A NOTE ON THE COMPUTATIONAL COMPLEXITY OF BRACKETING AND RELATED PROBLEMS

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IMA Preprint Series # 382

January 1988

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A NOTE ON THE COMPUTATIONAL COMPLEXITY OF BRACKETING AND RELATED PROBLEMS*

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Abstract. It is shown that the problem of finding the minimum number of bracketing transfers in order to transform one bracketing to another bracketing is an NP-complete problem. This problem is related to problems on random walks and to the problem of a comparison of two (labeled) rooted trees. The latter problem is studied with connection to cluster analysis. Finally, one polynomially solvable class of bracketing problems is obtained.

I. Introduction and background. Bracketing problems have a long history [8]. Though the main emphasis was mainly concentrated on enumeration problems we shall be interested in the computational complexity of evaluation of the distance between two given bracketings. Finally, using the concept of closed random walks we shall stress the connection of bracketing problems to the problem of comparison and evaluation of two labeled rooted trees. This type of problem is often investigated in cluster analysis [6].

More precisely, the word w in the alphabet $\Sigma = \{(,)\}$ is said to be a *bracketing* if it is generated by the following rules :

$$S \longrightarrow SS|(S)|\Lambda,$$

where Λ stands for an empty word. The set of all bracketings is often called the Dycklanguage and plays an important part in the theory of formal languages [3]. The abbreviation $l^i, i > 0, l \in \Sigma$, denotes $\underbrace{l \dots l}_{i \text{ times}}$. Let $\mathcal{B}$ ($\mathcal{B}_n$, resp.) be a set of all bracketings over Σ (...of length n, resp.). Note that n is even. A bracketing $b' \in \mathcal{B}$ is said to be a *sub-bracketing* of b, written $b' \subset b$, if b' is a proper subword of b. The *nesting level* of a sub-bracketing b' of b is the number of different sub-bracketings of b which contain b' as their sub-bracketing. Given two bracketings $b_1, b_2 \in \mathcal{B}_n$ we say that bracketing b_2 arises from bracketing b_1 by one *bracketing transfer* if there is a sub-bracketing b of b_2 such that

$$b_1 = xby \quad \text{and either} \quad b_2 = x_1bx_2y, \quad \text{where} \quad x_1x_2 = x,$$

$$\text{or} \quad b_2 = xy_1by_2, \quad \text{where} \quad y_1y_2 = y, \quad \text{for} \quad x, x_2, y, y_1 \in \Sigma^+, x_1, y_2 \in \Sigma^*.$$

By $\beta(b_1, b_2)$ the minimum number of bracketing transfers needed to transform b_1 to b_2 will be denoted, i.e.

$$\beta(b_1, b_2) = j \quad \text{if there is a sequence} \quad s_1, s_2, \dots, s_{j+1} \quad \text{of bracketings from } \mathcal{B}_n \text{ such that}$$

$$b_1 \equiv s_1, b_2 \equiv s_j, \beta(s_i, s_{i+1}) = 1 \quad \text{for} \quad i = 1, \dots, j.$$

First we have the following straightforward lemma :

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* Extended abstract

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LEMMA 1. *The function β is the distance measure on $\mathcal{B}_n$ and $(\mathcal{B}_n, \beta)$ is a metric space.* $\square$

The underlying computational problem **BR** in which lies our main interest is stated as follows :

INSTANCE : Two bracketings $b_1, b_2 \in \mathcal{B}_n$, positive integer k ;

QUESTION : Does it hold that $\beta(b_1, b_2) \leq k$?

Our NP-completeness terminology is that of [2].

II. Complexity results. First we shall prove the following.

THEOREM 1.

