Estimation of 3D-motion and structure from tracking 2D-lines in a sequence of images

T. VIEVILLE
INRIA Sophia, BP109 06561 Valbonne, France

We establish the motion equations for rigidly moving 3D lines, and the structure equations that relate a temporal match of 2D lines in three consecutive pictures in an image sequence. We also analyse in details the numerical stability of such estimations.

1 Introduction

The aim of this study is to develop a method of recovery of structure from motion and of camera displacement estimation in a situation where the following features are available :
(1) A sequence of view with a stationary background and one or more rigid objects in motion,
(2) Only one camera,
(3) A real-time acquisition every 20 *msec*,
(4) An odometric estimation of ego-motion.

This study extends previous works [3,2], on the problem of recovery of structure and motion from two or three instantaneous views, and is directly related with these studies.

Basis of the study

In a temporal sequence of views, early vision provides the estimation of 2D line segments in the picture, with their statistical covariance. The line support of each line segment (unbounded lines), will be considered, as elementary tokens. The *token-tracker Algorithm* designed by R. Deriche [5] provides *temporal matches* between the same 2D line segment in two or more consecutive views, a kind of "temporal stereo" algorithm. It is then well known [3] that a match between at least 3 views is required to have a constraint on the motion on lines, as it is the case here. Given such matches the structure **and** the motion of the line can be computed.

Then, we can formulate the problem studied in this paper as follows : *Given 3 consecutive views in a temporal sequence of images closely related in time, and matches between 2D lines, in these views, how do we compute the related 3D line parameters, and the instantaneous rigid 3D-motion of this line (angular velocity ω and linear velocity* $\mathbf{v}$*).*

What is this paper about.

This paper introduce the data representation and the equations used for the estimation of 3D motion and structure of 2D lines. We discuss the precision, stability and sensitivity of these equations when using real data.

2 Equations of 3D lines and 3D motion

2.1 Representation of lines and rigid motion

Camera geometry and camera motion

The camera is calibrated, and the geometric quality of actual CCD sensors, legitimates the use of pinhole model for the camera, with a unit focal distance. Every quantity is referred to the camera intrinsic coordinates. The origin of this frame of reference is the optical center of the camera (its image nodal point, in fact) and the $(\mathbf{X}, \mathbf{Y}, \mathbf{Z})$ vectors are oriented as shown on Fig. 1, the $\mathbf{Z}$ axis corresponding to the optical axis of the camera.

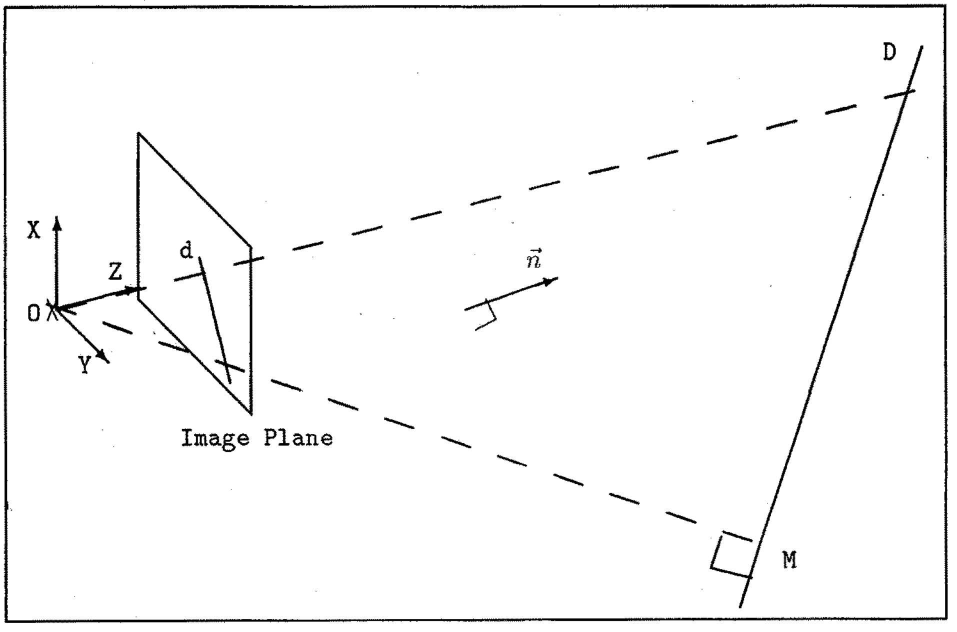

Figure 1: 2D and 3D lines representations

The camera motion is given by the kinematic screw $\{\mathrm{v}, \omega\}$. This 3D-motion will be assumed to be locally constant in time. The temporal sequence of views being taken at a high rate, the translation between two pictures will be approximated by :

