

One Bit Spectrum Sensing in Cognitive Radio Sensor Networks

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Received: date / Accepted: date

Abstract This paper proposes a spectrum sensing algorithm from one bit measurements in a cognitive radio sensor network. A likelihood ratio test (LRT) for the one bit spectrum sensing problem is derived. Different from the one bit spectrum sensing research work in the literature, the signal is assumed to be a discrete random correlated Gaussian process, where the correlation is only available within immediate successive samples of the received signal. The employed model facilitates the design of a powerful detection criteria with measurable analytical performance. One bit spectrum sensing criterion is derived for one sensor which is then generalized to multiple sensors. Performance of the detector is analyzed by obtaining closed-form formulas for the probability of false alarm and the probability of detection. Simulation results corroborate the theoretical findings and confirm the efficacy of the proposed detector in the context of highly correlated signals and large number of sensors.

Keywords Cognitive radio · Spectrum sensing · One bit measurements · Detection · Sensor network

1 Introduction

Cognitive radio (CR) [1] is an emerging technology to improve the spectrum access in wireless sensor networks. It allows unlicensed or secondary users (SUs) to detect and access any available radio spectrum unused by licensed or primary users (PUs) without causing harmful interference to PUs. Hence,

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Spectrum sensing [2]– [11] is a vital component of a CR system to identify the state of the PUs in the network.

Spectrum sensing techniques can be categorized into narrowband spectrum sensing and wideband spectrum sensing techniques [12]. Narrowband spectrum sensing [13] investigates the problem of identifying whether a particular slice of the spectrum is idle or not. In contrast, wideband spectrum sensing [14] aims to classify individual slices of a wide frequency range, i.e., megahertz (MHz) to gigahertz (GHz) range, to be either vacant or occupied. Therefore, in majority of existing wideband spectrum sensing techniques, a simple approach is to acquire the wideband signal samples using a standard Analog to Digital converter (ADC) and then utilize appropriate signal processing techniques to identify spectral opportunities. In these techniques, however, the samples of the signal should follow Shannon's theorem: the sampling rate must be at least twice the maximum available frequency in the signal, i.e., Nyquist rate, to avoid spectral aliasing. Hence, employing these wideband spectrum sensing techniques results in long sensing delays or leads to higher computational complexities and hardware costs. As a result, these techniques are inappropriate for a cognitive wireless sensor network (CWSN) [15], [16] with simple and affordable sensors.

A number of techniques have been proposed in the literature to address the challenges, including multiband sensing (FFT-based sensing), wavelet-based sensing, and filter-bank sensing [12]. However, these approaches still suffer from the practical issues such as power consumption, feasibility of ultra high sampling ADCs, sensing time and complexity. To avoid the high sampling rate or high implementation complexity in Nyquist systems, sub-Nyquist approaches have gained more attention, in which the sampling rates lower than Nyquist rate is employed to detect spectral opportunities. One of these sub-Nyquist approaches is compressive sensing (CS) [20], [21] for detection of sparse signals [22] or spectrum sensing in cognitive radio framework [23]. However, there are some limitations on CS techniques. For instance, the sensing matrix should be properly selected to satisfy some constraints (e.g., nearly orthonormal matrices). Further, the spectrum reconstruction part of CS approach is challenging [12].

To simplify the implementation of high sampling ADCs, it is preferred to use low precision ADCs. The extreme case is to use one bit ADCs utilizing sign measurement by a simple comparator [17], [18], [19]. In [17], an ultra low power wideband spectrum sensing architecture is suggested by utilizing a one bit quantization at the cognitive radio receiver. In [18], the same authors used a window-based autocorrelation to provide the power spectral density of the quantized signal. Recently, the authors in [19] considered the problem of detecting the presence or absence of a random wireless source with minimum latency utilizing an array of radio one bit sensors.

