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Observer Design for Nonlinear Descriptor Systems: A Survey on System Nonlinearities

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Abstract

In general, the construction of observers for nonlinear descriptor systems depends on the solvability of a linear matrix inequality involving system matrices, and it is based on the system's nonlinearity. Therefore, the type of nonlinearity present in the system heavily affects the observer design process. There are significant developments in the literature for observer design for descriptor systems with various types of nonlinearity. Motivated by this, the current work reviews the literature on observer design for nonlinear descriptor systems with an extensive discussion on the type of nonlinearities that are considered. Here, an analysis and the comparison on the most common nonlinearities is presented, providing a roadmap to all researchers in the field. Furthermore, less common nonlinearities have been identified, presenting under-explored areas within the literature, and can open new domains for future research.

Keywords Descriptor system · Nonlinear system · Observer · Singular system · Differential algebraic equations · Lipschitz · Quadratic constraints · Chaos synchronization · Signal estimation

1 Introduction

The problem of observer design is well known in control systems engineering and system analysis. It refers to designing a mathematical model of a secondary system (the observer) that outputs an estimate of the internal states of the original (master) system, using only its input and output measurements. The convergence of the estimated states to the real values depends upon the solvability of a linear matrix inequality (LMI). This LMI is generally formed through Lyapunov stability theory, by considering a candidate

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Lyapunov quadratic function of the error dynamics between the states of master system and the observer. A simplified outline of the observer design problem is shown in Fig. 1. The output of the master system is a linear and sometimes a nonlinear combination of its states and input, and it is fed into the observer. The system's estimated output $\hat{y}(t)$ from the observer may also be fed back into the observer system, as it can be used to design an appropriate feedback law for the estimation to converge.

Descriptor systems are also named as generalized state space systems, singular systems, or differential-algebraic equations (DAEs), which are described by implicit combinations of differential and algebraic equations. These systems arise in various engineering disciplines, including electrical circuits, mechanical systems, and chemical processes, where the presence of algebraic constraints or singularities is common [9, 17, 27]. Through observer design for descriptor systems, along with state estimation, problems like unknown input estimation, secure communications, fault diagnosis, sensor fusion, state feedback control, model-based control, and system identification are also addressed. It allows the estimation of unmeasured states, compensating for sensor limitations, and improving system performance and robustness [4, 23, 33]. For chaotic systems, the concept of observer design is the problem of having a secondary system following the trajectories of a master system is commonly termed synchronization [48], which is also a kind of observer design problem [4, 46].

When it comes to observer design for nonlinear systems, LMIs can be used to formulate observer design problems as convex optimization problems that can be solved efficiently. The formation of the resulting LMI depends on the system's nonlinearity. Thus, while studying the observer's convergence, it is important to specify the master system's nonlinearity and find an appropriate bound (upper or lower) for it. So far, many types of nonlinearities are being considered, each described by a different property.

The goal of this article is to compile most of the existing studies for nonlinear descriptor systems, categorized by the nonlinearity present in the system. The motivation for this is to serve as guidance for all researchers working on observer design problems for descriptor systems to navigate the existing nonlinearities already considered. Another goal is to identify nonlinearities that have not yet been addressed for descriptor systems, particularly. Our review identifies some less studied classes of nonlinearities, which can be fruitful grounds for future studies.

In the following, $\mathbb{R}^{m\times n}$ represents the $m \times n$ real matrix set. $A > 0$ ($A \geq 0$) is a positive definite (semi-definite) matrix. Transpose of a matrix X is denoted by X^T. I and 0 denote the identity and zero matrix of appropriate dimensions, respectively. $||x||$ denotes Euclidean norm of a vector x.

The rest of the work is structured as follows. Section 2 describes the underlying system structure and formulate general problems on it. Discussion on various types of nonlinearities and their comparison is done in Sect. 3. In Sect. 4, analysis of some less studied nonlinearities is carried out. Section 5 concludes the article along with some remarks on possible future works.

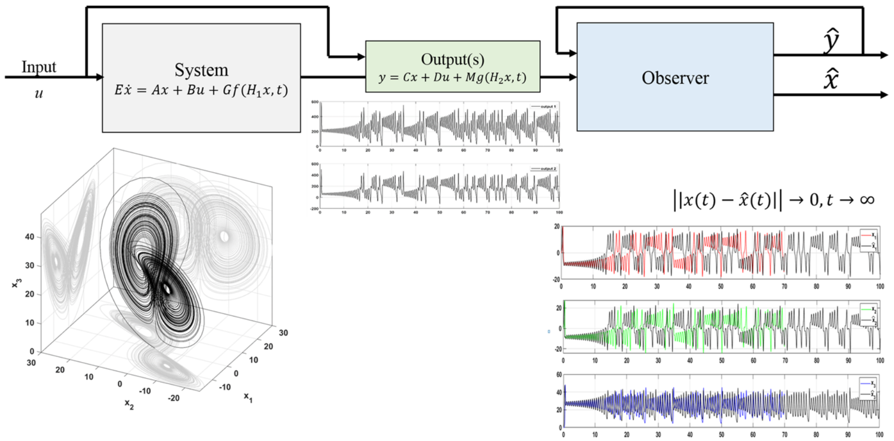

Fig. 1 A simplified outline of the observer design problem. The Lorenz chaotic system [37] is used as an example, and the observer feedback from the master system is activated at $t = 70$ seconds

2 Preliminaries

This section is devoted to the mathematical representation of the fundamental system and the current problem.

