The Dynamic Generalized Hough transform

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A new algorithm for the Generalized Hough transform is presented. The information available in the distribution of image points is used to optimize the computation of the transform. The calculated parameters are those associated with a single image point and all other image points in combinations of the minimum number of points necessary to define an instance of the shape under detection. The method requires only one dimensional accumulation of evidence. Using the algorithm, the transform of sparse images is more efficiently calculated. Dense images may be segmented and similarly processed. In two dimensions, the method provides a feedback mechanism between image and transform space whereby contiguity of feature points and endpoints of curves may be determined.

1 . Introduction

The Hough transform[1], [2] is a powerful tool in shape analysis. It is used to extract global features from shapes and gives good results even in the presence of noise or occlusion. While the theoretical potential of parametric transform methods has been demonstrated they have made little impact on large scale industrial applications because of supposed excessive storage requirements and computational complexity[3] The development of fast, efficient implementations of parametric transformation methods of shape detection has accordingly received much attention in the recent literature[4], [3], [5], [6], [7], [8]. An up-to-date and comprehensive review of the use of the Hough transform is given by Illingworth and Kittler[9].

The Hough transform works by grouping low level feature points (edge image data) into object specific intermediate features (e.g. line segments). This is accomplished by using the low level feature points to generate information concerning all possible groupings of points within the image. The corresponding transform plane is the accumulation of that evidence. The technique is computationally intensive because evidence is generated of all possible groupings of points in the image.

Previous suggested approaches may be divided into two categories. The first seeks to reduce the computational load by using information from the image to reduce the generation of evidence in the transform plane[2], [7], [8], [10]. The second class of methods involves absolute or iterative reductions in resolution of either the transform or the image space[5], [6], [4], [11].

Evaluation of the information generated in the transform space may present difficulties. Problems associated with the detection of maxima in the transform space may be partially solved by the use of matched filtering techniques to detect those maxima[12], [13]. However, a major shortcoming of the technique remains in that all information about feature points contributing to a maxima in the transform space is lost in the transformation process. It is therefore not possible to determine either contiguity of feature points nor end points of curves. Gerig[14] attempts to solve these problems, in the case of circle detection, using a technique which maps information from the parameter space back to the image space. In this way each image point has associated with it a most probable parametrisation. A second transformation is performed where, for each image point, only the cell in parameter space associated with the most probable parametrisation of that image point is incremented. The technique works well in that it is a reliable strategy for interpreting the accumulator space. It is however still computationally complex and offers no reduction in memory allocation.

The proposed method[15] seeks both to cut significantly the computational burden involved in the implementation of the transform, to provide an efficient feedback mechanism linking the accumulated boundary point evidence and the contributing boundary point data and to facilitate the detection of maxima.

2 . The Dynamic Generalized Hough transform

An expression for the Generalized Hough Transform, GHT, may be written in the form suggested by Deans[16]

$$f(\xi, p) = \iint_D F(x,y)\delta\left(p - C(x,y;\xi)\right) dx\, dy \tag{1}$$

where $F(x,y)$ is an arbitrary generalized function[17] defined on the xy plane D and the argument of the delta function defines some family of curves in the xy plane parametrized by the scalar p and the components $\xi_1, \xi_2, \ldots \xi_n$ of the vector ξ.

If, $F(x,y)$, represents a binary image the integral of equation 1 will have a value of 1 when the argument of the delta function evaluates to zero. In computational terms this occurs at all points that are solutions to the discrete equation:

$$p_j = C(x_i, y_i; \xi_j) \tag{2}$$

otherwise it will be zero. Equation 2 is used to calculate the standard GHT. The i, j subscripts refer to ordered pairs in the image and the transform space respectively. For every point, (x_i, y_i), of the image, i is fixed and the values p_j are calculated using stepwise increments of the components of ξ_j. Each point, (p_j, ξ_j), in the transform space will refer to a possible curve in image space which passes through the point (x_i, y_i). The SHT therefore provides a great redundancy of information concerning the image. This is because each image point is treated independently.

The present technique proposes that image points are tested for the most probable, as opposed to all possible, membership of a shape. (In two dimensions shape may refer to a curve and in three dimensions a surface). If, when the image is scanned for candidate feature points, a list of those feature points is maintained, then possible membership of curves/surfaces may be tested. Where n parameters are associated with the shape under detection then a minimum of n points are required to test the membership of a shape of any given point and any other $(n-1)$ image points. The equation of a shape, $p - C(x,y;\xi) = 0$, passing through n non-colinear points may be written in the following way:

$$\begin{pmatrix} c_1 & c_2 & \ldots & c_n & 1 \\ c_1^1 & c_2^1 & \ldots & c_n^1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ c_1^n & c_2^n & \ldots & c_n^n & 1 \end{pmatrix} = 0 \tag{3}$$

and used to calculate the parameters associated with the shape. For an image containing m points, to test each image point in this way would require $C_m^n = \frac{m!}{(m-n)!n!}$ computation cycles Thus, the computational effort is a function of the density of feature points in the image and increases sharply with the increase of m. If the number of feature points is large enough, the number of computations required far exceeds those required when using a standard GHT algorithm. This problem may be resolved in the following way.

Using equation (3), the n parameters associated with the shape under detection are calculated using a single fixed image point and all possible combinations of that image point with sets of $(n-1)$ other image points. The calculated parameters are accumulated in one dimensional histograms. Peaks in the histograms will indicate the parameters associated with the most probable instance of the shape in image space of which the fixed point is a member. This information may be used to detect that shape and to remove the points associated with it from the image. The process is then repeated using the shortened list. In two dimensions, the contiguity of points and endpoints of the detected curve or sections of it may be determined.

Many computer vision tasks require that a particular instance of a shape be recognised and located. Using the constraints inherent in such tasks computational savings may be made. If the calculated parameters are not within the range of possibilities suggested by the shape under detection, this pass of the algorithm may be abandoned. Such a strategy will deal with membership of extraneous shapes accidentally generated by random association of image points. Further computational savings may be made by segmenting the image[18]. The algorithm is inherently parallel thus offering the potential to further decrease computation times.

Particular examples of the algorithm, when used to detect straight lines or circles, are given in reference [18]. The algorithm offers a significant improvement to previous suggested implementations of the Hough transform. It is significantly computationally less intensive and is more efficient in memory utilization.

3 . Conclusion

A new algorithm for computing the Hough transform has been presented. It uses information present in the location of the feature points to reduce the generation of evidence in the transform plane. The algorithm gives improved performance compared with the standard Hough transform. The improvement is in computation time and memory allocation. Further advantages of using the algorithm are that peak detection is one dimensional and the end points of curves may be detected. The algorithm is also inherently parallel.

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