1.1 Contribution

In this paper, a likelihood ratio test (LRT) detector is derived for detection of a random source with one bit measurements. Unlike the above-mentioned research work on one bit spectrum sensing, to reduce the complexity of the one-bit model likelihood, we employ a correlated Gaussian random process for the received signal model, where the correlation is only available within the immediate successive samples of the received signal. The employed model enables use to design a powerful detection criteria with measurable analytical performance.

The detector performance is investigated in single sensor and multiple sensors scenarios. Then, theoretical analysis of the detector is performed by calculating closed-form formulas for the probability of detection and probability of false alarm. Simulation results show efficacy of the LRT detector and agreement between experimental and theoretical results. The proposed one bit spectrum sensing detector provides competitive capabilities to save the hardware, power and computing resources by minimizing the ADC output resolution for a large number of sensors in multiple sensor scenarios.

The rest of the paper is organized as follows. Section 2 introduces the model, the LRT detector and the theoretical analysis for the single sensor case. In section 3, the same steps are performed in the case of multiple sensors. Simulation results are presented in Section 4. Finally, conclusions are drawn in Section 5.

2 One bit spectrum sensing: single sensor case

Consider a random signal s_i for $1 \leq i \leq n$ in which n is the total number of samples. One bit measurements of the single sensor is modeled as

$$\begin{aligned} H_0 : \quad & y_i = \text{sgn}(w_i), \\ H_1 : \quad & y_i = \text{sgn}(s_i + w_i), \quad i = 1, 2, \dots, n \end{aligned}$$

where w_i is Gaussian noise with zero mean and variance σ^2 , H_0 and H_1 are hypotheses of absence and presence of the signal, respectively, and $\text{sgn}(x)$ is the indicator function ($\text{sgn}(x) = 1$ for $x \geq 0$ and $\text{sgn}(x) = 0$ for $x < 0$). Signal is assumed to be a correlated Gaussian random process with a covariance matrix which is toeplitz and banded with bandwidth 3. This means that correlation is present only between immediate successive samples. This is the case when sampling rate is less than or equal to twice the symbol rate of a digitally modulated signal Hence, we have $\mathbb{E}(s_i^2) = \sigma_s^2$ and $\mathbb{E}(s_i s_{i+1}) = r$ while $\mathbb{E}(s_i s_{i+k}) = 0$ for $|k| > 1$. The problem is to decide the true hypothesis (absence or presence of the signal) from one bit measurements $\mathbf{y} = [y_1, y_2, \dots, y_n]^T$.

The Neyman-Pearson LRT detector is defined as [24]:

$$\Lambda_{\text{LR}} = \frac{\mathbb{P}(\mathbf{y}|H_0)}{\mathbb{P}(\mathbf{y}|H_1)} \underset{H_1}{\overset{H_0}{\gtrless}} \lambda \quad (1)$$

where $\mathbb{P}(\cdot)$ is the probability mass function or probability depending on the context and λ is the detector's threshold. The likelihood under hypothesis H_0 is equal to $\mathbb{P}(\mathbf{y}|H_0) = (\frac{1}{2})^n$. The likelihood $\mathbb{P}(\mathbf{y}|H_1)$ is equal to

$$\mathbb{P}(y_1|H_1)\mathbb{P}(y_2|y_1, H_1)\mathbb{P}(y_3|y_2, H_1)\dots\mathbb{P}(y_n|y_{n-1}, H_1)$$

since we have only one sample dependence between measurements. Also, we have $\mathbb{P}(y_1|H_1) = \frac{1}{2}$. In Appendix A, the probability $\mathbb{P}(y_2|y_1, H_1)$ is calculated to be

$$\mathbb{P}(y_2|y_1, H_1) = p^{\mathbb{I}(y_1=y_2)}(1-p)^{\mathbb{I}(y_1 \neq y_2)} \quad (2)$$

where

$$\mathbb{I}(y_1 = y_2) = \begin{cases} 1 & y_1 = y_2, \\ 0 & \text{else} \end{cases} \quad (3)$$

and

$$p := \mathbb{P}(y_2 = 1|y_1 = 1, H_1) \quad (4)$$

A similar approach shows that $\mathbb{P}(y_{k+1}|y_k, H_1) = p^{\mathbb{I}(y_k=y_{k+1})}(1-p)^{\mathbb{I}(y_k \neq y_{k+1})}$ for $2 \leq k \leq n-1$. Replacing these conditional probabilities in logarithm of (1) followed by straightforward calculations lead to the final detection criterion:

$$\sum_{i=1}^{n-1} \mathbb{I}(y_i = y_{i+1}) \underset{H_0}{\overset{H_1}{\geq}} \eta \quad (5)$$