2.1 System Formulation

The general nonlinear descriptor system is described by

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t) + Gf(H_1x(t), t) \\ y(t) &= Cx(t) + Du(t) + Mg(H_2x(t), t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state of the system, $u(t) \in \mathbb{R}^q$ the input, and $y(t) \in \mathbb{R}^p$ the output. $E, A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{m\times q}$, $G \in \mathbb{R}^{m\times n_f}$, $H_1 \in \mathbb{R}^{h_1\times n}$, $C \in \mathbb{R}^{p\times n}$, $D \in \mathbb{R}^{p\times q}$, $M \in \mathbb{R}^{p\times n_g}$, $H_2 \in \mathbb{R}^{h_2\times n}$ are the system matrices of appropriate dimensions. $f(H_1x(t), t) \in \mathbb{R}^{n_f}$ is the dynamic nonlinearity and $g(H_2x(t), t) \in \mathbb{R}^{n_g}$ is the output nonlinearity of the system. If E is an invertible matrix, then (1) is well known as a state space system. In the rest of the paper, the explicit dependency of the state, input, and output vectors on time t is omitted for better readability.

In most works, the output is considered to be linear, but the general case of nonlinearity is assumed here. Note that various literature works consider nonlinearities under slightly different formulations, for example, either as exclusively state dependent $f(x)$, or as state and input dependent $f(x, u)$, state, input and output dependent $f(x, u, y)$ or as state, input, and time dependent $f(x, u, t)$. Here, to represent all of the above forms, we take $f(H_1x, t)$ as it includes all the aforementioned cases. Note that the dependence on time encompasses the dependence on the input $u(t)$ which is a function of time. The same holds for time dependent output $y(t)$. Moreover, the general rectangular case is assumed, where the system may be over- or under-determined.

2.2 Problem Formulation: States and Unknown Input Estimation

The observer design problem involves the estimation of the state $x(t)$ asymptotically especially when initial condition of the system (1) is not known in advance, i.e.,

$$||x(t) - \hat{x}(t)|| \to 0, \text{ as } t \to \infty, \ \forall\, x(0), \hat{x}(0), \tag{2}$$

where $\hat{x}(t)$ is the estimation of states.

Rectangular descriptor systems are especially useful for the problem of external input estimation and secure communications. In this problem, a descriptor system, or sometimes even simpler, a state space system is affected by an unknown input

$$\begin{aligned} \dot{x} &= Ax + Bu + Qs + Gf(H_1 x, t) \\ y &= Cx + Du + Rs + Mg(H_2 x, t) \end{aligned} \tag{3}$$

where $s(t) \in \mathbb{R}^{m_s}$ is an unknown input (UI), with $Q \in \mathbb{R}^{n \times m_s}$, $R \in \mathbb{R}^{p \times m_s}$. In secure communications, s is considered as a signal to be transmitted. The observer's goal is to estimate the UI along with the state x. This can be achieved by considering the external input s as an internal state of the system, leading to an augmented state vector $\mathbf{x} = \left(x^T \ s^T\right)^T$. The system (3) is now written in the descriptor form as

$$\begin{aligned} \mathbf{E}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + Bu + Gf(\mathbf{H_1}\mathbf{x}, t) \\ y \ &= \mathbf{C}\mathbf{x} + Du + Mg(\mathbf{H_2}\mathbf{x}, t) \end{aligned} \tag{4}$$

where

$$\mathbf{E} = \left(I \ 0\right), \mathbf{A} = \left(A \ Q\right), \mathbf{C} = \left(C \ R\right), \mathbf{H_1} = \left(H_1 \ 0\right), \mathbf{H_2} = \left(H_2 \ 0\right)$$

Sometimes s is also inserted in the nonlinear function. So, by designing an observer for (4), the state and UI can be estimated simultaneously. The above design is often used for applications in secure communications. In such cases, the signal s is not an undesired disturbance, but an information signal that is masked through a chaotic system (3), transmitted through the output, and then reconstructed at the observer end. The approach of converting system (3) into (4) is appropriate only when estimation of states and unknown inputs has to be done simultaneously. Gupta et al. [22, 23] show the disadvantage of the augmentation technique on algebraic constraints. In cases where only states are to be estimated, annihilation of unknown inputs should be done for milder algebraic constraints.

3 System Nonlinearities

In this section, we discuss some of the different nonlinearities that have been explored for observer designs for descriptor systems.

