suppression. Both multi-scale algorithms improve the detection of edge contours with little increase in the response to noise. The third also reduces the delocalization occurring at larger scales.

Further, in order to determine the size neighborhood where an edge point can appear at a different scale, a theoretical analysis of the movement of idealized edges is performed. Shah et al., developed equations for step pairs convolved with the Gaussian and its derivatives, and showed the general shape of the scale space curves. That work is extended to develop the equations of the scale space curves and analyze *quantitatively* the amount of the delocalization that occurs as images containing these steps are convolved with Gaussians having different values of σ.

2 Single Scale Edge Detection and Linking

In this section, we present an algorithm for finding a single good path through the set of gradient maximum points. In this method, the image is first convolved with a gradient of Gaussian operator. The set of gradient maximum points is placed in a priority queue with the point having largest magnitude on the top. Thus the strongest edge points will be extended into contours first.

The search for points to assign to a contour proceeds as follows. The first edge point is retrieved from the queue and gradient direction is used to determine the next edge point. The

*The research reported in this paper was supported by the Center for Research in Electro Optics and Lasers, University of Central Florida, grant number 20-52-043 and by NSF grant number IRI 87-13120.

†Now at Stetson University, DeLand, Florida 32720

point in the computed direction is examined first, then those in the adjacent directions on either side of it. Each branch is followed to the end and a weight assigned at each point based on four factors. The four factors are: (1) the difference between the gradient direction and the direction in which the point lies, (2) the difference between gradient direction of adjacent points, (3) gradient magnitude, (4) contour length. The weights are designed to favor the longest, strongest, straightest path. After points in the three primary directions have been examined, the path having the largest average weight is chosen and returned to the calling program. Contours are constructed in this manner until there are no more edges in the queue having magnitude greater than a given per cent of the maximum magnitude.

3 Maximum Movement of Edges

It is known that as edges undergo smoothing, they are delocalized. In this section the question of how far an edge undergoing Gaussian smoothing can move is analyzed. The edge model used is the ideal step edge, and the two cases considered are adjacent edges having the same and opposite parity. These are the *staircase* and the *pulse* edge types. If the unit step edge is $U(x)$ then the staircase and pulse with edges located at $-a$ and a have respective equations

$$S_a(x) = bU(x+a) + U(x-a) \text{ and } P_a(x) = bU(x+a) - U(x-a)$$

The relative heights of the two steps, b, satisfies $0 < b \leq 1$. Thus the weaker edge is at $x = -a$. After convolving with the derivative of the Gaussian, the equation for the staircase is

$$s_{\sigma,a}(x) = bg(x+a) + g(x-a) \text{ and that for the pulse is } p_{\sigma,a}(x) = bg(x+a) - g(x-a)$$

The equation of σ as a function of x at the maxima for the staircase is

$$\sigma = \left[\frac{2ax}{\ln(b(a+x)/(a-x))}\right]^{1/2} \text{ and for the pulse, } \sigma = \left[\frac{2ax}{\ln(b(x+a)/(x-a))}\right]^{1/2}$$

For derivations see [7]. The graphs for two values of b are given in Figure 1. The middle branch which appears for small values of σ in the graphs for the staircase represents a gradient minimum rather than a maximum so does not correspond to an edge. These will be referred to as the scale space images.

First we will consider the staircase. Notice that if $b = 1$ the edges move together until they meet when $\sigma = a$, then only one edge exists at $x = 0$ for $\sigma > a$. When $b < 1$ the stronger edge moves toward the middle and approaches the asymptote $a(1-b)/(1+b)$ as σ approaches ∞. The maximum corresponding to the weak edge on the left disappears when σ becomes sufficiently large. For example, when $b = .3$ the maximum movement of the weak edge occurs just before it disappears and is $(1 - .723)a = .277a$. The units on both axes are a.