In the derivation, it is assumed that $\ln \frac{p}{1-p} > 0$ which is equivalent to $p > \frac{1}{2}$ or $r > 0$. For the case of $p < \frac{1}{2}$ or equivalently $r < 0$, the detection criterion has the reverse direction. For the case of $r = 0$ in which the source samples like the noise samples are uncorrelated Gaussian random variables, the energy detector is the sole choice [24], [25]. Although, one bit measurements have no amplitude information, so there is no information to decide between the presence and absence of the signal.

In the following, detection probability and false alarm probability are calculated for the LRT detector of (5) in the case of $p > \frac{1}{2}$. Detection statistic is defined as $Y = \sum_{i=1}^{n-1} \mathbb{I}(y_i = y_{i+1})$, The decision is

$$\hat{d} = \begin{cases} 1 & Y \geq \eta \\ 0 & Y < \eta. \end{cases} \quad (6)$$

Hence, the false alarm probability $P_{\text{fa}} = \mathbb{P}(\hat{d} = 1|H_0) = \mathbb{P}(Y > \eta|H_0)$ is equal to

$$P_{\text{fa}} = \mathbb{P}\left(\sum_{i=1}^{n-1} \mathbb{I}(y_i = y_{i+1}) \geq \eta | H_0\right) \approx Q\left(\frac{\eta - \mu_0}{\sigma_0}\right) \quad (7)$$

where $Q(\cdot)$ is the Q-function, $\mu_0 = E(Y|H_0)$, σ_0^2 is the variance of Y subject to hypothesis H_0 and it is assumed that the detection statistic $Y = \sum_{i=1}^{n-1} \mathbb{I}(y_i =$

y_{i+1}) is Gaussian due to the Central Limit Theorem (CLT). In Appendix B, μ_0 and σ_0^2 are calculated to be:

$$\begin{aligned}\mu_0 &= \frac{1}{2}(n-1) \\ \sigma_0^2 &= \frac{1}{4}(n-1)\end{aligned}\quad (8)$$

The detection probability $P_d = \mathbb{P}(\hat{d} = 1|H_1) = \mathbb{P}(Y \geq \eta | H_1)$, we have:

$$P_d = \mathbb{P}\left(\sum_{i=1}^{n-1} \mathbb{I}(y_i = y_{i+1}) \geq \eta | H_1\right) \approx Q\left(\frac{\eta - \mu_1}{\sigma_1}\right) \quad (9)$$

where $\mu_1 = \mathbb{E}(Y|H_1)$, σ_1^2 is the variance of Y subject to hypothesis H_1 . In Appendix C, μ_1 and σ_1^2 are calculated as:

$$\mu_1 = 2p(n-1) \quad (10)$$

$$\sigma_1^2 = 2p(1-2p)(n-1). \quad (11)$$

3 One bit spectrum sensing: sensor network case

Consider a sensor network with N nodes. Each sensor performs a one bit measurement as

$$\begin{aligned}H_0 : \quad & y_{ki} = \text{sgn}(w_{ki}), \\ H_1 : \quad & y_{ki} = \text{sgn}(s_i + w_{ki})\end{aligned}$$

where $1 \leq i \leq n$ is the time index, $1 \leq k \leq N$ is the sensor index, N is the total number of sensors, w_{ki} is the Gaussian noise of k 'th sensor, and s_i is the signal sample with the same model as assumed in section 2.