3.1 Lipschitz Nonlinearities

The most common case, the Lipschitz nonlinearity for a function $f(x,t)$ is described by

$$||f(x_1,t) - f(x_2,t)|| \leq \lambda_f ||(x_1 - x_2)|| \;\; \forall \;\; x_1, x_2 \tag{5}$$

where $\lambda_f > 0$ is a positive scalar called the Lipschitz constant. Notice that the Lipschitz property is generally assumed to hold with respect to the first argument of f, which is the state x. The Lipschitz constant λ_f can be computed either by direct computation of the error bound when possible, or numerically. The constant can be numerically computed as the supremum of the norm of the Jacobian of the function over the region of interest $\mathcal{D}$ [42, 49]

$$\lambda_f = \limsup \left|\left|\frac{\mathrm{d}f}{\mathrm{d}x}\right|\right| \forall \, x \in \mathcal{D} \tag{6}$$

where $\frac{\mathrm{d}f}{\mathrm{d}x}$ is the Jacobian of f and $||\cdot||$ is the 2-norm.

When considering the convergence of the error dynamics $e = x - \hat{x}$, the resulting dynamics of the candidate Lyapunov function are usually formed in a quadratic formula involving e and $\Delta f = f(x,t) - f(\hat{x},t)$, with the following general form

$$\begin{pmatrix} e^T & \Delta f^T \end{pmatrix} \mathcal{M} \begin{pmatrix} e \\ \Delta f \end{pmatrix}. \tag{7}$$

To ensure the negative definiteness of (7), $\mathcal{M} < 0$ must be satisfied to guarantee convergence. Here $\mathcal{M}$ is a block matrix that generally involves the system matrices along with some other unknown matrices in its blocks. As a result, $\mathcal{M} < 0$ is resulted in the form of an LMI. $\mathcal{M}$ also takes contribution from the nonlinearity f using (5) as follows

$$\begin{aligned} &||f(x,t) - f(\hat{x},t)|| \leq \lambda_f ||(x - \hat{x})|| \\ &\quad \Rightarrow \Delta f^T \Delta f \leq \lambda_f^2 e^T e \\ &\quad\quad \Rightarrow 0 \leq \begin{pmatrix} e^T & \Delta f^T \end{pmatrix} \begin{pmatrix} \lambda_f^2 I & 0 \\ 0 & -I \end{pmatrix} \begin{pmatrix} e \\ \Delta f \end{pmatrix}. \end{aligned} \tag{8}$$

So, the contributing term in $\mathcal{M}$ is the matrix

$$\begin{pmatrix} \lambda_f^2 I & 0 \\ 0 & -I \end{pmatrix} \tag{9}$$

In some works, this matrix is scaled by an additional parameter $\epsilon > 0$ to improve the solvability of the LMI as

$$\begin{pmatrix} \epsilon\lambda_f^2 I & 0 \\ 0 & -\epsilon I \end{pmatrix}, \tag{10}$$

becomes the contributing term. Moreover, in some works, the nonlinearity is considered to be acting on a linear combination of the states and is written as $f(Hx, t)$, as in (1). This way, considering (5), the resulting matrix is

$$\begin{pmatrix} \epsilon\lambda_f^2 H^T H & 0 \\ 0 & -\epsilon I \end{pmatrix}. \tag{11}$$

In some earlier works [4, 14, 66, 67], the problem of secure communications was considered on state space systems. Input information signal was considered as a system state that augmented the system state as described in Sect. 2.2. This converts the system into a rectangular descriptor form, and then an observer is designed. The nonlinearity $f(x, s)$ is assumed to satisfy Lipschitz constraint with respect to $x(t)$ and $s(t)$, where $s(t)$ is the information signal. The feasibility of the observer is formulated under the solvability of an LMI and a set of algebraic constraints on the system matrices. Boutayeb et al. [4] use an important assumption, also present in other works, that the information signal is also transmitted through the output, that is $R \neq 0$ in system (3). The results are illustrated using the Lorenz system [37] to transmit a sinusoidal signal. For this, two cases are considered. In the first, the information is linearly injected in the system. In the second case, the information is also injected through the nonlinearity $f(x, s, y)$, which depends on the state, information signal, and output, and is assumed to be Lipschitz with respect to its first two arguments. In [14], the design is more complex, as the transmission system consists of two chaotic systems, one is used for signal encryption, and another for the synchronization process. The observer is adaptive, where the adaptive term is used to compensate for the unknown Lipschitz constant of the nonlinear term. In the numerical examples, the Rössler [58] and Lorenz systems are first considered to encrypt and transmit two sinusoidal signals simultaneously. In a second example, a modified Chua circuit [75] is taken to encrypt and transmit a binary image. In [66], the augmentation is done by adding a duplicate of the system of differential equations to make the resulting system square.