In practice, when an image is being examined, σ is known, but a is not. Thus, σ can be fixed, a can be allowed to vary, and the movement (m) can be plotted versus a. This is shown in Figure 2(a) and (b). On both axes the units are σ. When $a > 2\sigma$ the amount of movement is negligible. This corresponds to the part of the scale space image where the curve is near vertical and the edges are far enough apart to have little interaction. The interaction begins slowly as the edges appear closer together, then increases rapidly to the maximum, then decreases until a reaches 0. When $b < 1$, the weaker edge disappears. The largest possible movement, σ, occurs for equal edges which are 2σ apart.

A similar analysis can be performed for a pulse. Figure 3(a) and (b) show the movement versus a for a pulse. The maximum movement, when $b = 1$, is σ as it was for the staircase, but this value is now the limiting value as the edges become closer together. When $b < 1$ the strong edge in the scale space image approaches the vertical asymptote $a(1+b)/(1-b)$ and displays a well-defined maximum movement as in the staircase, for a value of a between 0 and σ. But the weak edge can move indefinitely as a becomes smaller. In the scale space image the weak edge approaches the horizontal parabola $x = (1/2a)(\ln\ b)\sigma^2$. But $p_{\sigma,a}(\frac{\ln b}{2a}\sigma^2) = 0$ for all values of σ, thus the parabola gives the location where the gradient value is zero. The gradient value of the weak edge becomes small, and falls below any threshold being used as σ increases, and for $0 < a < 1/2$ the conditions of the sampling theorem are not met, thus in practice the movement of the weak edge is limited. Figure 3(c) gives a graph of maximum movement for the stronger edge of a pulse as b varies.

In summary, for a staircase greatest movement is σ and occurs when edges are 2σ apart and have equal contrast. Movement decreases rapidly for edges closer or farther away and those having unequal stepsize. For pulse, maximum movement is σ for equal edges and for the stronger of two unequal edges and occurs when edges are very close together. The weaker of two edges can exhibit unbounded movement, but gradient magnitude decreases, so an edge will usually not be detected farther than σ from its original location.

4 Multiple Scale Algorithms

This section extends the algorithm given in Section 2 to one using multiple scales, as follows. Initially the image is convolved with gradient masks at three or more scales. The search for a contour proceeds as for the single scale, using the largest scale, until a best partial contour at that scale has been found. Then the next finer scale is chosen and the neighborhood around the ends of the contour is searched to see if the edge can be extended. The neighborhood searched is only one pixel in each direction, based on the analysis in Section 3. The original algorithm is then followed for each point that gives a good continuation of the contour, and the best is chosen as an extension to the original edge. While extending the edge, if any point is discovered to be a possible edge point at a coarser scale, the search scale is increased to that value. When the contour cannot be extended further the scale is decreased to the next finer scale, and the process is repeated until the contour cannot be extended at the finest scale. This algorithm resulted in a considerable improvement in the detection of some of the incomplete edge contours, with almost no degradation due to inclusion of noisy edge points.

A second algorithm combines the gradient information computed at several scales during non-maxima suppression. Non-maxima suppression is performed in the usual manner for the coarsest scale and the possible edge points are marked. Then non-maxima suppression is performed at successively smaller scales. If a point is being marked as a gradient maximum and an adjacent point was a maximum at a coarser scale, but not at the present scale, then the label for the coarser scale is moved to the present point. This had the effect of shifting a delocalized edge point to its location at the finer scale. An additional weight counts the number of scales at which a point was detected, similar to the Marr-Hildreth spacial coincidence assumption.

5 Experimental Results

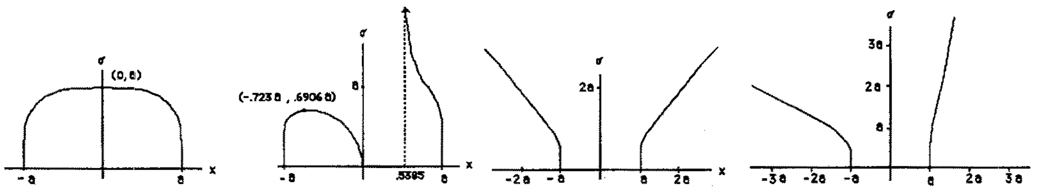
The algorithms were tested on several real images. The values of σ used were 1, $\sqrt{2}$, and 2 and a threshold of .08 was applied. For comparison, the images were also processed using the Canny operator. The results for two images, Part and Tiwanaku, are shown in Figure 5. The Canny operator is (a), the single scale algorithm is (b), the multiple scale algorithm is (c), and the multi-scale non-maxima suppression algorithm is (d).

