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# Information revelation and coordination using cheap talk in a game with two-sided private information

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## Abstract

We consider a version of the Battle of the Sexes with private information and allow cheap talk regarding the players' types before the game. We show that a desirable type-coordination property is achieved at the unique fully revealing symmetric equilibrium (when it exists). Type-coordination is also obtained in a partially revealing equilibrium that exists when the fully revealing equilibrium does not. We further prove that truthfully revealed messages, followed by actions that depend meaningfully on these messages, are not equilibrium profiles with one-sided cheap talk. Finally, fully revealing equilibria do not exist under sequential communication either.

**Keywords** Battle of the sexes · Revelation of information · Cheap talk · Symmetric equilibrium · Truthfulness · Coordination

**JEL Classification** C72

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## 1 Introduction

Following the seminal paper by Crawford and Sobel (1982), much of the cheap talk literature has focused on the sender-receiver framework in which only one player has private information who takes no action, while the other player is uninformed but is responsible for making a payoff-relevant decision. There indeed is a small but growing literature on games where both players have private information and may send cheap talk messages to each other (see the Related Literature in this paper below).

Our aim in this paper is to contribute to this literature by analysing some interesting cheap talk equilibria in a game with two-sided information and two-sided cheap talk. Although incentive compatible mediated mechanisms (as in Banks and Calvert, 1992) inform us about all achievable possibilities with strategic communication, it might be impractical to conceive of or employ an impartial mediator in a real-world situation. For instance, in a market entry game (as in Dixit and Shapiro, 1985) or in the adoption of product-compatibility standards (as in Farrell and Saloner, 1988), it is not clear how a mechanism involving an impartial mediator can be implemented. However, on the other hand, we know that firms do talk to each other and/or make public announcements from time to time. We believe that direct cheap talk communication among players might occur more naturally in a strategic situation; this is the motivation for studying cheap talk equilibria in our paper.

We use a simple version of the Battle of the Sexes (hereafter, BoS) with two-sided private information for our analysis. In such a game, it is not obvious at all whether truthful revelation and thereby separation of players' types can be achieved in an unmediated equilibrium; moreover, it is also not clear whether coordination using cheap talk, as in the theoretical and the experimental literature with the complete information BoS, would extend to the BoS with private information. To analyse the above two issues, namely, truthful revelation and coordination, by unmediated communication, we use the simplest possible version of the BoS, as in Banks and Calvert (1992), with two types ("High" and "Low") for each player regarding the payoff from the other player's favourite outcome. The question we ask is whether, in this game, players will (fully or partially) reveal their types in a direct cheap talk equilibrium and also coordinate on Nash equilibrium outcomes in different states of the world.

The main contributions of this paper are two-fold. We first prove that there exists a unique fully revealing symmetric cheap talk equilibrium of this game in which the players announce their types truthfully (Theorem 1). Theorem 1 also suggests that full revelation is not a cheap talk equilibrium when the probability of a player being High-type is too high or too low; the allowable range of the prior probability of the High-type for the fully revealing equilibrium to exist in Theorem 1 has to be moderately low (with the upper bound being strictly less than $\frac{1}{2}$). Our unique fully revealing cheap talk equilibrium has the desirable type-coordination property: when the players' types are different, it fully coordinates on the ex-post efficient pure Nash equilibrium.

The structure of the game we consider here has an in-built tension for each player between the desire to compromise in order to avoid miscoordination and the desire to force coordination on one's preferred Nash equilibrium outcome. With incomplete information, efficiency and coordination do not necessarily go together; however, one might find it desirable to coordinate on the (ex-post) efficient outcome when the two players are of different types, in which the compromise is made by the player who suffers a smaller loss in utility. In the game we consider here, it is not apriori clear at all whether either full information revelation or the desirable coordination can be achieved in an unmediated equilibrium.

To explain our second contribution, we consider partially revealing equilibria, particularly for situations when the fully revealing equilibrium does not exist. Keeping the spirit of the fully revealing equilibrium, we characterise a class of partially revealing cheap talk equilibria in which only the High-type is not truthful, while the Low-type is truthful. We analyse this particular type of partial revelation because in the fully revealing equilibrium, the High-type is expected to compromise and coordinate on his less preferred outcome when the other player claims to be of Low-type. We identify the unique partially revealing cheap talk equilibrium with the type-coordination property in this set of equilibria and prove its existence based on the prior probability of the High-type being within a range that turns out to be non-overlapping and higher than that for the fully revealing equilibrium (Theorem 2). We also characterise the complete set of partially revealing cheap talk equilibria in which only the High-type is not truthful while the Low-type is truthful (Proposition 3).