The problem **BR** *is NP-complete.*

Proof. Clearly, the problem **BR** is in the class NP. We shall exhibit a polynomial transformation from the problem **BIN PACKING** which is known to be strongly NP-complete [2]. **BIN PACKING** has been introduced as follows :

INSTANCE : Positive integers $i_1, \ldots, i_s, B, r$ such that $\sum_{j=1}^{s} i_j = rB$;

QUESTION : Is there a partition of $\{i_1, \ldots, i_s\}$ into r classes $I_1, \ldots, I_r$ such that $\sum_{j \in I_m} i_j = B, m = 1, \ldots, r$?

Given an instance of **BIN PACKING** the instance of the problem **BR** is constructed in polynomial time by putting

$$b_1 \stackrel{def}{=} \underbrace{(^B)^B \ldots (^B)^B}_{r \text{ times}}, \quad b_2 \stackrel{def}{=} (^{i_1})^{i_1}(^{i_2})^{i_2} \ldots (^{i_s})^{i_s}, \quad k \stackrel{def}{=} s - r.$$

Now, the equivalence

$$\beta(b_1, b_2) = s - r \Leftrightarrow \textbf{BIN PACKING} \text{ has "yes"-solution}$$

is easily verified and the theorem is proved. $\square$

Theorem 1 says that it is very unlikely that there exists a polynomial algorithm for the problem **BR**. Therefore we would like to exhibit a polynomial approximation for the problem **BR**. Notice that proof of Theorem 1 does not exclude the possible existence of such an algorithm. The so-called "next fit" approximation algorithm has been believed to provide a "good" polynomial approximation for **BR** since **BR** generalizes in some way the **BIN PACKING** problem. Recall that the next fit algorithm was proved to be a "good" approximation for **BIN PACKING** both from the worst and average case complexity viewpoint [2,4]. Formally the approximation algorithm $\mathcal{A}$ for **BR** is encoded as follows :

Algorithm $\mathcal{A}$:

(Step 1.) $s_1 := b_1$; $s_2 := b_2$;

(Step 2.) **do** $2n$ **times**

Scan and compare current letters of s_1 and s_2;

if they are different **then**

{*suppose that scanned letter in* s_1 *is* "(" , *i.e.* $s_1 = x(y, \quad x, y \in \Sigma^*$}

(Step 3.) find in s_2 a "next" sub-bracketing b (minimal/maximal with respect to the current nesting level) such that $s_2 = x)y_1 b y_2, \quad y_1, y_2 \in \Sigma^*$;

$s_2 := xb)y_1 y_2$;

endif

endo

endalgorithm

The correctness and time analysis of the algorithm $\mathcal{A}$ is established in the following theorem :

THEOREM 2. *Algorithm $\mathcal{A}$ runs in polynomial time and solves problem* **BR** *using $O(n)$ bracketing transfers.*

Proof. Rough time estimate for Step 3 is $O(n)$. This yields $O(n^2)$ time complexity of the algorithm $\mathcal{A}$. The algorithm $\mathcal{A}$ transforms bracketing b_1 into bracketing b_2. This is observed from the fact that eventually both words s_1, s_2 produced by $\mathcal{A}$ are equal. As possibly both b_1, b_2 and consequently s_1, s_2 are changed the sequences $b_1 \longrightarrow s_1$ and $s_2 \longrightarrow b_2$ provide the sequence of bracketing transfers required for transforming b_1 to b_2. In the worst case the number of bracketing transfers is proportional to the corresponding number of sub-bracketings of b_1 (b_2, resp.) and thus it is $O(n)$. □

Remark. Using so-called search trees [7] as a data structure for the representation of bracketings, Step 3 can be implemented in $O(\log n)$ time. Asymptotically $O(n \log n)$ upper bound is the best possible for polynomially solvable instances of **BR** since the well-known **SORTING** problem is linearly transformable (assuming an unary representation of input numbers) to the following instance of **BR** :

$$b_1 = ()(()) \ldots (^n)^n, b_2 = (^{x_1})^{x_1}(^{x_2})^{x_2} \ldots (^{x_n})^{x_n} \quad \text{where} \quad \{x_1, x_2, \ldots, x_n\} = \{1, \ldots, n\}$$

constitute an instance of **SORTING**. Recall that **SORTING** is solvable in $\Theta(n \log n)$ time [7]. □

Let us deal with the question of how good the approximation produced by $\mathcal{A}$ is. Regrettably no constant bounded worst case error ratio is guaranteed.