Researchers [13, 24] use the augmentation technique for unknown inputs estimation on state space systems by converting them into a rectangular descriptor system. Ha and Trinh [24] decomposed the system nonlinearity $f((x, u), y)$ into two parts, with the first being Lipschitz, and the second being unknown. The observer is designed in order to nullify the effect of the unknown nonlinearity and estimate the state and UI. In the numerical example, a 4D system is considered, with an unknown sinusoidal input acting on it. The unknown nonlinearity is a product of the two states injected in the second differential equation. Rössler system was used, and two unknown inputs were estimated successfully [13].

In [32], full-order observer was designed for descriptor systems under UIs. Two approaches are considered, based on the available information on the UIs. A proportional observer is designed for the case where the spectral domain of the unknown input is unknown, and a proportional integral (PI) observer is designed when the spectral domain of the UI is in the low-frequency range. In the second case, the addition of the integral part is possible under the assumption that the UI is a piecewise constant.

Square descriptor systems were considered to design full and reduced order observers [38]. In this article, full-order observers are given in the descriptor form.

These observers were further converted to reduced order observers in the state space form. In the numerical example, a one-dimensional observer is designed for a two-dimensional system. Gupta et al. [21] designed an observer for square descriptor systems. The authors show that the detectability of the linear part is necessary for the solvability of the LMI that is required for observer convergence. Using systems with sinusoidal and cosinusoidal nonlinearities, two numerical examples are then presented. The authors extended these results for rectangular descriptor systems with UIs in [22]. The UIs were removed from the output first and then a reduced observer is appropriately designed to nullify their effect from the dynamic equation. An application to secure communications was described on Lorenz system, with a sinusoidal information signal that is estimated, and an unknown input sinusoidal signal acting on the system. Lorenz system is used for secure communications and the nonlinearity is multiplied by a full column rank matrix converting it to $Rf(x)$, which helps simplifying the error dynamics [7]. The reference [22] also discusses the benefits and drawbacks of augmentation technique presented in Sect. 2.2 while considering UIs.

Darouach et al. [10, 11] minimize algebraic constraints and provide maximally reduced order observers for rectangular descriptor systems with unknown inputs and external disturbances. In [2], descriptor observers were designed through the behavioral approach. A recent work [44] considers output nonlinearities in rectangular descriptor systems. Full and reduced order observers were developed, under the condition that the derivative of the output nonlinearity also satisfies a Lipschitz constraint. The application to secure communications was considered, using a 4D system for the transmission of a QR code image. Anti-synchronization was considered along with parameter estimation, for systems with coexisting attractors [45].

3.2 One-Sided Lipschitz and Quadratically Inner-Bounded Nonlinearities

In contrast to classical Lipschitz systems, the one-sided Lipschitz (OSL) nonlinear systems are more general [64, 79]. Lipschitz functions lead to an inequality in a simple quadratic form, while the one-sided Lipschitz functions lead to a weighted bilinear form which creates important challenges in constructing the Lyapunov derivative negative-definite function [59]. One-sided Lipschitz nonlinearity was first considered in 80 s and 90 s [12, 61, 68]. For observer design-related problem, Hu [28] considered OSL condition for the first time with the fact that when the Lipschitz constant becomes large, more results fail to provide a solution. Thus, in order to overcome this drawback, the Lipschitz continuity has been generalized to one-sided Lipschitz continuity. The region of one-sided Lipschitz constant is much larger than that of Lipschitz constant, because the one-sided Lipschitz constant can be zero or even negative, while the Lipschitz constant must be positive. This clarifies the advantages of the one-sided Lipschitz over the classical Lipschitz condition.

The one-sided Lipschitz nonlinearity is described by

$$(x_1 - x_2)^T (f(x_1, t) - f(x_2, t)) \leq \rho ||x_1 - x_2||^2 \; \forall \, x_1, x_2 \tag{12}$$

where $\rho \in \mathbb{R}$ is called the one-sided Lipschitz constant. Note that in contrast to the Lipschitz constant, ρ is not required to be positive. The constant ρ can be specified as [1]

$$\rho = \lim\sup \left[\mu \left(\frac{\partial f}{\partial x} \right) \right] \forall\, x \in \mathcal{D}, \tag{13}$$

where the function $\mu(\mathbf{F})$ is logarithmic matrix norm (matrix measure) which is defined as

$$\mu(A) = \lim_{\epsilon \to 0} \frac{||I + \epsilon \mathbf{F}|| - 1}{\epsilon}. \tag{14}$$

Any Lipschitz nonlinearity is a one-sided Lipschitz nonlinearity but the converse is not true, see [64, 79] for examples. The one-sided Lipschitz constant ρ is usually smaller than λ_f.

