Construction of Basic Match Schedules for Sports Competitions by Using Graph Theory

Arjen van Weert[1], Jan A.M. Schreuder[2]

[1] Agrotechnological Research Institute ATO-DLO, P.O. box 17,
NL-6700 AA Wageningen, The Netherlands
a.vanweert@ato.dlo.nl
[2] University of Twente, Faculty of Mathematical Sciences, P.O. box 217,
NL-7500 AE Enschede, The Netherlands
j.a.m.schreuder@math.utwente.nl

Abstract. Basic Match Schedules are important for constructing sports timetables. Firstly these schedules guarantee the fairness of the sports competitions and secondly they reduce the complexity of the problem. This paper presents an approach to the problem of finding Basic Match Schedules for sports competitions. The approach is clarified by applying it to the Dutch volleyball competition. By using graph representation and theory this approach has the potential to classify sports competitions and to build libraries with Basic Match Schedules for each class of sports competitions. As an example we present an overview of some Basic Match Schedules for the volleyball competition.

1 Introduction

Constructing sports timetables used to be a task for volunteers. These volunteers used their own approaches and had to face just a few constraints. This situation is changing. Sport leagues are organized more professionally and more commercial interests are involved. For this reason, sport leagues are demanding structural methods to construct their timetables.

Constructing sports timetables is a complex task. Constraints are often closely related to each other. Above all, many potential timetables have to be checked in most cases. By using computers this can be performed more accurately. Some examples of computerized timetabling can be found in [1] or [10].

To handle the complexity it is common practice to decompose the construction of sports timetables. This construction is usually decomposed in two phase's [5]. In the first phase a Basic Match Schedule (BMS) is constructed. A BMS determines, for each competition round, the matches and groups the teams have to play in and thus gives the technical outline of the specified sports competition. In the second phase individual teams are assigned to the matches and groups of the constructed BMS. In this phase constraints connected to the individual teams are taken into account. Each phase needs a specific approach.

In this paper we will focus on the construction of Basic Match Schedules for the Dutch volleyball league. In Section 2 we present the characteristics of the Dutch volleyball competition. Section 3 presents the general approach used to find Basic Match Schedules for the volleyball competition. The execution of this approach is presented in Section 4. Section 5 concludes the paper and indicates how the approach can be used for the classification of all sports competitions.

2 The Dutch Volleyball Competition as Example

A volleyball competition consists of a number of sport teams competing with each other. The volleyball competition is subjected to competition rules and external wishes. The competition rules guarantee the fairness of the competition. The external wishes guarantee more or less the practicability of the competition and are focused on specific teams.

The following rules and wishes apply for the Dutch volleyball competition [11]:

Competition rules

1. Each team plays against two opponents in one group in each round.
2. The group sizes are fixed and are the same in each round.
3. The competition consists of two halves. In each half the teams play once against each other.
4. The second competition half is a copy of the first competition half.

The group structure is based on the fact that in a round a group of teams meets in a specific sport venue to play their matches.

Possible external wishes

1. Two teams do not want to play against each other in a specific round.
2. A specific team wants to play in a specific round close to home.

The number of external wishes tends to increase every year. The wishes expressed above are examples of the vast amount of possible external wishes.

In the following Sections we show how to construct a Basic Match Schedule for the described volleyball competition.

3 Construction of Basic Match Schedules

First we present a definition for a Basic Match Schedule.

Definition 3.1

A *Basic Match Schedule (BMS)* determines for each competition round the matches and groups the teams have to play in. A Basic Match Schedule is denoted as a four dimensional matrix W, where W_{ijkl} defines the opponent k for team i in round j played in group l.

Table 3.1. Example of BMS for round 1 and group 1

4 teams, 1 round and 1 group (W_{i1k1})		
Team	**Opponent 1 (k=1)**	**Opponent 2 (k=2)**
1	2	3
2	1	4
3	4	1
4	3	2

Starting from the Dutch volleyball competition description, we use a three-step approach to reach a Basic Match Schedule. This approach is depicted in Figure 3.1.

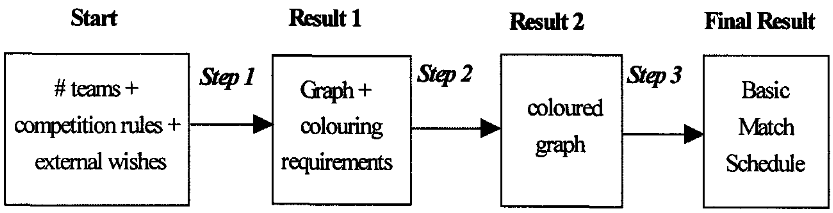

Figure 3.1: Three-step approach to the construction of Basic Match Schedule.