THEOREM 3. *Algorithm $\mathcal{A}$ has a $\Theta(n)$ worst case error ratio.*

Proof. Let $b_1 = xy, b_2 = yx, x = (), y = ()(\underbrace{()\ldots()}_{O(n)\text{ times}})$. In this case $\beta(b_1, b_2) = 1$. However algorithm $\mathcal{A}$ constructs a sequence of $O(n)$ bracketing transfers regardless of the nesting level of the "next" sub-bracketing in Step 3. □

The failure of algorithm $\mathcal{A}$ is due to the fact that $\mathcal{A}$ does not search for identical sub-bracketings in b_1 and b_2. Therefore its behavior could be slightly improved by pre-processing, i.e. by decomposing b_1 and b_2 into their corresponding sub-bracketings, say, maximal up to inclusion. This approach supposes setting up a data structure where nesting level and sub-bracketing can be directly accessed. This way we avoid pathological behavior of $\mathcal{A}$ on the current nesting level but complexity problems remain unchanged when dealing with identical sub-bracketings on different nesting levels.

Let us conclude this section by a remark that some preliminary calculations indicate that algorithm $\mathcal{A}$ also has the average case error ratio of order $\Theta(n)$. The details will appear in a full paper.

III. Random walks and rooted trees. The aim of this section is to discuss a 1-1 correspondence between bracketings, random walks and rooted trees. It will enable us to extend the results of the previous section to trees embedded to the plane. Our exposition is based on [5].

Random walk of length n is a $(n+1)-$tuple $\Phi = (\varphi(0), \varphi(1), \ldots \varphi(n))$ where φ is a mapping to non-negative integers such that

$$\varphi(0) = \varphi(n) = 0, \varphi(i) \in \{\varphi(i-1) - 1, \varphi(i-1) + 1\}, i = 1, \ldots, n.$$

LEMMA 2. *There is a one-to-one correspondence between $\mathcal{B}_n$ and the set of all random walks of length n.*

Proof. Let $b \in \mathcal{B}_n$. Define a random walk Φ of length n as follows

$$\varphi(i) = \begin{cases} 0, & \text{if } i \in \{0, n\} \\ \varphi(i-1) + 1, & \text{if } i\text{-th letter of } b \text{ is "("} \\ \varphi(i-1) - 1, & \text{if } i\text{-th letter of } b \text{ is ")"}. \ \square \end{cases}$$

Let T be a rooted tree on n vertices embedded into the plane. Let us consider a topological ordering $\omega_T = v_0 v_1 \ldots v_{2n-2}$ of its vertices which is recursively defined as follows :

(1) If $T = \{v_0\}$ then $\omega_T = v_0$,

(2) If T has a root v_0 with the subtrees $T_1, T_2, \ldots, T_k$ then $\omega_T = v_0 \omega_{T_1} v_0 \omega_{T_2} v_0 \ldots \omega_{T_k} v_0$.

The set of all trees on n vertices with a root v_0 embedded into the plane will be denoted by $\mathcal{T}_n$.

Let $T_1, T_2 \in \mathcal{T}_n$. T_1 is said to be obtained from T_2 by one *subtree modification* if

$$T_1 \approx T_2 - uv + uw, \text{ where } uv \in E(T_2), uw \notin E(T_2),$$

where symbol $\approx$ expresses that both trees have the same topology of plane embedding, i.e. they are *topologically isomorphic.* The *subtree distance* $\tau(T_1, T_2)$ between T_1 and T_2 is defined as the minimum number of subtree modifications needed to be performed on T_2 in order to obtain a tree which is topologically isomorphic to T_1. Notice that the root v_0 is supposed to be fixed. The following observations are quite straightforward:

LEMMA 3. *The pair $(\mathcal{T}_n, \tau)$ forms a metric space.* □

LEMMA 4. *There is a one-to-one correspondence between $\mathcal{T}_n$ and random walks of length $2n-2$.*

Proof. Let $T \in \mathcal{T}_n$. Let $d(v_i, v_0)$ be the distance of the i-th vertex v_i of ω from v_0. The corresponding random walk $\Phi = (\varphi(0), \ldots, \varphi(2n-2))$ is defined as follows :

$$\varphi(i) = \begin{cases} 0, & \text{if } i \in \{0, 2n-2\} \\ d(v_i, v_0), & \text{otherwise. } \square \end{cases}$$

Now we are ready to prove the following :

THEOREM 4. *Given $T_1, T_2 \in \mathcal{T}_n$, the underlying decision problem of computing $\tau(T_1, T_2)$ is NP-complete.*

Proof. Choose $b_1, b_2 \in \mathcal{B}_n$ and by virtue of Lemma 4 consider two corresponding trees $T_1, T_2 \in \mathcal{T}_n$. By the aid of Lemma 2 and Lemma 4 we have

$$\beta(b_1, b_2) = \tau(T_1, T_2).$$

The use of Theorem 1 completes the proof. □

Trees from $\mathcal{T}_n$ are very often constructed by hierarchical clustering procedures. The study of the consensus between these trees is one of the most important problems encountered in cluster analysis [6]. However, special attention is mostly paid to binary trees [1]. The concept of random walks can be used for proving similar NP-completeness results for binary trees, too.

Let $T \in \mathcal{T}_{2n-1}$ be a binary rooted tree on n leaves, i.e. having all internal vertices of degree 3 except of the root v_0 which is of degree 2. A given binary rooted tree T induces a topological ordering $\omega_T = v_0 v_1 \ldots v_{2n-2}$ which is defined recursively as follows :

(1) If $T = \{v_0\}$ then $\omega_T = v_0$,

(2) If T has a root v_0 with the subtrees T_1, T_2 then $\omega_T = \omega_{T_1} \omega_{T_2} v_0$.

The set of all binary trees on n leaves and with the root v_0 embedded into the plane will be denoted by $\mathcal{T}_n^b$. Given $T_1, T_2 \in \mathcal{T}_n^b$ we say that T_1 is obtained by one *(binary) subtree modification* if

$$T_1 \approx T_2 - \{\{u_1, v\}, \{u_1, w\}\} + \{\{u_2, v\}, \{u_2, w\}\}$$

where

$$\{u_1, v\}, \{u_1, w\} \in E(T_2) \text{ and } \{u_2, v\}, \{u_2, w\} \notin E(T_2).$$

Notice that u_2 is a leaf. The *(binary) subtree distance* τ^b is defined as the minimum number of subtree modifications required to obtain a tree T_1 from T_2. Similarly as in the general case the following propositions hold

LEMMA 5. *The pair $(\mathcal{T}_n^b, \tau^b)$ forms a metric space.* □

LEMMA 6. *There is a one-to-one correspondence between binary rooted trees on n leaves and random walks of length $2n-2$.*

Proof. Let $T \in \mathcal{T}_n, \omega = v_0 v_1 \ldots v_{2n-2}$. The corresponding random walk Φ is defined as follows

$$\varphi(i) = \begin{cases} 0, & \text{if } i \in \{0, 2n-2\} \\ \varphi(i-1)+1, & \text{if } v_i \text{ is a leaf in } T \\ \varphi(i-1)-1, & \text{if } v_i \text{ is an internal vertex in } T. \end{cases}$$ □

Combining Lemma 2, Lemma 6 and Theorem 1 we get

THEOREM 5. *Given $T_1, T_2 \in T_n^b$ the underlying decision problem of computing $\tau^b(T_1, T_2)$ is NP-complete.* □

