### Lecture Notes in Artificial Intelligence

Subseries of Lecture Notes in Computer Science Edited by J. Siekmann

Lecture Notes in Computer Science Edited by G. Goos and J. Hartmanis

#### Editorial

Artificial Intelligence has become a major discipline under the roof of Computer Science. This is also reflected by a growing number of titles devoted to this fast developing field to be published in our Lecture Notes in Computer Science. To make these volumes immediately visible we have decided to distinguish them by a special cover as Lecture Notes in Artificial Intelligence, constituting a subseries of the Lecture Notes in Computer Science. This subseries is edited by an Editorial Board of experts from all areas of AI, chaired by Jörg Siekmann, who are looking forward to consider further AI monographs and proceedings of high scientific quality for publication.

We hope that the constitution of this subseries will be well accepted by the audience of the Lecture Notes in Computer Science, and we feel confident that the subseries will be recognized as an outstanding opportunity for publication by authors and editors of the AI community.

Editors and publisher

# Lecture Notes in Artificial Intelligence

Edited by J. Siekmann

Subseries of Lecture Notes in Computer Science

395

## M. Schmidt-Schauß

## Computational Aspects of an Order-Sorted Logic with Term Declarations



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### Preface

In this volume the logical foundations and the computational aspects of a rather general order-sorted logic are investigated. This sorted logic extends first order logic by a partially ordered set of sorts, such that every term is of a particular sort or type and such that only well-sorted substitutions are permitted.

Several sorted unification algorithms are described which work for different signatures with or without equational theories. The type and the complexity of sorted unification is determined for different types of signatures. The type can range from unary to infinitary and the time complexity ranges from linear up to undecidable, even without defining equations.

Different sorted resolution-based calculi for clause sets with and without equations are described and their refutation-completeness is investigated.

#### Statement

This paper is a shortened and slightly edited version of my thesis [Sch88]. In particular, the part on sort-generation is completely omitted.

#### Acknowledgements

I would like to thank my supervisor Jörg Siekmann. He introduced me into the field of Artificial Intelligence and Automated Deduction. His enthusiasm, guidance and critics were indispensible for writing down this thesis. I thank him for his final revision.

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