This approach starts with an instance of one half of the volleyball competition as defined in Section 2. The second half is copied from the second half as stated in Competition Rule 4. For the Basic Match Schedules the external wishes are ignored. Only the competition rules are taken into account.

Step 1: Represent the competition in graph

In Step 1 the teams and matches are represented in a graph. Each team is represented in a graph by a vertex. Each match between two teams can be represented by an edge between these vertices. The advantage of this graph is the compact representation of teams and matches. Graph models are already used for other timetabling problems. For an example, see [8] or [9].

Step 2: Represent the rounds in a graph

By colouring the edges of the graph constructed in Step 1 the rounds for the matches can be indicated. A specific round is represented by one colour. We can assign each edge to one of these colours. This colour indicates the round of the match related to the edge. Depending on the characteristics of a sport competition (e.g. group sizes) certain graph theoretic properties will hold for the graph colouring. These properties can be used to prove that a BMS can be constructed or can help to construct a BMS.

Step 3: coloured graph to BMS
The resulting total colouring of Step 2 represents a BMS.

This approach is clarified by applying it to the volleyball competition.

4 Results for Volleyball

4.1 Odd number of teams

Step 1
If we have a volleyball competition with an odd number $2m+1$ of teams (where m is an integer), the resulting graph will be a complete graph with $2m+1$ nodes denoted as K_{2m+1}. The completeness of the graph results from Competition Rule 3: Each team is connected only once with all other teams. Figure 4.1 shows the resulting graph for a volleyball competition with 7 teams.

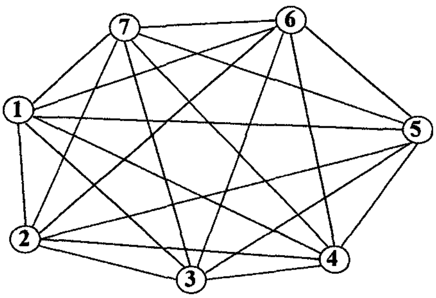

Figure 4.1. complete graph for 7 teams

Step 2
The edges of the complete graph resulting from Step 1 must be coloured with m colours, each colour representing one round. Competition Rules 1 and 2 do not allow just any colouring. A graph induced by one colour must be spanning and 2-regular. This means that all vertices must be connected with two other vertices by two edges of that specific colour. A graph with such properties is an example of a 2-factor. In general a 2-factor of a graph is defined as a 2-regular spanning subgraph. Each group of k teams must be represented in the 2-factor by a set of cycles spanning k vertices. Note that for $k < 6$ this set of cycles is always reduced to one single cycle. The results in this paper are based on single cycles for all k.

We show that only for a competition with an odd number of teams it is possible to find such a colouring.

Theorem 4.1
If K_n can be coloured resulting in edge-disjunct isomorphic 2-factors, then n is odd.

Proof.
Let F_k be a 2-factor for K_n. Then the number of edges in F_k is n. The total number of edges in K_n is $n(n-1)/2$, so $(n-1)/2$ edge-disjunct 2-factors can be coloured. This proves that n must be odd.

In Figure 4.2 the fat lines indicate one round in the graph of Figure 4.1. One group of four teams and one group of three teams are represented.

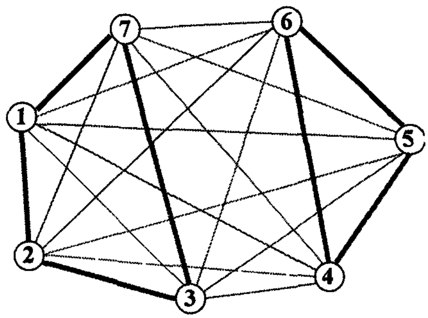

Figure 4.2. Round with group sizes 4 and 3.

We formulate the problem of constructing a BMS for sport competitions with an odd number of teams as the graph theoretic Oberwolfach problem.

Definition 4.1
The *Oberwolfach problem* is defined as finding for a K_{2m+1} a splitting of the edges in m coloured edge-disjunct isomorphic 2-factors F_k. Each F_k consists of vertex disjunct cycles $C_{k1}, C_{k2},.., C_{ks}$ where $k_1 + k_2 + .. + k_s = 2m + 1$, k_1 to k_s are the group sizes. The problem is denoted as $OP(k_1,..k_s)$.

The number of rounds is equal to m. The number of teams in group i is equal to k_i $(i=1,..,s)$. For some m, a construction of these 2-factors is known [2]; for some m it is known that no construction exists. As an example we will show an unpublished construction of 2-factors for $OP(4,4,5)$ in a structural way.

