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CORRECTION

Correction to: Lorenz comparisons of nine rules for the adjudication of conflicting claims

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Mirás Calvo et al. (2021) point out that the adjusted proportional rule violates order preservation under claims variations. This property was wrongly used to prove the part of Theorem 1 according to which the adjusted proportional rule Lorenz dominates the minimal overlap rule. Mirás Calvo et al. (2021), moreover, provide an alternative proof of the disputed part of Theorem 1. The lemma below, however, establishes that the adjusted proportional rule satisfies a restricted version of order preservation under claims variations that allows a simple correction of our original proof.

Lemma *Let* (c, E) *be a claims problem with* $0 \leq c_1 \leq c_2 \leq \cdots \leq c_n$ *and* $E \leq C/2$. *Let* $c' = (0, c_2, c_3, \ldots, c_n)$ *and let* $i < j$. *Then,*

$$A_i(c', E) - A_i(c, E) \leq A_j(c', E) - A_j(c, E). \tag{1}$$

Proof Let $C_n = c_1 + c_2 + \cdots + c_{n-1}$. We first consider a claims problem (c, E) with $C_n < E \leq c_n$. Then, only individual n has a positive minimal right. The adjusted proportional rule allocates this minimal right (equal to $E - C_n$) to individual n and uses the proportional rule to solve the revised problem $(c_1, c_2, \ldots, c_{n-1}, C_n; C_n)$. Consequently,

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$$A(c,E) \;=\; (c_1/2, c_2/2, \dots, c_{n-1}/2, E - C_n/2). \tag{2}$$

Now, let (c, E) be a claims problem with $E \leq C/2$. We distinguish two cases.

Case $(i)\,:\, C_n < E$. Since $2E \leq C$, we have $E < c_n$. Use (2) and obtain

$$A(c,E) \;=\; (c_1/2, c_2/2, \dots, c_{n-1}/2, E - C_n/2).$$

Consider (c', E). Let $C'_n = 0 + c_2 + \cdots + c_{n-1}$. Since $C'_n \leq C_n < E$ and $E < c_n$, we can use (2) and we obtain

$$A(c',E) \;=\; (0, c_2/2, \dots, c_{n-1}/2, E - C_n/2 + c_1/2).$$

Hence, moving from (c, E) to (c', E), the adjusted proportional rule shifts the amount $c_1/2$ from individual 1 to individual n, and the inequalities in (1) are satisfied.

Case $(ii)\,:\, E \leq C_n$. No individual has a positive minimal right in (c, E). Let $\bar{c} = (\bar{c}_1, \bar{c}_2, \dots, \bar{c}_n)$ with $\bar{c}_i = \min\{c_i, E\}$ for each i. Then, $A(c,E) = P(\bar{c}, E)$.

We distinguish three cases.

Case (ii, a): in (c', E) no individual has a positive minimal right.

Then, $A(c', E) = P(\bar{c}', E)$ and the inequalities in (1) are satisfied.

Case (ii, b): in (c', E) only individual n has a positive minimal right.

Then, $C'_n < E$, and either $E \leq c_n$ or $c_n < E$. In case $E \leq c_n$, we have $\bar{c}_n = E$ and $A(c,E) = (E/(C_n + E)) \times \bar{c}$ with $E/(C_n + E) \leq 1/2$. Next, use (2) and obtain

$$A(c',E) \;=\; (0, c_2/2, \dots, c_{n-1}/2, E - C_n/2 + c_1/2).$$

The inequalities in (1) are satisfied.

In case $c_n < E$, we have $A(c,E) = (E/C) \times c$ with $E/C \leq 1/2$. Furthermore,

$$A(c',E) \;=\; P(0, c_2, \dots, c_{n-1}, c_n - E + C'_n, C'_n) + (0, 0, \dots, 0, E - C'_n). \tag{3}$$

The factor of proportionality in the P-term in (3) is equal to $C'_n/(2C'_n + c_n - E) > 1/2$. Again, the inequalities in (1) are satisfied.

Case (ii, c): in (c', E) two individuals have a positive minimal right.

We claim that $n = 3$. Assume the contrary: let $n \geq 4$ and let individuals n and $n-1$ have positive minimal rights. Then, $C'_n < E$ and $C' - c_{n-1} < E$. Therefore, $C \leq C'_n + C' - c_{n-1} < 2E$. This conflicts with the assumption that $C \geq 2E$ and our claim follows. Hence, we consider the claims problem (c_1, c_2, c_3, E) with individuals 2 and 3 having positive minimal rights in $(0, c_2, c_3, E)$. Then $c_3 < E$ and

$$A(0, c_2, c_3, E) = (0, (E + c_2 - c_3)/2, (E - c_2 + c_3)/2).$$

Since $A(c_1, c_2, c_3, E) = (E/C) \times (c_1, c_2, c_3)$, the inequalities in (1) hold. □

We note that in case $E > C/2$, the inequalities in (1) may be violated. The following example, in the spirit of Mirás Calvo et al. (2021), provides an illustration:

$$(c,E) = (\mathbf{1}, 1, 2, 6, 6, E = 10) \text{ and } A(c,E) = (10, 10, 20, 60, 60)/16, \text{ and}$$
$$(c',E) = (\mathbf{0}, 1, 2, 6, 6, E = 10) \text{ and } A(c',E) = (0, 8, 16, 53, 53)/13.$$

Counter to the inequalities in (1), individual 2 gains more (loses less) than individual 3 when moving from claims problem (c, E) to (c', E).

We now formulate and prove the disputed part of Theorem 1.

Theorem *The adjusted proportional rule Lorenz dominates the minimal overlap rule.*

Proof Let $E \leq C/2$. The original proof (pp. 802-803 (d) $A \rightarrow MO$) uses order preservation under claims variations in the final paragraph of Proposition 4 (p. 803) to compare claims problems of the form (c, E) and (c', E) as defined in the above lemma. Hence, the full power of the property is not needed. The restricted version of order preservation under claims variations is sufficient for the original proof to hold. Next, if $E > C/2$, then the adjusted proportional rule Lorenz dominates the Talmud rule, and the Talmud rule Lorenz dominates the minimal overlap rule (p. 805 (c), and p. 802 (d)). The transitivity of the Lorenz dominance relation entails that the adjusted proportional rule Lorenz dominates the minimal overlap rule. □

Reference

Mirás Calvo MÁ, Núñez Lugilde I, Quinteiro Sandomingo C, Sánchez Rodríguez E (2021) The adjusted proportional and the minimal overlap rules restricted to the lower-half, higher-half, and middle domains. ECOBAS Working Papers, 2021-02. Universidade de Vigo