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Hyperedge Replacement: Grammars and Languages

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Foreword

First approaches to graph transformations, graph grammars and graph languages have been introduced about twenty years ago by several authors in the U.S.A. and Europe in order to model different application areas within Computer Science. Graph grammars can be considered as a generalization of classical Chomsky grammars where strings are replaced by graphs. While in the string case it is obvious how to replace a substring by another one, replacement is no longer trivial in the graph case. Given a graph production with left hand side L and right hand side R, an application of this production to a graph G means to find an occurrence of L in G, to remove L from G and to embed R into the remaining part of G leading to a derived graph H. The main problem is how to construct this embedding. In fact, there have been various different solutions for this embedding problem leading to a variety of graph grammar approaches in the literature.

One of the main influences on this volume has been the algebraic approach developed in Berlin in the early seventies and the NLC (node label controlled) grammars from the school in Leiden built up by G. Rozenberg in the late seventies. The school in Berlin was initiated by H.J. Schneider, M. Pfender and myself in 1973 and mainly influenced by B.K. Rosen (IBM Research, Yorktown Heights), H.-J. Kreowski since about 1975 and Annegret Habel, the author of this volume. She joined our group as a student at the end of the seventies and followed H.-J. Kreowski to Bremen about five years later. In her diploma thesis she provided a most elegant new proof for the "Concurrency Theorem" for graph grammars and a technique for the decomposition of graph productions into atomic parts. She continued this line of research together with two colleagues in Berlin leading to the important concept of amalgamation of graph transformations, an elegant algebraic version of synchronization concepts for graph productions in distributed systems developed by U. Montanari and P. Degano in Pisa.

Influenced by the NLC-school in Leiden, she became interested in context-free graph grammars and languages. In the early eighties it was an open problem how far the rich theory of context-free Chomsky grammars and languages could be generalized to the graph case. In contrast to node replacement in the NLC approach, she joined H.-J. Kreowski investigating edge replacement as a special case of the algebraic approach, where the left hand side of a production consists of a single edge.

Although nice results were obtained in this framework, I was still somewhat sceptical at that time because there were only a few interesting applications of this edge replacement approach. The range of applications, however, becomes significantly larger when hyperedges instead of edges are considered. Hyperedges are allowed to have sequences of source and target nodes instead of just one source and one target node for each edge in a graph. This allows one to model functional expressions, Petri nets, flow diagrams and chemical molecule structures using hyperedge replacement grammars while non-context-free productions are necessary in the framework of usual graphs and graph grammars to model these applications.

Annegret Habel was not only able to generalize the theory of edge to hyperedge replacement system which is presented in this volume but also to find an answer to the open problem from the early eighties: Several major constructions and results known from the classical theory of context-free Chomsky grammars and languages remain valid for hyperedge replacement grammars and corresponding hypergraph languages. This includes closure properties, a Kleene-type characterization of languages, a fixed-point theorem, a pumping lemma, Parikh's theorem, boundedness properties and results concerning the generative power of languages. Although most of these results and their proofs in the graph case are much more sophisticated than those in the string case the presentation by Annegret Habel is significantly clear and complete in contrast to several other publications in the area of graph grammars.

Another important part of this volume is the general theory of decidability concerning graph theoretic properties. In fact, the notion of "compatible properties" includes connectivity, the existence of Eulerian paths and cycles, edge colorability, and several other interesting properties and it is shown that all these properties are decidable for hyperedge replacement grammars. Similar results were obtained in the literature by M. Bauderon and B. Courcelle as well as T. Lengauer and E. Wanke using different approaches.

Last but not least it should be mentioned that this volume is based on the Ph.D. thesis of Annegret Habel and constitutes an important contribution within the ESPRIT Basic Research Working Group "Computing by Graph Transformation". I am very glad that this significant part of the theory of graph grammars and their applications in Computer Science is published by Springer Verlag in this volume of Lecture Notes in Computer Science, which is well recognized in the Computer Science community.

Berlin, July 1992 Hartmut Ehrig

Preface

The area of graph grammars is theoretically attractive and well motivated by various applications. More than 20 years ago, the concept of graph grammars was introduced by A. Rosenfeld as a formulation of some problems in pattern recognition and image processing as well as by H.J. Schneider as a method for data type specification. The four proceedings volumes of the Workshop on Graph Grammars and Their Applications to Computer Science series (Lecture Notes in Computer Science) provide a rich record of the development of this field.

Within graph-grammar theory one may distinguish among several approaches:

- the set-theoretical approach,
- the algebraic approach, and
- the logical approach

(see the overview papers by Nagl, Ehrig, and Courcelle [Na 87, Eh 79+87, Co 89c]). These approaches differ in the method in which graph replacement is described. Specific approaches are

- the node-replacement approach and
- the hyperedge-replacement approach,

concerning the basic units of a hypergraph, nodes and hyperedges.

This monograph is mainly concerned with the hyperedge-replacement approach reviving some ideas from the early seventies which can be found in Feder's [Fe 71] and Pavlidis' [Pa 72] work. Hyperedge-replacement grammars are introduced as a device for generating hypergraph languages including graph languages and string languages (where the strings are uniquely represented as certain graphs). The concept combines a context-free rewriting with a comparatively large generative power. The former is indicated, for example, by a fixed-point theorem and a pumping lemma, the latter by the examples such as the refinement of petri nets, the analysis of flow diagrams, the structural description of molecules, and some typical non-context-free string languages. Hence, we can be confident that this framework combines the theoretical attractivity of a class of context-free grammars and languages with the prospect for nice applications.

This book is a revised, enlarged and updated version of my doctoral dissertation accepted by the University of Bremen. I started to work on it while I was a member of the Theoretical Computer Science group at the Technical University of Berlin; it was finished while I was a member of the Computer Science Department at the University of Bremen. Special thanks are due to my thesis supervisors Hartmut Ehrig and Hans-Jörg Kreowski. They introduced me to the fascinating world of graph grammars, taught me to do research in theoretical computer science, and gave me an environment in which I enjoyed to do creative work. Working in their groups, doing common research, writing joint papers, as well as a feeling of liking and esteem has played a central role in starting and carrying out the research described here.

Additionally, I would like to express my thanks to all those people without whose help this book would not be what it is now. Special thanks are due to Walter Vogler and Clemens Lautemann for many stimulating discussions that we have had during the preparation of joint papers, Peter Padawitz, Udo Hummert, and Detlef Plump for a number of fruitful discussions, numerous suggestions, and constructive comments during the long period of preparing the Ph.D. Thesis, Frank Drewes, Sabine Kuske, Geoff Simmons, and Stefan Taubenberger for their helpful feedback as I revised the manuscript, and all good friends for their general moral support. Finally, let me mention Ingeborg Mayer and Hans Wössner from Springer-Verlag who invited me to submit the manuscript for publication and took care of the production of this book.

I also wish to thank my husband Christopher for helping me throughout this adventure in more ways than I can list here. He suggested me to study theoretical computer science and encouraged me to focus on formal language theory while he was writing his doctoral dissertation in linguistics entitled Aspects of Valuating Grammars. During the time writing my doctoral dissertation he contributed a number of suggestions and — more important than anything else — sound advice. I could not have done it without him.

Bremen, July 1992 — Annegret Habel

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