$$\mathrm{t} = \Delta T \cdot \mathrm{v}$$

where ΔT is the sampling period between two pictures. The rotation between two pictures will be approximated by [1] :

$$R = e^{\Delta T \cdot \tilde{\omega}} \simeq I + \Delta T \cdot \tilde{\omega}$$

2D and 3D lines representations

A straight line D, in 3D space, is represented by its unitary direction vector δ and a point M on the line D. The point M is chosen to be the closest point to the optical center O of the camera (Fig.1) . It is equivalent to say that M is chosen such that $\vec{OM}$ is orthogonal to δ.

A 2D-line d in the retina plane is represented by a unitary vector $\mathbf{n} = (u,\ v,\ w)^T$ giving its equation :

$$u\,x + v\,y + w = 0 \tag{1}$$

The interpretation of $\mathbf{n}$ is that it is the normal to the plane defined by the 2D line and the optical center of the camera, as represented on Fig.1. The vector $\mathbf{n}$ is then orthogonal to δ and $\vec{OM}$.

The unitary vector $\mathbf{n}$ is the output of our version of the token-tracker. Each $\mathbf{n}$ estimation is - in fact - the mean value of a statistical estimation. A covariance matrix noted Wn is also provided by the token-tracker.

Summarizing, the unitary vector δ and the point M are constrained by the following equations :

$$\begin{aligned} \mathbf{n}^T \cdot \mathbf{n} &= 1 \\ \mathbf{n}^T \cdot \delta &= 0 \\ \delta^T \cdot \delta &= 1 \\ \delta^T \cdot \vec{OM} &= 0 \\ \mathbf{n}^T \cdot \vec{OM} &= 0 \end{aligned}$$

Matching between 3 views

Let us consider three views: the *present view* (subscript 0), the *previous view* (subscript $-$), the *next view* (subscript $+$), as represented on Fig.2. The $\mathbf{n}$ vectors components, as computed by the token-tracker, are given in a frame of reference attached to each view, while we want all quantities to be expressed in the same frame of reference, let us say, in the present view.

We are going to use the subscript /0 (only when necessary), to state that a quantity is expressed in the present view frame of reference, as shown on Fig.1.

In order to simplify the notations let us take $\Delta T = 1$. This will not modify the nature our derivations.

[1] I will denote the 3×3 identity matrix, and $\tilde{\omega}$ the antisymmetric matrix such that :

$$\forall \mathbf{x},\ \tilde{\omega} \cdot \mathbf{x} = \omega \wedge \mathbf{x}$$

where $\wedge$ denotes the cross product of two vectors.

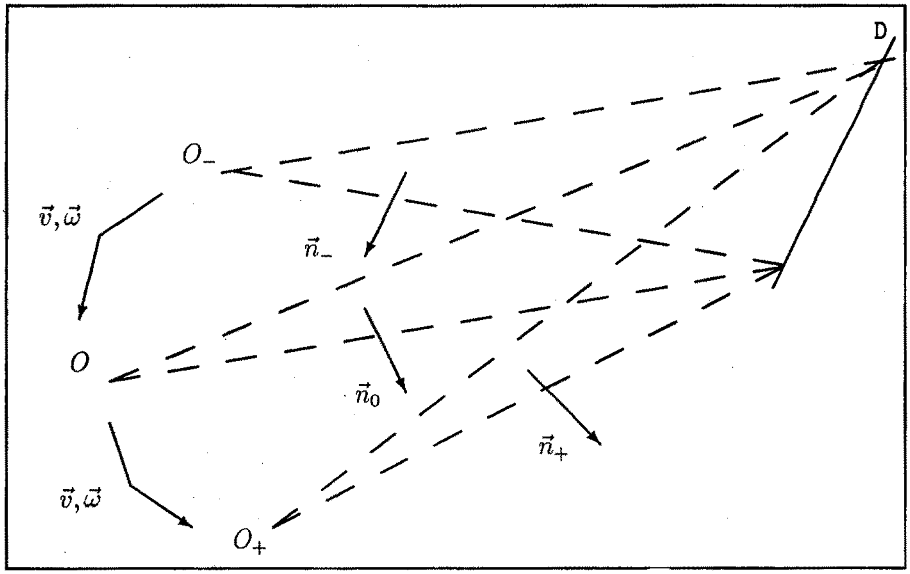

Figure 2: Schematic representation of a segment matching over 3 views

The transformations are then given by :