The LRT detector will be [24]:

$$\Lambda_{\text{LR}} = \frac{\mathbb{P}(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n | H_0)}{\mathbb{P}(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n | H_1)} \underset{H_1}{\overset{H_0}{\gtrless}} \lambda \quad (12)$$

where $\mathbf{x}_i = [y_{1i} \ y_{2i} \ \dots \ y_{Ni}]^T$ is the measurements of all sensors at i 'th time instant. We will have $\mathbb{P}(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n | H_0) = (\frac{1}{2})^{nN}$. Also, $\mathbb{P}(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n | H_1)$ is equal to:

$$\mathbb{P}(\mathbf{x}_1 | H_1) \mathbb{P}(\mathbf{x}_2 | \mathbf{x}_1, H_1) \mathbb{P}(\mathbf{x}_3 | \mathbf{x}_2, H_1) \dots \mathbb{P}(\mathbf{x}_n | \mathbf{x}_{n-1}, H_1) \quad (13)$$

where $\mathbb{P}(\mathbf{x}_1 | H_1) = (\frac{1}{2})^N$. Also $\mathbb{P}(\mathbf{x}_2 | \mathbf{x}_1, H_1)$ is equal to $\prod_{k=1}^N \mathbb{P}(y_{k2} | y_{k1}, H_1)$ where $\mathbb{P}(y_{k2} | y_{k1}, H_1) = p^{\mathbb{I}(y_{k2}=y_{k1})} (1-p)^{\mathbb{I}(y_{k2} \neq y_{k1})}$. We will have

$$\mathbb{P}(\mathbf{x}_2 | \mathbf{x}_1, H_1) = (1-p)^N \left(\frac{p}{1-p}\right)^{\sum_k \mathbb{I}(y_{k2}=y_{k1})} \quad (14)$$

Similar calculations lead to $\mathbb{P}(\mathbf{x}_{i+1}|\mathbf{x}_i, H_1) = (1-p)^N \left(\frac{p}{1-p}\right)^{\sum_k \mathbb{I}(y_{k,i+1}=y_{ki})}$. Replacing these conditional probabilities into (13) and (12), leads to the following criterion for $p > \frac{1}{2}$:

$$Y := \sum_{i=1}^{n-1} \sum_{k=1}^N \mathbb{I}(y_{k,i+1} = y_{ki}) \underset{H_0}{\overset{H_1}{\gtrless}} \eta \quad (15)$$

which is a direct generalization of detection criterion in single sensor case in (5). For the case of $p < \frac{1}{2}$, the direction of the detection criterion in (15) is reversed. The case of $p = \frac{1}{2}$ is the same as that in the single sensor case where there is no information to detect the presence of the signal.

In the next step, detection probability and false alarm probability are calculated for the LRT detector of (15) when $p > \frac{1}{2}$. False alarm probability $P_{\text{fa}} = \mathbb{P}(\hat{d} = 1|H_0) = \mathbb{P}(Y > \eta|H_0)$ is equal to

$$P_{\text{fa}} = Q\left(\frac{\eta - m_0}{s_0}\right) \quad (16)$$

where $m_0 = \mathbb{E}(Y|H_0)$ and s_0^2 is the variance of Y subject to hypothesis H_0 and it is assumed that the detection statistic Y is Gaussian due to the CLT. In Appendix D, m_0 and s_0^2 are calculated to be:

$$\begin{aligned} m_0 &= \frac{1}{2}(n-1)N \\ s_0^2 &= \frac{1}{4}(n-1)N \end{aligned} \quad (17)$$