IV. Labeled rooted trees. In this section a polynomially solvable class of bracketing problems will be explored by means of labeled rooted trees. Let $T \in \mathcal{T}_n$ and let ω_T be its topological ordering. Let us define on the set of vertices of T a *labeling* ξ, $\xi : \{v_0, \ldots, v_{n-1}\} \rightarrow \{0, \ldots, n-1\}$ as follows :

$$\xi(x) = \begin{cases} 0, & \text{if } x \equiv v_0 \\ i, & \text{if vertex } x \text{ occurs as the } i\text{-th new vertex in } \omega_T. \end{cases}$$

Let us suppose that we are given a fixed labeling ξ on $\{v_0, \ldots, v_{n-1}\}$. Let $\mathcal{T}_n^\xi$ denote the set of all labeled trees on n vertices with the root v_0 and with the same labeling ξ. Now we can define a subtree modification distance τ^ξ between labeled rooted trees from $\mathcal{T}_n^\xi$ formally in the same way as in the unlabeled case with the only exception that now the labeling ξ must be preserved by subtree modifications. Clearly Lemma 3 and Lemma 4 can be rewritten as follows :

LEMMA 5. *The pair* $(\mathcal{T}_n^\xi, \tau^\xi)$ *forms a metric space.* □

LEMMA 6. *There is a one-to-one correspondence between* $\mathcal{T}_n^\xi$ *and random walks of length* $2n-2$. □

Let us define two graphs

$$\mathcal{G}_1 = (\mathcal{T}_n, E_1), \quad \mathcal{G}_2 = (\mathcal{T}_n^\xi, E_2)$$

where

$$\{T_1, T_2\} \in E_1 \Leftrightarrow \tau(T_1, T_2) = 1 \quad \text{for} \quad T_1, T_2 \in \mathcal{T}_n,$$

$$\{T_1, T_2\} \in E_2 \Leftrightarrow \tau^\xi(T_1, T_2) = 1 \quad \text{for} \quad T_1, T_2 \in \mathcal{T}_n^\xi.$$

It is easy to see that $\mathcal{G}_2$ is a proper subgraph of $\mathcal{G}_1$. This observation justifies the following theorem :

THEOREM 6. *Given* $T_1, T_2 \in \mathcal{T}_n^\xi$, *the problem of the computation of* $\tau^\xi(T_1, T_2)$ *is polynomially solvable.*

Proof. Let us consider the following algorithm :

(Step 1.) **do** traverse the tree T_2 using so-called breath-first search [7]

(Step 2.) **if** childrens of the current vertex of T_1 and T_2 are different

then update locally the tree T_2 by T_1

endo

The loop involved in Step 1 requires $O(n)$ time, Step 2 can be implemented in $O(\log n)$ time using search trees as data structures for the fast search and update in T_1 and in T_2. □

It is left to the reader to find an example which shows that the algorithm outlined above has $\Theta(n)$ worst case error ratio if it is used as an approximation for a general bracketing problem.

REFERENCES

[1] K. ČULÍK II. AND D. WOOD, *A note on some tree similarity measures*, Information Processing Letters, 15 (1982), pp. 39–42.

[2] M.R. GAREY AND D.S. JOHNSON, *Computers and intractability*, W.H. Freeman, San Francisco, 1979.

[3] M.A. HARRISON, *Introduction to formal language theory*, Addison-Wesley, 1978.

[4] M. HOFRI, *Probabilistic analysis of algorithms*, Springer-Verlag, 1987.

[5] R. KEMP, *Fundamentals of the average case analysis of particular algorithms*, Wiley-Teubner, Stuttgart, 1984.

[6] M. KŘIVÁNEK, *The computational complexity of the consensus between hierarchical trees*, Proc. IMYCS' 84, Smolenice, pp. 119–125.

[7] K. MEHLHORN, *Data structures and algorithms*, Springer-Verlag, 1984.

[8] SCHRÖDER, *Vier Combinatorische Probleme*, Z. für M. Phys., 15 (1870), pp. 361–376.

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