$$\begin{aligned}
\mathbf{n}_{-/0} &= R \cdot \mathbf{n}_- \simeq (I + \tilde{\omega}) \cdot \mathbf{n}_- = \mathbf{n}_- + \omega \wedge \mathbf{n}_- \\
\mathbf{n}_{+/0} &= R^{-1} \cdot \mathbf{n}_+ \simeq (I - \tilde{\omega}) \cdot \mathbf{n}_+ = \mathbf{n}_+ - \omega \wedge \mathbf{n}_+ \\
\mathbf{n}_{0/0} &= \mathbf{n}_0 \\
\vec{OO}_{-/0} &= -\mathbf{v} \\
\vec{OO}_{+/0} &= +\mathbf{v} \\
O_{/0} &= O
\end{aligned}$$

2.2 Constraints on the angular velocity and 3D-line direction

Derivating equations

All our developments will be based on the following geometrical property : *since in the three views the* $\mathbf{n}$ *vectors are orthogonal to the 3D-line direction* δ*, these three vectors are in the same plane.* This condition can be written as :

$$| \, \mathbf{n}_0, \mathbf{n}_{-/0}, \mathbf{n}_{+/0} \, | = 0 \tag{2}$$

Expanding equation 2 as a function of ω, we have [2] :

$$| \mathbf{n}_0, \mathbf{n}_-, \mathbf{n}_+ | + (\omega, \beta) - (\omega, \mathbf{n}_0) \cdot (\omega, \mathbf{n}_- \wedge \mathbf{n}_+) = 0 \tag{3}$$

where the vector $\beta = A \cdot \mathbf{n}_0$. The A matrix is defined as :

$$A = \mathbf{n}_-\mathbf{n}_+^T + \mathbf{n}_+\mathbf{n}_-^T - 2(\mathbf{n}_+, \mathbf{n}_-)I$$

Looking at the previous transformation equation 2 can be equivalently formulated as follows : the orientation of the 3D line (δ), computed from the previous and next views ($\delta = \mathbf{n}_{-/0} \wedge \mathbf{n}_{+/0}$), should be orthogonal to $\mathbf{n}_0$. This statement is, of course, not dependent upon permutations of the $\mathbf{n}$ vectors. It is, in fact, the only one equation one can obtain on ω from the $\mathbf{n}$ vectors, in three views. The vector β defines the direction along which ω can be computed using equation 2.

The matrix A is a symmetric matrix. Its eigen values are easy to compute, since we have :

$$A \cdot \mathbf{x} = \lambda \cdot \mathbf{x} \Leftrightarrow (\mathbf{n}_-, \mathbf{x})\mathbf{n}_+ + (\mathbf{n}_+, \mathbf{x})\mathbf{n}_- = (\lambda + 2(\mathbf{n}_-, \mathbf{n}_+))\mathbf{x}$$

and the three solutions are :

$$\begin{aligned} x &= \mathbf{n}_- \wedge \mathbf{n}_+, & \lambda_1 &= -2(\mathbf{n}_-, \mathbf{n}_+) \\ x &= \mathbf{n}_- - \mathbf{n}_+, & \lambda_2 &= -1 - (\mathbf{n}_-, \mathbf{n}_+) \\ x &= \mathbf{n}_- + \mathbf{n}_+, & \lambda_3 &= 1 - (\mathbf{n}_-, \mathbf{n}_+) \end{aligned}$$

This result will be useful to study the stability of equation 2, and the conditioning of A. In addition, one can see that A is the sum of two projections. Since the matrix :

$$P_{\mathbf{u}^\perp \| \mathbf{v}} = \mathbf{u}\mathbf{v}^T - (\mathbf{u}, \mathbf{v})I$$

defines the projection on to the vector plane orthogonal to $\mathbf{u}$ along the direction of $\mathbf{v}$ (Note that we have : $P^T_{\mathbf{u}^\perp \| \mathbf{v}} = P_{\mathbf{v}^\perp \| \mathbf{u}}$), we have : $A = P_{\mathbf{n}_-^\perp \| \mathbf{n}_+} + P_{\mathbf{n}_+^\perp \| \mathbf{n}_-}$