The contribution of the nonlinearity to (7) is the term

$$\begin{pmatrix} \rho I & -\frac{1}{2} I \\ -\frac{1}{2} I & 0 \end{pmatrix}. \tag{15}$$

In order to achieve viable outcomes, along with the one-sided Lipschitz condition, it is often necessary to include the quadratic inner-boundedness condition within a design paradigm [56]. The nonlinear matrix function $f(x, u)$ is called quadratic inner-boundedness in the region $\tilde{D}$ if $\forall\, x_1, x_2 \in \tilde{D}$, $\exists\, \beta, \gamma \in \mathbb{R}$ such that

$$||f(x_1, t) - f(x_2, t)||^2 \leq \beta ||x_1 - x_2||^2 + \gamma (x_1 - x_2)^T (f(x_1, t) - f(x_2, t)). \tag{16}$$

The Lipschitz function is also quadratically inner-bounded with $\gamma = 0$ and $\beta > 0$. That means if $f(x, t)$ satisfies the Lipschitz condition, then it is also one-sided Lipschitz and inner-bounded. However, the converse is not true [59]. The contribution of the above formula to the quadratic formula (7) is

$$\begin{pmatrix} \beta I & \frac{1}{2} \gamma I \\ \frac{1}{2} \gamma I & -I \end{pmatrix}. \tag{17}$$

Zulfiqar et al. [79] consider regular descriptor systems with state and output disturbances. Additionally, output nonlinearities are considered, which are also one-sided Lipschitz. The observer is constructed in descriptor form. The feasibility is achieved under a bilinear matrix inequality, which can be transformed into LMIs. Two numerical examples are considered. In the first, the dynamics of a moving particle is considered with sinusoidal disturbance, and also output nonlinearity that models the sensor characteristics. The second example studies two coupled synchronous FitzHugh–Nagumo neurons system with both the unidirectional and the bidirectional gap junctions in the medium between them.

In [64], full and reduced order unknown input observers (UIO) are designed for nonlinear descriptor systems satisfying OSL and quadratic inner-boundedness using generalized inverses and LMI approach. Tian and Ma [62, 63] design H_∞ observers for nonlinear continuous-time singular Markov jump systems satisfying OSL and quadratic inner-boundedness conditions. Hao and Dong [26] create an observer for descriptor time delay systems leveraging OSL and quadratic internal boundary conditions. The authors formulate a condition based on LMI that effectively enhances the system's resilience against disturbances bounded by the L_2 norm. The observer methodology in [16] addresses systems with time-varying delays, uncertain parameters, and disturbances.

3.3 (Generalized) Monotone Nonlinearities

Generalized monotone nonlinearities are described by

$$\frac{\partial f}{\partial x} + \left(\frac{\partial f}{\partial x}\right)^T \geq \mu I, \tag{18}$$

where $\mu \in \mathbb{R}$ [34]. If $\mu = 0$, then it is called monotone nonlinearity [18]. The mathematical representation of generalized monotone nonlinearities reveals that these functions have a minimum slope, indicating a lower bound. In contrast, the Lipschitz property (5) ensures that the absolute value of the slope is bounded above. If (18) holds, then the nonlinearity satisfies

$$(x_1 - x_2)^T (f(x_1, t) - f(x_2, t)) \geq \frac{1}{2}\mu ||x_1 - x_2||^2. \tag{19}$$

The above nonlinearity contributes to (7) the term

$$\begin{pmatrix} -\mu I & I \\ I & 0 \end{pmatrix}. \tag{20}$$

Gupta et al. [23] designed reduced order observers for rectangular descriptor systems with unknown inputs. Output nonlinearities satisfying (18) are also considered here, as a decoupled case, where the system has a linear, and a purely nonlinear output. Examples of a nonlinear LCR circuit from [73] and a rectangular system with exponential nonlinearity are demonstrated. Recent work [3] somewhat unifies the design of descriptor observers for Lipschitz and monotone using the behavioral framework. The authors have supplied ample conditions that ensure the presence of asymptotic observers and state estimators. By restricting the solution of a linear matrix inequality to a subspace determined by the Wong sequences, the design parameters of the observer are derived.

3.4 Quadratic Constraints

Quadratic nonlinearities assume the general form

$$(f(x_1, t) - f(x_2, t))^T \, \Theta \, (x_1 - x_2) \geq 0, \;\; \forall \; x_1, x_2, \tag{21}$$

where Θ is a matrix of appropriate dimensions. Full and reduced order descriptor observers are designed for square systems [71, 73] with an additional condition $\Theta > 0$. The nonlinearities (21) are assumed to have a decoupled form, so matrix Θ is taken as block-diagonal. The error dynamics are formulated as a Lur'e descriptor system. The nonlinearity is also assumed to be locally Lipschitz. Two examples are considered, a non-physical and a circuit. The quadratic inequality given by (21) may not satisfy the globally Lipschitz condition; one such example is given in [73]. The quadratic inequality is more general than the monotone one. Nonlinearity (21) can easily be generalized to the following generalized quadratic nonlinearity

$$(f(x_1, t) - f(x_2, t))^T\, \Theta\, (x_1 - x_2) \geq \mu I, \tag{22}$$

where Θ may not have any additional restrictions. To the authors' knowledge, no prior work has specifically tackled the observer design for descriptor systems with nonlinearity (22). Moreover, research on rectangular descriptor systems with nonlinearities (21) and (22) is not available.