Fully revealing equilibria have been a natural point of interest in extensions of the sender-receiver framework of Crawford and Sobel (1982) and numerous other models of cheap talk. To name a few, fully revealing equilibria have been studied in situations where there are two senders and an unidimensional state space (Krishna and Morgan 2001), two senders and a multidimensional Euclidean state space (Battaglini 2002) and two senders and a general multidimensional state space that is a closed subset of the whole Euclidean space (Ambrus and Takahashi 2008).[1] Fully revealing equilibria are also studied in related games of verifiable information disclosure. There is a large literature on voluntary disclosure of certifiable private information by informed agents (started by Grossman, 1981; see Milgrom, 2008 for a survey) that focus on conditions for the existence (and, sometimes, uniqueness) of fully revealing equilibria. In our paper, without assuming verifiability, we show that full revelation can happen in equilibrium under certain conditions.

We consider a symmetric communication process since the players are identical, facing an identical symmetric situation ex-ante. One could argue that it is independently meaningful to study symmetric strategy profiles in a symmetric game like ours; since the game is ex-ante symmetric with the two players facing an identical strategic situation, it is as if they are copies of each other. In the absence of any identifiers or characteristics to distinguish their positions, it is very natural to assume that they will behave and react in a symmetric manner. It is debatable whether

[1] We should note that these papers and some other extensions of Crawford and Sobel (1982) focus on fully revealing equilibria as they maximise the receiver's or decision-maker's expected payoff. This motivation does not apply to our setting as there is two-sided private information and two-sided cheap talk. We thank an anonymous referee and an associate editor for emphasising this.

we should allow asymmetric strategies to begin with, as Farrell (1987; p. 36), commented “we are concerned with the problem of how initially symmetric firms achieve asymmetric coordination: it would be begging the question to have them use asymmetric strategies.” Similar arguments are used also by Bhaskar (2000; p. 249) and Kuzmics et al (2014; p. 26) while studying repeated versions of similar coordination games. We thus consider it reasonable to study (type and player) symmetric cheap talk equilibria, following the tradition in the literature (as in Farrell, 1987 and Banks and Calvert, 1992). We however justify our assumption of symmetry in a stronger manner by formally proving that this assumption does not make us lose much! Having checked each of the potential candidate equilibrium strategy profiles, we formally prove that our symmetric equilibrium is the only responsive and non-degenerate equilibrium among the truthful equilibria of this game (Lemmata 1 and 2 and Proposition 2).

One may argue that simultaneous unmediated communication is perhaps rare to find in reality and that in real-life, people talk sequentially. In this paper, within our set-up (game), we prove that there is no meaningful cheap talk equilibrium involving truth-telling when the players talk sequentially (Theorem 4). Thus, Theorems 1 and 4 could potentially hint at an explanation why simultaneous unmediated cheap talk should be more prevalent than sequential talk in real-life coordination problems; this result perhaps provides a negative platform for the prospects of generating full revelation of information in (unmediated) sequential communication.

Having established our two main contributions in this paper, we further show that these cheap talk equilibria are more efficient than the Bayesian-Nash equilibrium of the game without any communication (Proposition 4) which suggests it is better to talk! We also analyse the scenario when only one of the players is allowed to talk in our game, to understand the difference between one-sided and two-sided cheap talk. We find that truthful cheap talk is not possible in any meaningful equilibrium by one player only (Theorem 3). Finally, we consider non-canonical message spaces at the cheap-talk stage; we identify a new equilibrium with a bigger message space (including the types) and find that, with the help of more messages, truthfulness and desirable coordination may be achieved together even when the direct truthful cheap talk equilibrium does not exist (Theorem 5).

### 1.1 Related literature

Our paper is definitely not the first to analyse two-sided cheap talk. Two-sided cheap talk using multiple stages of communication where only one of the players has incomplete information has been studied, among others, by Aumann and Hart (2003) and Krishna and Morgan (2004). Examples of information transmission using two-sided cheap talk under two-sided incomplete information can also be found in the literature. Farrell and Gibbons (1989) show that in a bargaining model, cheap talk between two privately informed parties can lead to distinctly new equilibria that could not arise without cheap talk. There is separation of types to some extent in the communication phase because different types trade off differently their bargaining position against the chance of continued negotiation.