Equation 2 is a quadratic equation on ω and the related quadric Q. Taking the non-orthogonal frame of reference (x, y, z) defined by the equations :

$$\begin{array}{ccc} (\mathbf{x}, \mathbf{n}_0) = 1 & (\mathbf{y}, \mathbf{n}_0) = 0 & (\mathbf{z}, \mathbf{n}_0) = 0 \\ (\mathbf{x}, \mathbf{n}_- \wedge \mathbf{n}_+) = 0 & (\mathbf{y}, \mathbf{n}_- \wedge \mathbf{n}_+) = 1 & (\mathbf{z}, \mathbf{n}_- \wedge \mathbf{n}_+) = 0 \\ (\mathbf{x}, \beta) = 0 & (\mathbf{y}, \beta) = 0 & (\mathbf{z}, \beta) = 1 \end{array}$$

which have a unique solution if only if :

$$| \mathbf{n}_0, \mathbf{n}_- \wedge \mathbf{n}_+, \beta | = ((\mathbf{n}_0, \mathbf{n}_+) + (\mathbf{n}_0, \mathbf{n}_-)) \cdot ((\mathbf{n}_0, \mathbf{n}_+) - (\mathbf{n}_0, \mathbf{n}_-)) \neq 0$$

[2] We used the following notations and relations in our computations :

$$\begin{gathered} (\mathbf{x}, \mathbf{y}) = \mathbf{x}^T\mathbf{y} = \mathbf{y}^T\mathbf{x} \quad (\mathbf{x}, \mathbf{x}) = \|x\|^2 = x^2 \\ (\mathbf{x} \wedge \mathbf{y}, \mathbf{z}) = | \mathbf{x}, \mathbf{y}, \mathbf{z} | = | \mathbf{y}, \mathbf{z}, \mathbf{x} | = - | \mathbf{y}, \mathbf{x}, \mathbf{z} | = \ldots \\ \mathbf{x} \wedge (\mathbf{y} \wedge \mathbf{z}) = (\mathbf{x}, \mathbf{z})\mathbf{y} - (\mathbf{x}, \mathbf{y})\mathbf{z} \\ (\mathbf{x} \wedge \mathbf{y}) \wedge (\mathbf{x} \wedge \mathbf{z}) = | \mathbf{x}, \mathbf{y}, \mathbf{z} | \cdot \mathbf{x} \\ \tilde{\mathbf{x}} \cdot \mathbf{y} = \mathbf{x} \wedge \mathbf{y} \\ (\mathbf{x} \wedge \mathbf{y}, \mathbf{x} \wedge \mathbf{z}) = (\mathbf{y}, \mathbf{z})(\mathbf{x}, \mathbf{x}) - (\mathbf{x}, \mathbf{y})(\mathbf{x}, \mathbf{z}) \end{gathered}$$

and the multilinearity of $| \mathbf{x}, \mathbf{y}, \mathbf{z} |$.

and if we note $\omega = (a, b, c)^T$ in this frame of reference we simply get :

$$\mathbf{x} = (a, b, c)^T \in Q \Leftrightarrow a \cdot b + c + q_0 = 0$$

This is obviously the equation of an hyperbolic cone (see for example [4], Chap *XI*), a quadric of rank 3. In the case where there is not a unique solution, the degenerate case, the quadric is of rank 2, and it is known to be a pair of two planes.

Finally, one should notice that *only the component of ω orthogonal to the 3D-line direction δ can be computed.* The component of ω aligned with δ, induces a rotation for which δ is invariant, and the components of the $\mathbf{n}$ vectors in their local frame of references are not modified by it. (One should also notice that, since we have no information a priori on the relative location of two views, we have no information on the relative location of the $\mathbf{n}$ vectors in space, but only on their location with respect to each view frame of reference. This is a fundamental difference with stereo).