We use lemmas from [4]. These lemmas show how permutations of vertices can be used to find a union of disjunctive cycles for classes of complete graphs.

Notations:

i: Integer

σ_i : Permutation i of vertices

$\sigma_i = (1,2,3,4)(5)$ means vertex 1 is permuted to vertex 2, vertex 2 is permuted to vertex 3, vertex 3 is permuted to vertex 4, vertex 4 is permuted to vertex 1 and vertex 5 is permuted to vertex 5.

C_i: Cycle of i vertices

K_i: Complete graph on i vertices

$K_{i,j}$: Complete bipartite graph with two independent sets of i vertices and j vertices: All possible edges between the sets of vertices are present, within the sets of vertices no edges are present.

Lemma 4.1. For all m :

$$K_{2m+1} = \bigcup_{h=0}^{m-1} \sigma_1^h (C_{2m+1})$$

with $C_{2m+1} = (1;2;\ 2m;\ 3;\ 2m\text{-}1;\ 4;\ 2m\text{-}1;5;\ \ldots\ ;m+3;\ m;\ m+2;\ m+1;\ 2m+1)$
$\sigma_1 = (1,2,..,2m)(2m+1)$

In Figure 4.3 and Figure 4.4 results of Lemma 4.1 are depicted for $m = 2$, $C_5 = (1;2;4;3;5)$ and $\sigma_1 = (1,2,3,4)(5)$.

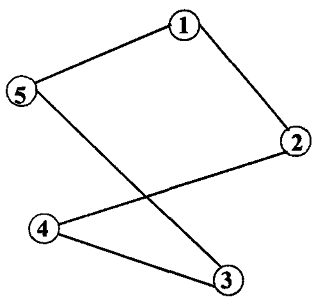

Figure 4.3. $\sigma_1^0 (C_5) = C_5$

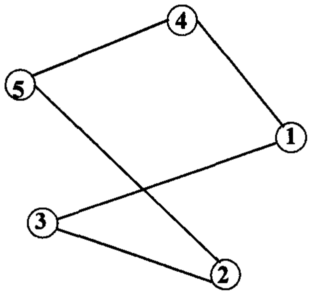

Figure 4.4. $\sigma_1^1 (C_5)$

Lemma 4.2. For all m,n :

$$K_{nm,nm} = \bigcup_{j=0}^{n-1} \sigma_2^{j} \left(K_{m,m}^{1} \cup \ldots \cup K_{m,m}^{n} \right)$$

With $\sigma_2 = (1,m+1,2m+1,..,(n-1)m+1)(2,m+2,..,(n-1)m+1)..(m,2m,..,nm)(nm+1)..(2nm)$

and $K_{m,m}^{i} := \{\{a,b\} \mid a \in A^{i}, b \in B^{i}\}$
with $A^i := \{(i-1)m+1,..,im\}$, $B^i := \{nm+x|x,A^i\}$

Lemma 4.3. For all m,n :

$$K_{m(2n-1)+1} = \bigcup_{k=0}^{2n-2} \sigma_3^{k} (K_{m+1} \cup K_{m,m}^{1} \cup .. \cup K_{m,m}^{n-1})$$

With $K_{m+1} := \{1,2n,2(2n-1)+1,3(2n-1)+1,..,(m-1)(2n-1)+1, m(2n-1)+1)\}$,
For $K_{m,m}{}^{i}$, $\{e^i{}_1,e^i{}_2,..,e^i{}_{2m}\}$ is the set of vertices,
where $e^i{}_{2k-1} := (k-1)(2n-1)+i+1$,
$e^i{}_{2k} := k(2n-1)-i+1$,
and $\{\{e^i{}_\nu, e^i{}_\mu\}|\nu+\mu\equiv 1(\text{mod } 2)\}$ is the set of edges
$\sigma_3 = (1,2,..,2n-1)(2n,..,2(2n-1))...((m-1)(2n-1)+1,..,m(2n-1))(m(2n-1)+1)$.

With the lemma's it can be verified that

$$K_{13} = \bigcup_{j=0} \sigma_3^{j}((\bigcup_{i=0} \sigma_1^{i}(C_5)) \cup (\bigcup_{k=0} \sigma_2^{k}(K_{2,2} \cup K_{2,2})))$$

with $K_{2,2}$ isomorphic to C_4

From this it follows that K_{13} is the union of the following cycles:
(1;4;7;10;13) (3;5;9;11) (6;2;12;8) (= F_1)
(4;10;1;7;13) (6;5;12;11) (3;2;9;8) (= F_2)
(2;5;8;11;13) (1;6;7;12) (4;3;10;9) (= F_3)
(5;11;2;8;13) (4;6;10;12) (1;3;7;9) (= F_4)
(3;6;9;12;13) (2;4;8;10) (5;1;11;7) (= F_5)
(6;12;3;9;13) (5;4;11;10) (2;1;8;7) (= F_6)
Each row represents one 2-factor, so K_{13} is the union of six 2-factors.