The single scale edge linking algorithm cleans up the Canny edges and in addition produces a set of linked lists corresponding to the contours found. The multiple scale algorithm is able to improve detection of edges that are close together and interact at scales which are large enough to remove noise and fine texture. It also improves detection of weak, but well defined edges, such as those of the shadows in the Part image. Thus a number of fragmented contours have been completed. Best results in all cases occurred with the multiple scale non-maxima suppression algorithm. Edges which had been delocalized were moved back to their location at a smaller scale, separating edges which had become too close together to differentiate and some contours were extended farther than with the multiple scale linking algorithm.

The weights used in the linking were chosen heuristically. Experiments varying the weights indicated that the actual values were not critical as long as higher weights were given to the points in the primary direction having the same direction as the current point, high magnitude, and longer length contour. Experiments in which each one of the factors in turn was removed, however, indicated that no three gave as good results as using all four.

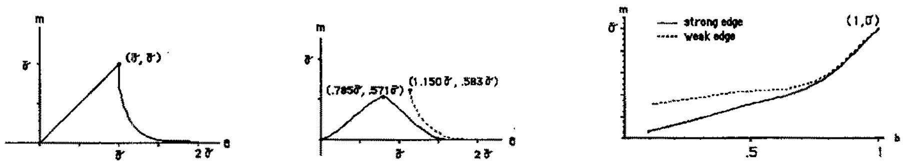
6 Conclusions

An edge linking agorithm is presented that first computes gradient magnitude then uses non-maxima suppression to reduce the search space. The gradient magnitude and direction information is used to assign weights to paths through the set of points, and the best path is chosen. The single scale algorithm uses a depth first search, but each point is allowed to occur in only one



(a) (b) (c) (d)

Figure 1: Scale Space Image: Location of Gradient Extrema for Staircase when (a) $b = 1$, (b) $b = .3$ and for Pulse when (c) $b = 1$, (d) $b = .3$



(a) (b) (c)

Figure 2: (a)Movement vs Distance between edges, Staircase, $b = 1$, (b) $b = .8$, (c) Maximum movement in terms of σ vs b for weaker and stronger edges.

subtree. This algorithm is extended to one which links edge points detected at multiple scales. A second variation is presented which uses the gradient information at multiple scales in the non-maxima suppression operation. A modified version of the single scale algorithm then links these edge points into contours. The first multi-scale method fills in gaps in single-scale contours, however the second method provides better localization and separation of nearby edges. Both improve detection of edges over the single-scale algorithm without introducing the noisy edges detected at the smaller scales.

References

[1] V. Berzins Accuracy of laplacian edge detectors. *CVGIP*, 27:195–210, 1984.

[2] F. Bergholm. Edge focusing. *IEEE Trans. on PAMI*, 9(6):726–741, 1987.

[3] J. F. Canny. *Finding Edges and Lines in Images.* Master's thesis, MIT, 1983.

[4] D. Marr and E. Hildreth. Theory of edge detection. In *Proc. R. Soc. Lond.*, pages 187–217, 1980.

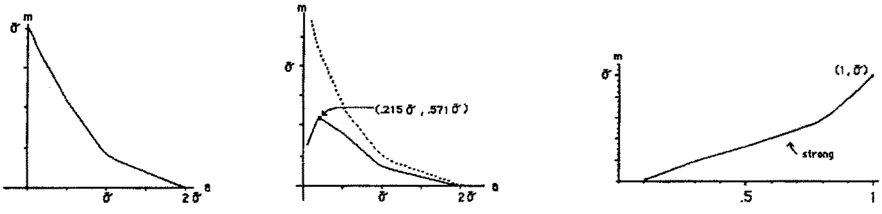
[5] R. Nevatia and K. R. Babu. Linear feature extraction and description. *Computer Graphics and Image Processing*, 13:257–269, 1980.