One can then assume ω to be orthogonal to δ, that is in the plane of the $\mathbf{n}$ vectors. These additional constraints are :

$$\begin{array}{rcccl} \mid \omega, \mathbf{n}_{-/0}, \mathbf{n}_{+/0} \mid &=& (1-\omega^2)(\omega, \mathbf{n}_- \wedge \mathbf{n}_+) + 2(\omega \wedge \mathbf{n}_-, \omega \wedge \mathbf{n}_+) &=& 0 \\ \mid \omega, \mathbf{n}_0, \mathbf{n}_{-/0} \mid &=& \mid \omega, \mathbf{n}_0, \mathbf{n}_- \mid + (\omega \wedge \mathbf{n}_0, \omega \wedge \mathbf{n}_-) &=& 0 \\ \mid \omega, \mathbf{n}_0, \mathbf{n}_{+/0} \mid &=& \mid \omega, \mathbf{n}_0, \mathbf{n}_+ \mid - (\omega \wedge \mathbf{n}_0, \omega \wedge \mathbf{n}_+) &=& 0 \end{array} \quad (4)$$

Numerical stability of the previous equations

It is useful to consider the order of magnitude of the terms in the expansions of equation 2. Since the instantaneous rotation between the three views has a small angle, and is assumed to be locally constant, the relative angles between the $\mathbf{n}$ vectors, and the norm of ω are small quantities. We are now going to used Taylor expansions to study the numerical stability of the previous equations.

Our discussion will be based on the following statements :

1. Since $\mathbf{n}_0$, $\mathbf{n}_{-/0}$, and $\mathbf{n}_{+/0}$ are in the same plane we have :

$$(\widehat{\mathbf{n}_{-/0}, \mathbf{n}_{+/0}}) = (\widehat{\mathbf{n}_{-/0}, \mathbf{n}_0}) + (\widehat{\mathbf{n}_0, \mathbf{n}_{+/0}})$$

2. Since the translation is small, the angles $(\widehat{\mathbf{n}_{-/0}, \mathbf{n}_0})$ and $(\widehat{\mathbf{n}_0, \mathbf{n}_{+/0}})$ have the same order of magnitude. This is illustrated on Fig.3. The angles $(O_-, \widehat{M}, O_0)$ and $(O_0, \widehat{M}, O_+)$ are roughly equal, since $\mathbf{v}$ is small with respect to $\|\vec{O_-M}\|$, $\|\vec{O_0M}\|$, and $\|\vec{O_+M}\|$. Since $\mathbf{n}_{-/0}$, $\mathbf{n}_0$, and $\mathbf{n}_{+/0}$ are orthogonal respectively to $\vec{O_-M}$, $\vec{O_0M}$ and $\vec{O_+M}$, there relative angles are equal. We can then conclude that $(\widehat{\mathbf{n}_{-/0}, \mathbf{n}_0})$ and $(\widehat{\mathbf{n}_0, \mathbf{n}_{+/0}})$ are roughly equal. Precisely this means that they differ only at the second order. We will use this result though the following notations :

$$\begin{array}{rcl} (\widehat{\mathbf{n}_{-/0}, \mathbf{n}_0}) &=& \varepsilon_- = \varepsilon + \epsilon^2 \\ (\widehat{\mathbf{n}_0, \mathbf{n}_{+/0}}) &=& \varepsilon_+ = \varepsilon - \epsilon^2 \end{array}$$

3. Since ω is in the same plane P of the vectors $\mathbf{n}$, its orientation is entirely defined one angle, tel us say $(\widehat{\omega, \mathbf{n}_0})$. Since the norm of ω is the angle of the rotation, and since $\mathbf{n}_{+/0}$ is related to $\mathbf{n}_0$ by this rotation, and $\mathbf{n}_0$ is also related to $\mathbf{n}_{0/-}$ by this rotation,

the norm of ω has the same order of magnitude as $(\widehat{\mathbf{n}_{-/0}, \mathbf{n}_0})$ and $(\widehat{\mathbf{n}_0, \mathbf{n}_{+/0}})$. We will summarize these two points by the following notations :

$$\begin{aligned} (\widehat{\omega, \mathbf{n}_0}) &= \alpha \\ \|\omega\| &= w\varepsilon \end{aligned}$$

where $w = \frac{2\|\omega\|}{(\widehat{\mathbf{n}_{-/0}, \mathbf{n}_{+/0}})}$ is the ratio between $\|\omega\|$ and ε. We have :

$$w = o(1)$$

since $\|\omega\|$ and ε have a similar order of magnitude.