3.5 Incremental Quadratic Constraints

This subsection not only explores a broader range of nonlinearities than previous ones, but also consolidates and unifies earlier related findings. To be specific, the nonlinear function $f(x, t)$ adheres to satisfy the following incremental quadratic constraint (δQC)

$$\left((x_1 - x_2)^T (f(x_1, t) - f(x_2, t))^T\right) \mathcal{Q} \begin{pmatrix} x_1 - x_2 \\ f(x_1, t) - f(x_2, t) \end{pmatrix} \geq 0, \tag{23}$$

where $\mathcal{Q}$ is a symmetric matrix called *incremental multiplier matrix* (IMM). The above nonlinearities cover many others as special cases, for example Lipschitz, one-sided Lipschitz, monotone, generalized monotone, and quadratic [43]. Given that $\Delta f = f(x_1, t) - f(x_2, t)$ and $\Delta x = x_1 - x_2$, Table 1 establishes the fact that these nonlinearities fall under the general nonlinearity δQC. A method for computing $\mathcal{Q}$ is described in [77].

Moysis et al. [43] designed observers for rectangular systems under the presence of UIs. The special case of output nonlinearities was considered. Application to secure communications was illustrated on a hyperchaotic Lorenz master system [31]. First, a sinusoidal signal was transmitted, and secondly, a grayscale image was first transformed into a binary signal and encrypted using a chaos-based pseudo-random bit generator.

Reference [5] applies the augmentation method to design sliding mode observers for state space nonlinear systems via the descriptor approach. A single joint flexible robot was explained under a square and sawtooth exogenous input signals. A second example of a system with random matrices is also considered.

Systems with disturbances that originate from an external and unknown exogenous system have been studied in [35]. It introduces state and adaptive disturbance observers for three distinct scenarios, i.e., linearities, decoupled nonlinearities, and coupled nonlinearities within the output equation. Simulations are presented to demonstrate and confirm the effectiveness of the suggested observers by taking a model of a single-link robotic manipulator and the Lorenz system.

Wen et al. [69] investigated H_∞ synchronization problem considering both external and stochastic disturbances for descriptor systems characterized by nonlinearities that conform to incremental quadratic constraints. Diverging from the observer-based design methodology, an intermittently periodic controller is formulated. This involves tuning the control period and width, leading to a potential reduction in control expenses. Subsequently, a set of adequate conditions is derived, and the controller gain is determined through the solution of linear matrix inequalities. Finally, examples were given to validate the result.

4 Under-Explored Nonlinearities

In this section, we outline some nonlinearities that are mostly worked upon state space systems and can be explored on descriptor systems in future.

4.1 Quasi-One-Sided Lipschitz Nonlinearities

Quasi-one-sided Lipschitz nonlinearity is an extension of one-sided Lipschitz nonlinearity and extended by Hu [29] which is used frequently and plays an immense role in the stability of dynamical systems. Quasi-one-sided Lipschitz condition is given as,

$$(x_1 - x_2)^T P(f(x_1, t) - f(x_2, t)) \leq \left(x_1 - x_2\right)^T R \left(x_1 - x_2\right) \ \forall\, x_1, x_2, \tag{24}$$

where P is a positive definite matrix. R is a real symmetric matrix, typically depends on the choice of P, and is called as quasi-one-sided Lipschitz constant matrix. Note that when $R = \rho I_n$ with ρ being a constant, the condition (24) becomes the one-sided Lipschitz condition (12). Hence, quasi-one-sided Lipschitz nonlinearity is less conservative than one-sided Lipschitz nonlinearity and more effective in terms of stabilizing a Lyapunov equation while designing an observer. Furthermore, [30] designs an observer for a system that satisfies quasi-one-sided Lipschitz condition but observer design constraints are not feasible using one-sided Lipschitz condition. This shows the importance of nonlinearity (24).

Recent studies [15, 30] investigate quasi-one-sided Lipschitz condition in the following form

$$(x - \hat{x})^T P(f(x, y) - f(\hat{x}, \hat{y})) \leq \begin{pmatrix} x - \hat{x} \\ y - \hat{y} \end{pmatrix}^T \begin{pmatrix} R & 0 \\ 0 & S \end{pmatrix} \begin{pmatrix} x - \hat{x} \\ y - \hat{y} \end{pmatrix} \ \forall\, x, \hat{x}, y, \hat{y} \tag{25}$$

where P is a positive definite matrix. R and S are quasi-one-sided Lipschitz constant matrices for nonlinearity $f(x, y)$.