The detection probability $P_d = \mathbb{P}(\hat{d} = 1|H_1) = \mathbb{P}(Y \geq \eta|H_1)$. we have:

$$P_d = Q\left(\frac{\eta - m_1}{s_1}\right) \quad (18)$$

where $m_1 = \mathbb{E}(Y|H_1)$ and s_1^2 is the variance of Y subject to hypothesis H_1 . In Appendix E, m_1 and s_1^2 are calculated as

$$m_1 = 2p(n-1)N \quad (19)$$

$$s_1^2 = 2p(1-2p)(n-1)N \quad (20)$$

4 Simulation Results

This section presents the simulation results. Correlated random signal is generated as described in section 2, with parameters r and $\sigma_s = 1$. Noise is generated as a Gaussian uncorrelated random process with zero mean and variance $\sigma = 10^{-2}$. In all simulations, number of time samples are assumed to be $n = 20$. For comparison of the detectors, the detection probability versus false alarm probability known as Receiver Operating Characteristic (ROC) is depicted. For the monte carlo simulation, the experiments are repeated 20000

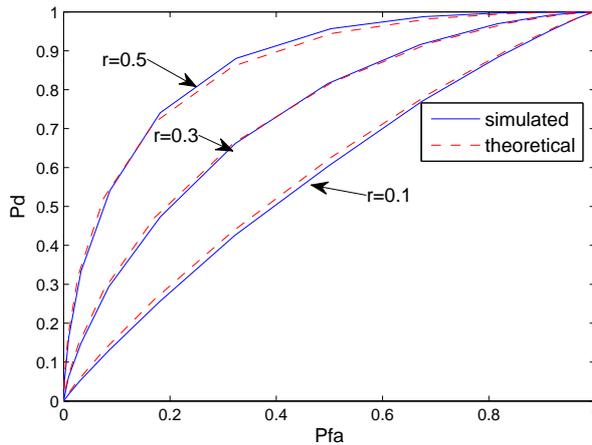


Fig. 1 ROC curve of the LRT detector for single sensor case.

times and detection probability and false alarm probability are averaged over all the trials. Moreover, to verify the theoretical analysis, we compared the experimental results with the theoretical results. Two experiments are performed for single sensor and multiple sensor cases.

In the first experiment, a single sensor is used for spectrum sensing. Four signals are examined with correlation coefficients $r = 0.1, 0.3, \text{ and } 0.5$. A good agreement between experimental and theoretical ROC curves are shown in Fig 1. Also, it shows that by increasing the correlation coefficient, the performance of the detector improves.

In the second experiment, we utilize a cognitive sensor network with 1, 2, and 3 sensors. The correlation coefficient of signal is $r = 0.5$. ROC curves are sketched in Fig. 2. It shows that by increasing the number of sensors, the detector performance improves. It also demonstrates a good agreement between theoretical and experimental ROC curves.

5 Conclusion

In this paper, we have derived the LRT detector for one bit spectrum sensing problem in single sensor and multiple sensor cases for a correlated Gaussian signal model. The detectors utilize correlation available within successive samples of the received signal to obtain the detection criteria. Closed-form detection and false alarm probabilities are derived in single and multiple sensor scenarios. Simulation results show the efficacy of the detector specially when the correlation coefficient is large or the number of sensors increases. Moreover, the simulations results corroborate the theoretical analysis. The proposed one bit spectrum sensing detector provides competitive capabilities to save the

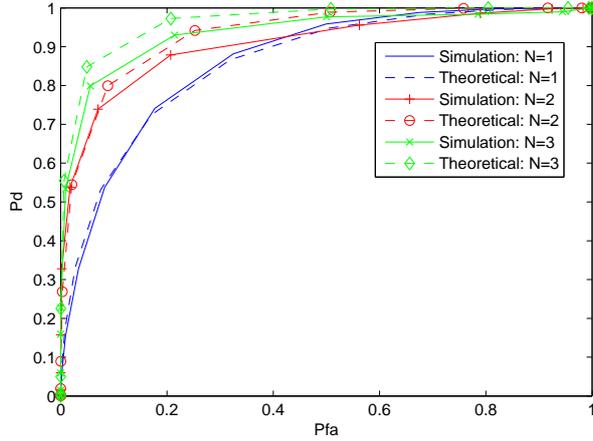


Fig. 2 ROC curve of the LRT detector for multiple sensor case.

hardware, power and computing resources by minimizing the ADC output resolution for a large number of sensors in multiple sensor scenarios.