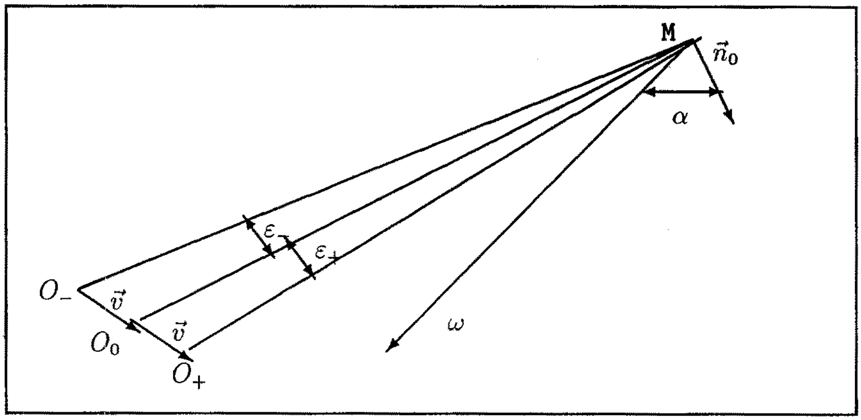

Figure 3: Schematic representation of the angles in the plane P defined by the vectors $\mathbf{n}$

With these notations, and using equations 4 one can easily derive the orders of magnitude of all quantities related to equation 2. The following results are then obtained :

- The quadratic term in equation 2 is not negligible since :

$$\begin{aligned} \|\beta\| &= \sqrt{3 + (2 - 10\cos(\alpha)^2) w^2 \varepsilon} + o(\varepsilon^2) \\ |\, \mathbf{n}_0, \mathbf{n}_-, \mathbf{n}_+ \,| &= o(\varepsilon^2) \\ (\omega, \mathbf{n}_- \wedge \mathbf{n}_+)(\omega, \mathbf{n}_0) &= o(\varepsilon^2) \end{aligned}$$

  and we will have to deal with a quadratic equation, in term of ω, even for small angles of rotation.

- The vectors $\mathbf{n}$ do not form a stable frame of reference, since their relative angles are very small.

$$\begin{aligned} (\widehat{\mathbf{n}_-, \mathbf{n}_0}) = (\widehat{\mathbf{n}_0, \mathbf{n}_+}) &= \sqrt{1 + 2\sin(\alpha)^2 w^2 \varepsilon} + o(\varepsilon^2) \\ (\widehat{\mathbf{n}_-, \mathbf{n}_+}) &= \sqrt{(6 + 2 * \cos(\alpha)^2)}\|\omega\| + o(\varepsilon^2) \end{aligned}$$

- The matrix A is not well conditioned since one eigen-value is very small with respect to the others, while the two other eigen-values are very similar. We precisely have :

$$\begin{array}{rcl} \lambda_3 & = & (3+\cos(\alpha)^2)\|\omega^2\| + o(\varepsilon^3) \\ \lambda_1 & = & -2 + 2\,\lambda_3 + o(\varepsilon^3) \\ \lambda_2 & = & -2 + \lambda_3 + o(\varepsilon^3) \end{array}$$

- The direction of the vector β is not trivial, but we have :

$$\begin{array}{rcl} (\widehat{\beta, \mathbf{n}_- \wedge \mathbf{n}_+}) & = & o(\varepsilon) \\ (\widehat{\beta, \mathbf{n}_{-/0} \wedge \mathbf{n}_{+/0}}) & = & \frac{\Pi}{2} + o(\varepsilon) \\ (\widehat{\beta, \mathbf{n}_0}) & = & \Phi\; o(\varepsilon) \;\; with \;\; 0 << \Phi << \frac{\Pi}{2} \end{array}$$

  This defines the direction along which ω can be estimated which is roughly aligned with $\mathbf{n}_- \wedge \mathbf{n}_+$ but not with δ or $\mathbf{n}_{-/0} \wedge \mathbf{n}_{+/0}$, as expected.

This has two consequences :
(1) On one hand, we are going to use the found quadratic constraint to recursively estimate ω. This equation can be used as a measurement equation in an extended Kalman Filter, used for the recovery of the 3D line motion.

(2) On the other hand, we are not going to try to reconstruct ω using five, or more than five, views, since even if the reconstruction is, in principle, possible, the computation will not be numerically stable.

In addition, the use of a first estimate of ω from odometric cues will be very useful in this approach, and the previous constraint will be used only to correct the a priori estimate.