Table 1 Various types of nonlinearities and their IMM

Type of nonlinearity	Mathematical condition satisfied	IMM	Key references
Lipschitz nonlinearity	$\Delta f^T \Delta f \leq \gamma^2 \Delta x^T \Delta x : \ \gamma > 0$	$\begin{pmatrix} \gamma^2 I & 0 \\ 0 & -I \end{pmatrix}$	Gupta et al. [21, 22]
One-sided Lipschitz nonlinearity	$\Delta f^T \Delta x \leq \rho \Delta x^T \Delta x : \rho \in \mathbb{R}$	$\begin{pmatrix} \rho I & -\frac{1}{2} I \\ -\frac{1}{2} I & 0 \end{pmatrix}$	Zulfiqar et al. [79]
Monotone Nonlinearity	$\Delta x^T \Delta f \geq 0$	$\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$	Fan and Arcak [18]
Generalized monotone nonlinearity	$\Delta x^T \Delta f \geq \frac{1}{2} \mu \Delta x^T \Delta x : \mu \in \mathbb{R}$	$\begin{pmatrix} -\mu I & I \\ I & 0 \end{pmatrix}$	Gupta et al. [23]
Quadratic inequality	$\Delta x^T \Theta \Delta f \geq 0 : \Theta > 0$	$\begin{pmatrix} 0 & \frac{1}{2}\Theta \\ \frac{1}{2}\Theta & 0 \end{pmatrix}$	Yang et al. [71, 73]

Remark 1 Matrix R in Eq. (25) need not to be positive- or negative-definite; rather, it can be any symmetric matrix. Again, when $x = \hat{x}$, the inequality (25) is reduced to $(y - \hat{y})^T S(y - \hat{y}) \leq 0 \; \forall \; y, \; \hat{y}$. Therefore, the matrix S is required to be positive semi-definite.

Now, (25) can be rewritten as:

$$\begin{aligned} &\Delta x^T P \Delta f \leq \Delta x^T R \Delta x + \Delta y^T S \Delta y \\ \Leftrightarrow\; &\Delta x^T R \Delta x + \Delta y^T S \Delta y - \Delta x^T P \Delta f \geq 0 \\ \Leftrightarrow\; &\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta f \end{pmatrix}^T \begin{pmatrix} R & 0 & -P/2 \\ 0 & S & 0 \\ -P/2 & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta f \end{pmatrix} \geq 0 \end{aligned}$$

Hence, we can say that

$$\begin{pmatrix} R & 0 & -P/2 \\ 0 & S & 0 \\ -P/2 & 0 & 0 \end{pmatrix}, \tag{26}$$

contributes to an LMI formulation in Eq. (7) to stabilize the error dynamics term.

Furthermore, if a nonlinearity $f(x, y)$ satisfies

$$x^T P f(x, y) \leq \begin{pmatrix} x \\ y \end{pmatrix}^T \begin{pmatrix} R & 0 \\ 0 & S \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, \tag{27}$$

then it is defined as *weak quasi-one-sided Lipschitz condition* [15]. Just as in (26), one can formulate the matrix which contributes in LMI formulation. While numerous observers have been developed for state space systems considering the condition of quasi-one-sided Lipschitz nonlinearity [15, 19, 30, 54, 60], there is limited research on observers designed for descriptor systems. Observer design for descriptor systems, especially rectangular ones, incorporating quasi-one-sided Lipschitz nonlinearity is a relatively new and less explored area of research.

4.2 Slope Restricted Nonlinearities

The slope restriction is given by

$$0 \leq \frac{f(x_1, t) - f(x_2, t)}{x_1 - x_2} \leq \lambda, \tag{28}$$

where $\lambda > 0$. If a function satisfies the slope restricted nonlinearity, it also satisfies the Lipschitz condition. Yet, as can be observed from the above literature review, the Lipschitz assumption is more commonly considered. Zhou et al. [78] designed full and reduced order observers for square systems with slope restricted nonlinearities. In the numerical examples, sinusoidal nonlinearities are considered, so the factor

λ is easy to specify. Yang et al. [72] tackled a specific class of descriptor systems with nonlinearities having slope restrictions in both the system dynamics and output equations. The paper introduces a method for designing H_∞ observers based on LMIs.

Reference [40] studies the slope restricted nonlinearities on discrete-time descriptor systems in the following form

$$\begin{pmatrix} \Delta f \\ x_1 - x_2 \end{pmatrix}^T \begin{pmatrix} 2I & -\Lambda \\ -\Lambda & 0 \end{pmatrix} \begin{pmatrix} \Delta f \\ x_1 - x_2 \end{pmatrix} \leq 0, \tag{29}$$

where $\Delta f = f(x_1, t) - f(x_2, t) \ \forall \ x_1, x_2 \in \mathbb{R}^n$. Moreover, $\begin{pmatrix} 2I & -\Lambda \\ -\Lambda & 0 \end{pmatrix}$ aids in formulating an LMI within Eq. (7) to ensure the stability of the error dynamics term. The nonlinearity (29) is yet to be explored for continuous-time descriptor systems.

