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The State of the Art

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Preface

A workshop on the competitive analysis of on-line algorithms was held at Schloss Dagstuhl (Germany) during the last week of June 1996. Many survey talks were presented at this workshop. We then asked the speakers to prepare survey chapters that covered and extended the contents of their talks. This volume now is the compilation of the survey chapters. We thank all contributors who so generously gave of their time to make the book a success.

Amos Fiat and Gerhard J. Woeginger
May 1998

Sooner or later in life everyone discovers that perfect happiness is unrealizable, but there are few who pause to consider the antithesis that perfect unhappiness is equally unattainable. The obstacles preventing the realization of both these extreme states are of the same nature; they derive from our human condition which is opposed to everything infinite. Our ever-insufficient knowledge of the future opposes it; and this is called in the one instance, hope, and in the other, uncertainty of the following day.

(Primo Levi comments on the on-line nature of human life.
From: Primo Levi, *Se questo è un uomo*, 1958)

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