Estimation of the 3D-line direction

Since $\mathbf{n}_0$ is constrained to be in the same plane as $\mathbf{n}_{-/0}$ and $\mathbf{n}_{+/0}$, as discussed previously, and since δ is orthogonal to this plane, the 3D-line direction is simply given by :

$$\delta \parallel \mathbf{n}_{-/0} \wedge \mathbf{n}_{+/0} = \mathbf{n}_- \wedge \mathbf{n}_+ + A\omega + o(\omega^2).$$

In the previous section, we studied in details the properties of the matrix A, and the direction β along which ω is estimated. Let us remind that β is very close to the direction of $\mathbf{n}_- \wedge \mathbf{n}_+$ which is a eigen direction of A. Then, ω estimation for this line is mainly performed in the direction on $\mathbf{n}_- \wedge \mathbf{n}_+$, only. We then are going to consider $\omega \parallel \mathbf{n}_- \wedge \mathbf{n}_+$, and we have $A\omega \parallel \mathbf{n}_- \wedge \mathbf{n}_+$ also, and finally :

$$\delta \wedge (\mathbf{n}_- \wedge \mathbf{n}_+) \;\; = \;\; 0 \tag{5}$$

or $\delta = \frac{\mathbf{n}_- \wedge \mathbf{n}_+}{\|\mathbf{n}_- \wedge \mathbf{n}_+\|}$.

However, since we have

$$\|\mathbf{n}_- \wedge \mathbf{n}_+\| \;\; = \;\; \sqrt{2 * \lambda_3} + o(\varepsilon^2)$$

where λ_3 has been defined previously, this direct estimate is not numerically stable, and equation 5 is to be used instead.

2.3 Constraints on the translation and the 3D-line distance

The translation velocity $\mathbf{v}$ and the 3d-line distance characterized by the location of the point M can be computed from the following relations :

$$\left\{ \begin{array}{ccc} \vec{O_-M} & \perp & \mathbf{n}_{-/0} \\ \vec{O_+M} & \perp & \mathbf{n}_{+/0} \\ \vec{OM} & \perp & \mathbf{n}_0 \\ \vec{OM} & \perp & \delta \\ & where : & \\ \vec{O_-M} & = & \mathbf{v} + \vec{OM} \\ \vec{O_+M} & = & \mathbf{v} - \vec{OM} \end{array} \right.$$

It is immediate to derive :

$$\begin{array}{rcl} (\mathbf{v}, \mathbf{n}_{-/0}) & = & -(\vec{OM}, \mathbf{n}_{-/0}) \\ (\mathbf{v}, \mathbf{n}_{+/0}) & = & +(\vec{OM}, \mathbf{n}_{+/0}) \end{array}$$

Since $\vec{OM}$ is orthogonal to δ, $\vec{OM}$ is in the plane P of the $\mathbf{n}$ vectors. Since $\vec{OM}$ is also orthogonal to $\mathbf{n_0}$, M is on the unique line of the plane P, orthogonal to $\mathbf{n_0}$, and going through the origin. Using this remark, and after some algebra, the previous set of relations is equivalent to :

$$(\mathbf{v}, \gamma) = 0 \tag{6}$$

and :

$$\begin{array}{rcl} \vec{OM} & \parallel & \zeta \\ (\vec{OM}, \mathbf{n}_{-/0} - \mathbf{n}_{+/0}) & = & -(\mathbf{v}, \mathbf{n}_{-/0} + \mathbf{n}_{+/0}) \end{array} \tag{7}$$

or :

$$\vec{OM} = \frac{(\mathbf{v}, \mathbf{n}_{-/0} + \mathbf{n}_{+/0})}{(\zeta, \mathbf{n}_{-/0} - \mathbf{n}_{+/0})} \zeta$$

The vectors γ and ζ are two vectors of the plane P containing the three $\mathbf{n}$ vectors, precisely :

$$\begin{array}{rcl} \gamma & = & a\mathbf{n}_{-/0} - b\mathbf{n}_{+/0} \\ \zeta & = & (\mathbf{n}_{+/0}, \mathbf{n}_0)\mathbf{n}_{-/0} - (\mathbf{n}_{-/0}, \mathbf{n}_0)\mathbf{n}_{+/0} \end{array}$$

where :

$$\begin{array}{rcl} a & = & (\mathbf{n}_{-/0}, \mathbf{n}_0) - (\mathbf{n}_{-/0}, \mathbf{n}_{+/0})(\mathbf{n}_{+/0}, \mathbf{n}_0) \\ b & = & (\mathbf{n}_{+/0}, \mathbf{n}_0) - (\mathbf{n}_{-/0}, \mathbf{n}_{+/0})(\mathbf{n}_{-/0}, \mathbf{n}_0) \end{array}$$