4.3 Ellipsoidally (Locally) Lipschitz

Ellipsoidally Lipschitz constraints are also termed as locally Lipschitz [25, 55] or less conservative Lipschitz [51, 52]. A function is said to be ellipsoidally Lipschitz if it satisfies

$$||f(x_1, t) - f(x_2, t)|| \leq ||\Lambda(x_1 - x_2)|| \tag{30}$$

for an ellipsoidal region

$$x_1, x_2 \in x^T R^{-1} x \leq 1 \ \forall \ x \in \mathbb{R}^n, \tag{31}$$

where R is symmetric positive definite and Λ is a constant matrix. The Lipschitz condition (5) can be considered a special case of the above, if Λ is taken as a scalar instead of a matrix, and it is assumed that $R = \lim_{r\to\infty} rI$. The contribution of (30) to the error dynamics LMI is

$$\begin{pmatrix} \epsilon \Lambda^T \Lambda & 0 \\ 0 & -\epsilon I \end{pmatrix}. \tag{32}$$

4.4 Integral Quadratic Constraints

This is one of the most general cases, when there exists constant $d \geq 0$ such that the nonlinear function $f(x, t)$ satisfies

$$\int_0^t ||f||^2 \mathrm{d}t \leq \int_0^t ||x||^2 \mathrm{d}t + d \tag{33}$$

This has been considered in recent works [8, 74] for state space systems. This property could be effective in PI observers, in which the integral of the error is of use. Descriptor systems under these conditions remain unexplored.

4.5 Bounded Jacobian

Many nonlinear systems exist which require larger value of Lipschitz constant. In such cases a smaller Lipschitz constant won't work out. This essentially boils down to a bounded Jacobian nonlinearity. The observer method that incorporates a bounded Jacobian employs the scalar mean value theorem and the differential mean value theorem to articulate the nonlinear estimation error dynamics as a convex combination of matrices. Hence, it can be said that the concept of the bounded Jacobian approach emerged through observation and consideration of both mean value theorems [50, 52, 53]. Observer design for descriptor systems, incorporating bounded Jacobian nonlinearity, has yet to be investigated.

4.6 Other Types of Synchronization

Traditionally, the goal of the observer is to estimate the internal states of the master system and achieve synchronization. This is the standard problem considered, as knowledge of the system's state is usually what is sought after. Nonetheless, this problem can be generalized to a wider category, where the observer estimates to some function of the master system's state. This generalization is certainly interesting from a mathematical standpoint, as the problem may be more complex, but could also be of practical use. For example, in secure communication applications, the information could be masked in such a way so that classic synchronization may fail to decrypt the signal. Examples include anti-synchronization [45, 57], where the observer synchronizes with the opposite of the state of the master system

$$||x(t) + \hat{x}(t)|| \to 0, \tag{34}$$

lag synchronization [70], which synchronizes with a delayed state

$$||x(t-\tau) + \hat{x}(t)|| \to 0, \tag{35}$$

projective synchronization [20], which synchronizes with a scaled state

$$||ax(t) + \hat{x}(t)|| \to 0, \tag{36}$$

and function projective synchronization [36], which synchronizes with a scaled function state

$$||a(t)x(t) + \hat{x}(t)|| \to 0. \tag{37}$$

5 Conclusions and Future Works

In this article, a review on advances in observer design for nonlinear descriptor systems is presented. On the one hand, considering wide classes that can encompass many others, like the δQC case, is of importance, as the results are applicable to many cases, and can also unify previous results from many works. On the other hand, studying narrower and specialized types of nonlinearities can help in deriving softer observer design conditions, albeit they will be applicable for fewer cases. Hence, both research directions bring merit to the observer design study.

This review aims to serve as guidance to researchers on identifying research gaps on the types of nonlinearities that have so far been considered. Slope restricted, integral type, and ellipsoidally Lipschitz constraints seem promising grounds for future studies. Nonlinearities in the output equations have been considered rarely, but the problem is gaining attention in the last years.

Of course, with so many developed works, a question that naturally arises is whether it is possible to make any comparison as to which of the available observers is better than the rest. For this, there are several different evaluation criteria that can be considered. Examples include the ease of feasibility of the LMI linked to the observer's convergence, the algebraic conditions required to derive the LMI, and the observer's convergence rate. The problem with such comparison though is that in many cases these criteria may be conflicting. For example, an observer with a faster convergence rate, may require the feasibility of a stricter LMI, compared to another one that is slower, but with a softer feasibility condition. Thus, it is not possible to consider an observer that is universally superior to the rest in the literature. Comparison can only be made with respect to specific design aspects each time.

An important observation from the works studied is that there are very few comments on identifying the type of nonlinearity a system belongs to, or how to compute the corresponding bounds in each case. Thus, future works should accompany their results with a discussion on this; otherwise, it may be hard to identify the applicability of the derived results.

Finally, the works reviewed here follow a Luenberger structure for observer design. To deal with more complex problems, like uncertainty in the system parameters or disturbance, different observers can be considered, like H_∞ observers [41], generalized dynamic observers [47], functional observers [65], robust variations [6], and fractional order systems [39, 76] of the above.

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