Appendix A Calculating $\mathbb{P}(y_2|y_1, H_1)$

We first calculate the four probabilities $\mathbb{P}(y_2 = 1|y_1 = 1, H_1) =: p$, $\mathbb{P}(y_2 = 0|y_1 = 1, H_1) = 1 - p$, $\mathbb{P}(y_2 = 1|y_1 = 0, H_1) = p'$, and $\mathbb{P}(y_2 = 0|y_1 = 0, H_1) = 1 - p'$. The probability $p = \mathbb{P}(y_2 = 1|y_1 = 1, H_1)$ is equal to

$$p = \mathbb{P}(w_2 + s_2 \geq 0 | w_1 + s_1 \geq 0) =$$

$$\frac{\mathbb{P}(z_1 \geq 0, z_2 \geq 0)}{\mathbb{P}(z_1 \geq 0)} = 2 \mathbb{P}(z_1 \geq 0, z_2 \geq 0) \quad (21)$$

where $z_1 = s_1 + w_1$, $z_2 = s_2 + w_2$ and $p(z_1 \geq 0) = \frac{1}{2}$. To calculate $p(z_1 \geq 0, z_2 \geq 0)$, note that z_1 and z_2 are correlated Gaussian random variables with covariance matrix \mathbf{C} with elements $C_{11} = E(z_1^2) = \sigma_s^2 + \sigma^2$, $C_{12} = C_{21} = E(z_1 z_2) = r$ and $C_{22} = E(z_2^2) = \sigma_s^2 + \sigma^2$. Therefore, joint probability density function (pdf) is $f(z_1, z_2) = \frac{1}{2\pi\sqrt{\det(\mathbf{C})}} \exp(-\frac{1}{2}\mathbf{z}\mathbf{C}^{-1}\mathbf{z}^T)$ where $\mathbf{z} = [z_1 \ z_2]$. Hence, we have $p = 2 \int_0^{+\infty} \int_0^{+\infty} f(z_1, z_2) dz_1 dz_2$. To calculate the other probability p' , we consider that $p' = \mathbb{P}(y_2 = 1|y_1 = 0, H_1) = \frac{\mathbb{P}(y_2=1, y_1=0|H_1)}{\mathbb{P}(y_1=0|H_1)} = 2 \mathbb{P}(y_2 = 1, y_1 = 0|H_1) = 2(\frac{1}{2} - \frac{p}{2}) = 1 - p$ which leads to (2).

Appendix B Calculating μ_0 and σ_0^2

We have $\mu_0 = \sum_{i=1}^{n-1} \mathbb{E} \mathbb{I}(y_i = y_{i+1} | H_0) = \frac{1}{2}(n-1)$. Also, we have $\sigma_0^2 = \mathbb{E}(Y^2 | H_0) - \mathbb{E}^2(Y | H_0)$ in which $\mathbb{E}(Y | H_0) = \frac{1}{2}(n-1)$ and

$$\mathbb{E}(Y^2 | H_0) = \sum_{i,i'} \mathbb{E}(\mathbb{I}(y_i = y_{i+1}) \mathbb{I}(y_{i'} = y_{i'+1}) | H_0) \quad (22)$$

where the expectation is equal to

$$\mathbb{P}((y_i = y_{i+1}) \wedge (y_{i'} = y_{i'+1}) | H_0) = \begin{cases} \frac{1}{2} & : i = i' \\ \frac{1}{4} & : i \neq i' \end{cases}$$

Replacing (B) into (22) results in (8).