In equations 6 and 7 the estimation of $\mathbf{v}$ and M are decoupled. We have one equation for $\mathbf{v}$, which constrains its direction but not its amplitude, as expected in a monocular system. One additional constraint could be : $\mathbf{v}^2 = 1$. In our case, since we have an odometric estimation of $\mathbf{v}$, we are going to use equation 6 only to correct this estimate.

An estimation of $\mathbf{v}$ being provided the point M is uniquely defined using the pair of linear equations 7. The numerical stability of these equations can be studied as previously, and we have the following results :

- The vectors $\mathbf{n}_{-/0} + \mathbf{n}_{+/0}$ and $\mathbf{n}_{-/0} - \mathbf{n}_{+/0}$ are orthogonal, and can be used as an orthogonal frame of reference in the plane P.

- The vector ζ is roughly aligned with $\mathbf{n}_{-/0} - \mathbf{n}_{+/0}$ and its norm has the order of magnitude of the $\mathbf{n}$ vectors angles :

$$\begin{aligned} \|\zeta\| &= 2\epsilon + o(\varepsilon^4) \\ (\widehat{\zeta, \mathbf{n}_{-/0} - \mathbf{n}_{+/0}}) &= o(\varepsilon) \end{aligned}$$

- As for δ, the direct estimation of M is not numerically stable. However using equation 7, since ζ is almost aligned with $\mathbf{n}_{-/0} + \mathbf{n}_{+/0}$, the second line of equation 7 provides a stable constrains on the norm of $\vec{OM}$.

- The vector γ is roughly aligned with $\mathbf{n}_{-/0} + \mathbf{n}_{+/0}$ and its norm is very small which is acceptable, since it is used in a homogeneous equation (eq 6) :

$$\begin{aligned} \|\gamma\| &= 4\epsilon^2\varepsilon + o(\varepsilon^5) \\ (\widehat{\gamma, \mathbf{n}_{-/0} + \mathbf{n}_{+/0}}) &= o(\tfrac{\epsilon^2}{\varepsilon(1/2)}) \end{aligned}$$

3 Conclusion

Given a match between three 2D-lines in three consecutive views, we can compute one quadratic equation on the angular velocity (eq 3), and one linear equation on direction of the linear velocity (eq 6). These equations are based on quantities having a small order of magnitude ($o(\varepsilon)$) but which are numerically stable since these order of magnitude are compatible.

The computation of the velocity torque should be done either in cooperation with other matches and/or in cooperation with odometric cues, since the velocity torque is evaluated only along a given direction (β or γ).

Given a match between three 2D-lines in three consecutive views, with an estimation of the velocity torque, the direction of the 3D-line and its location with respect to the optical center of the camera can be directly evaluated from equations 5 and 7.

In this study we do not use estimations of parameters velocity, as it was done in [2]. We then avoided the computations of time derivative, which have the drawbacks to be noise-sensitive, while the choice of a good derivative estimator is a complex problem. One can consider our study as a "discrete version" of the approach in [2], where we implicit estimate velocity and acceleration related quantities, since we use 3 consecutive views. However, the derivated equations are slightly different and seem to be much stable in our case.

In comparison to the study of [5], where a similar problem has been investigated, we made profit of the fact that pictures are very closed, while the rotation between two pictures can be approximated by a simple cross-product. We then come to linear equations, and could study in details the precision of our method. In addition, we are here dealing with the line support of the segments, instead of their extremities, or other points of interest. Line-tokens are less noisy geometrical primitives in a picture, since they are estimated from several points, and correspond to recognizable features in the visual environment.

Other methods proposed in the literature reach their limit very quickly as the noise in the data increases [1], or are based on rather heavy computations [6], or implies important restrictions on the type of visual environment [7].

The use of odometric cues in cooperation with one camera provides a solution to the scale factor problem, for a stationary background. In the future the cooperation between vision and odometry will be developed from this initial study.

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