In Table 4.1 all known constructions are listed for K_n ($n < 16$), with n odd.

Table 4.1. Overview of constructions for the Oberwolfach problem

Number of teams	Group sizes	Construction possible?	Reference
7	3,4	yes	[4]
9	3,3,3	yes	[3]/[4]
	4,5	no	[4]
11	3,3,5	not known	
	5,6	yes	[4]
	3,4,4	yes	[4]
13	6,7	yes	[4]
	5,8	yes	[4]
	4,9	yes	[4]
	3,10	not known	
	4,4,5	yes	This paper
	3,5,5	not known	
	3,4,6	not known	
	3,3,7	yes	[4]
	3,3,3,4	not known	
15	all combinations	yes	[4]

Step 3

As stated in Step 2 each coloured edge in the graph represents a match in a round and the extra graph properties guarantee the group sizes. This means that each construction for the Oberwolfach problem can be translated to a BMS for a specific volleyball competition.

As an example, Table 4.2 and Table 4.3 present two groups in the first round for OP(4,4,5), resulting from the 2-factors presented in Step 2.

Table 4.2. W_{t1k1} , BMS for round 1 and group 1

Team	Opponent 1	Opponent 2
1	4	13
4	1	7
13	10	1
7	10	4
10	7	13

Table 4.3. W_{t1k2} , BMS for round 1 and group 2

Team	Opponent 1	Opponent 2
2	6	12
6	2	8
12	8	2
8	12	6

4.2 Even number of teams

Step 1

If we have a volleyball competition with an even number of teams, the resulting graph will be a complete graph with $2m$ nodes denoted as K_{2m}.

Step 2

Following from the results of Step 2, Paragraph 4.2 there is no colouring possible obeying all competition rules. To find a BMS for this specific competition with an even number of teams we have to define slightly different competition rules. The rounds are divided into *normal* rounds and one *intermediate* round. In a normal round each team plays two matches against different opponents within a group. In the intermediate round each team plays only one match. After all normal rounds and the intermediate round each pair of teams has encountered each other once.

We will formulate the problem of constructing such a BMS as the problem to find for a K_{2m} a splitting of the edges in $(2m-2)/2$ coloured isomorphic edge-disjunct 2-factors F_k and a perfect matching. Each F_k consists of vertex disjunct cycles C_{k1}, C_{k2},.., C_{ks} where $k_1 + k_2 + .. + k_s = 2m$. The problem is denoted as $KP(k_1,k_2,..,k_s)$. $2m$ is equal to the number of teams, k_i $(i=1,..s)$ are equal to the number of teams in group i. The perfect matching corresponds to the intermediate round.

It can be proved for a K_{2m} that if $(n-2)/2$ isomorphic edge-disjunct 2-factors exist, a perfect matching is left over. In Table 4.4 all known constructions are listed for K_n ($n < 16$), with n even.

Step 3

Step 3 can be performed in the same way as Step 3 for an odd number of teams.

Table 4.4. Overview of constructions for the KP problem

Number of teams	Group sizes	Construction possible?	Reference
6	3,3	no	[3]
8	4,4	yes	[7]
	3,5	unknown	
10	5,5	unknown	
	3,4,3	yes	[7]
12	6,6	unknown	
	5,7	unknown	
	4,8	unknown	
	3,9	unknown	
	4,4,4	yes	[7]
	3,4,5	unknown	
	3,3,6	unknown	
	3,3,3,3	no	[3]
14	all combinations	unknown	

5 Discussion and Conclusion

The presented three-step approach makes it possible to find Basic Match Schedules for the defined class of volleyball competition problems. These Basic Match Schedules are used successfully by the organizers of the Dutch volleyball competition. In our opinion, this approach is applicable to many sports timetabling problems.

In many sports competitions home and away patterns for the matches must be met and multiple matches between two teams have to be scheduled. In these cases, a directed graph model or a multi graph model can be explored as in Step 1. In Step 2 the graph properties have to be defined. In our case of the volleyball competition, the 2-factor properties are used. Comparable results are expected for football competitions [5]. In this way, sports competitions can be divided into classes each with its own graph properties and results. Using these classes in libraries will allow quick construction of Basic Match Schedules for new competitions. This will give timetable constructors more time to invest in the second phase of constructing sports timetables: Assigning individual teams to the Basic Match Schedules subjected to external wishes [6].

Future research will focus on the unknown constructions and on the analysis of other sports competitions.

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