[6] M. Shah, A. Sood, and R. Jain. Pulse and staircase edge models. *Computer Vision, Graphics, and Image Processing*, 34:321–341, June, 1986.

[7] D. Williams and M. Shah. *Edge Contours.* Technical Report CS-TR-88-18, University of Central Florida, Computer Science Department, Sept, 1988.

[8] D. Williams and M. Shah. Multiple Scale Edge Linking. In Trivedi, M. M., editor, *Applications of Artificial Intelligence VII*, pages 13–24, SPIE, 1989.

[9] A. Witkin Scale-space filtering. In *Internat. Joint Conf. on A.I.*, pages 1019–1021, 1983.



(a) (b) (c)

Figure 3: (a)Movement vs Distance between edges, Pulse, $b = 1$, (b) $b = .8$, (c) Maximum movement in terms of σ vs b for stronger edge.

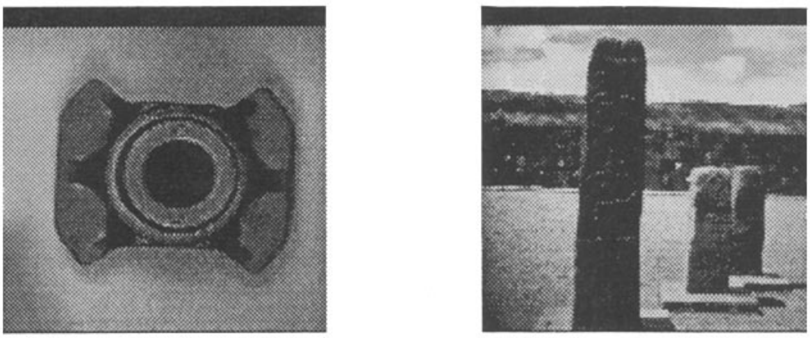
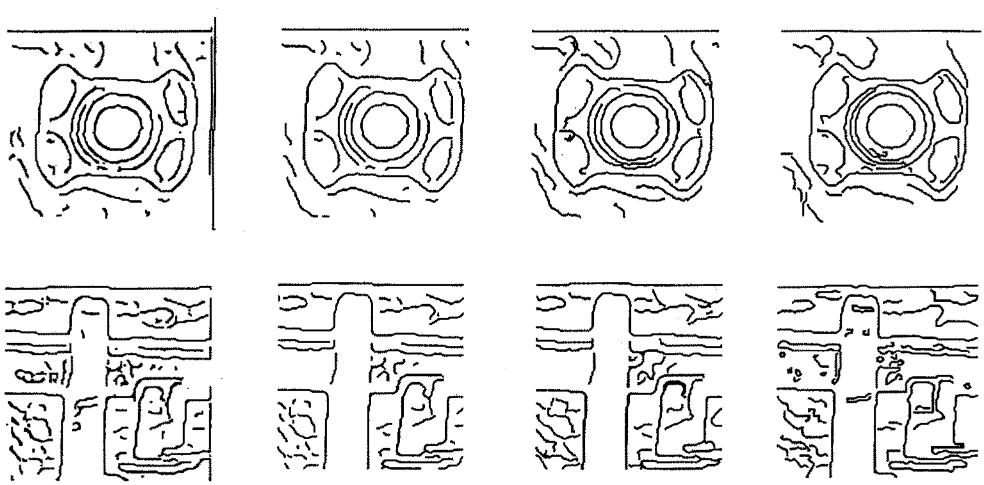


Figure 4: Original Images: Part, Tiwanaku.



(a) (b) (c) (d)

Figure 5: (a) Canny Operator, (b) Single Scale Edge Linking Algorithm, (c) Multiple Scale Algorithm, (d) Multi-Scale Non-maxima Suppression.