Appendix C Calculating μ_1 and σ_1^2

We have $\mu_1 = \sum_{i=1}^{n-1} \mathbb{E} \mathbb{I}(y_i = y_{i+1} | H_1) = \sum_{i=1}^{n-1} \mathbb{P}(y_i = y_{i+1} | H_1) = 2p(n-1)$. Also, we have $\sigma_1^2 = \mathbb{E}(Y^2 | H_1) - \mathbb{E}^2(Y | H_1)$ in which $\mathbb{E}(Y | H_1) = 2p(n-1)$ and

$$\mathbb{E}(Y^2 | H_1) = \sum_{i,i'} \mathbb{E}(\mathbb{I}(y_i = y_{i+1}) \mathbb{I}(y_{i'} = y_{i'+1}) | H_1) \quad (23)$$

where the expectation is equal to

$$\mathbb{P}((y_i = y_{i+1}) \wedge (y_{i'} = y_{i'+1}) | H_1) = \begin{cases} \mathbb{P}(y_i = y_{i+1} | H_1) = 2p & : i = i' \\ \mathbb{P}(y_i = y_{i+1} | H_1) \mathbb{P}(y_{i'} = y_{i'+1} | H_1) = 4p^2 & : i \neq i' \end{cases} \quad (24)$$

Replacing (24) into (23) results in (11).

Appendix D Calculating m_0 and s_0^2

We have $m_0 = \sum_{i=1}^{n-1} \sum_{k=1}^N \mathbb{E} \mathbb{I}(y_{ki} = y_{k,i+1} | H_0) = \frac{1}{2}(n-1)N$. Also, we have $s_0^2 = \mathbb{E}(Y^2 | H_0) - \mathbb{E}^2(Y | H_0)$ in which $\mathbb{E}(Y | H_0) = \frac{1}{2}(n-1)N$ and

$$\mathbb{E}(Y^2 | H_0) = \sum_{i,k,i',k'} \mathbb{E}(\mathbb{I}(y_{ki} = y_{k,i+1}) \mathbb{I}(y_{k'i'} = y_{k',i'+1}) | H_0) \quad (25)$$

where the expectation is equal to

$$\mathbb{P}((y_{ki} = y_{k,i+1}) \wedge (y_{k'i'} = y_{k',i'+1}) | H_0) = \begin{cases} \frac{1}{2} & : i = i' \wedge k = k' \\ \frac{1}{4} & : i \neq i' \vee k \neq k' \end{cases} \quad (26)$$

Replacing (26) into (25) results in (17).

Appendix E Calculating m_1 and s_1^2

We have $m_1 = \sum_{i=1}^{n-1} \sum_{k=1}^N \mathbb{E} \mathbb{I}(y_{ki} = y_{k,i+1} | H_1) = \sum_{i,k} \mathbb{P}(y_{ki} = y_{k,i+1} | H_1) = 2p(n-1)N$. Also, we have $s_1^2 = \mathbb{E}(Y^2 | H_1) - \mathbb{E}^2(Y | H_1)$ in which $\mathbb{E}(Y | H_1) = 2p(n-1)N$ and

$$\mathbb{E}(Y^2 | H_1) = \sum_{i,k,i',k'} \mathbb{E}(\mathbb{I}(y_{ki} = y_{k,i+1}) \mathbb{I}(y_{k'i'} = y_{k',i'+1}) | H_1) \quad (27)$$

where the expectation is equal to

$$\begin{aligned} & \mathbb{P}((y_{ki} = y_{k,i+1}) \wedge (y_{k'i'} = y_{k',i'+1}) | H_1) = \\ & \mathbb{I}(i = i' \wedge k = k') \mathbb{P}(y_{ki} = y_{k,i+1} | H_1) + \\ & \mathbb{I}(i \neq i' \vee k \neq k') \mathbb{P}(y_{ki} = y_{k,i+1} | H_1) \mathbb{P}(y_{k'i'} = y_{k',i'+1} | H_1) \end{aligned}$$

which is

$$\begin{aligned} & \mathbb{P}((y_{ki} = y_{k,i+1}) \wedge (y_{k'i'} = y_{k',i'+1}) | H_1) \quad (28) \\ & = \begin{cases} 2p & : i = i' \wedge k = k' \\ 4p^2 & : i \neq i' \vee k \neq k' \end{cases} \end{aligned}$$

Replacing (28) into (27) results in (20).

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