Matthews and Postlewaite (1989) also analyse communication before a similar double auction but their focus is on comparing mediated and unmediated communication and their unmediated cheap talk is about coordinating different types on different equilibria of the game without talk. Our model differs from these papers in the underlying game structure as well as in the focus of the results that we obtain. Chen (2009) studies an extension of the standard sender-receiver game by allowing both the expert and the decision maker to have private information about the state of the world. With one-sided communication from the expert, some information transmission takes place via non-monotonic equilibria despite preferences satisfying the single-crossing property. With two-sided cheap talk where first the decision maker and then the expert sequentially communicate, information revelation from the decision maker is impossible. In this paper, although there is two-sided private information as well as two-way cheap talk, we note that only the decision maker can take an action unlike in our model. Our game also differs from the Hawk-Dove game studied in Baliga and Sjöström (2012) and the Cournot game in Goltsman and Pavlov (2014) where a player's preference over the other player's action does not depend on his type or action.

The primary focus of Banks and Calvert (1992) was to study communication in a similar game using an impartial mediator and the efficiency implications of such mediated communication, allowing more general message spaces (i.e., not restricted to only two types) in the communication phase. They identified conditions (Proposition 2, Sect. 4 in their paper) under which the outcome of an ex-ante efficient incentive compatible mediated mechanism can be achieved as the equilibrium of an unmediated communication process. In contrast, the focus of our current paper is to identify conditions under which (full) revelation occurs at the cheap talk stage and some form of coordination property holds. Obviously, these objectives are different from those studied in Banks and Calvert (1992). Okuno-Fujiwara et al. (1990), which provides a general analysis of such a problem, argue that studying information revelation is important regardless of whether the full information outcome is "good" or not. One of the reasons given in that paper is that public policies are formulated sometimes to enhance and in other circumstances to restrict information disclosure by market agents, based on perceptions of too little or too much information sharing in equilibrium (see discussion on p. 46 in their paper).[2]

Similarly, coordination is another important issue in this literature. In a seminal paper, Farrell (1987) showed that multiple rounds of cheap talk regarding the intended choice of play reduces the probability of miscoordination; the probability of coordination on one of the two pure Nash equilibria increases with the number of rounds of communication (although, at the limit, may be bounded away from 1). Park (2002) identified conditions for achieving efficiency and coordination in a similar game with three players. Parallel to the theory, the experimental literature also shows that cheap-talk and any pre-play non-binding communication can significantly improve coordination in games like BoS (Cooper et al. 1989; Crawford 1998; Costa-Gomes 2002; Camerer 2003; Burton et al. 2005; Cabrales et al. 2018).

[2] The motivation for studying fully revealing equilibria here arises also because it might be the unique equilibrium of these models.

A few other relevant papers are worth mentioning in this context. Baliga and Morris (2002) identify sufficient conditions under which a fully revealing equilibrium coordinates on efficient Nash equilibria of an underlying complete information game. However, they mainly analyse one-sided cheap talk in two player games with one-sided incomplete information; they also present a particular example where there is two-sided incomplete information and two-sided cheap talk and show that some information transmission (i.e., partial revelation) can take place in a symmetric equilibrium with cheap talk. Doraszelski et al. (2003) consider a scenario where two players vote to decide whether to form a partnership with uncertain rewards, and before voting, the players can talk to each other. Under both one-sided and two-sided cheap talk, the paper shows how some information transmission takes place. Baliga and Sjöström (2004) consider a game with private information and show that under certain conditions, two-sided cheap talk can help; their underlying model is an arms race game and they prove that the probability of an arms race comes down close to zero, even when the unique Bayesian-Nash equilibrium without cheap talk involves an arms race with probability one. However, it's worth emphasising that full revelation is not possible in their model. Horner et al. (2015) study conflict resolution and peace negotiations amongst two sovereign countries, each with private information about their relative strengths. This paper mainly focuses on comparing optimal third-party arbitration mechanisms with enforcement power to optimal mediation mechanisms without enforceability. It also discusses unmediated cheap talk and shows that optimal mediation leads to strictly higher welfare than the optimal truth-telling equilibrium of the unmediated communication game.

The complete information BoS has many economic applications (see the Introduction in Cabrales et al., 2000); the corresponding game of incomplete information is not just a natural extension but is also relevant in many of these economic situations where the intensity of preference and its prior probability are important factors. BoS-type games may be more complicated with incomplete information, where each player has private information about the "intensity of preference" for the other player's favourite outcome. Apart from its applications, the BoS with private information is clearly of interest to theorists and experimentalists (see the Introduction in Cabrales et al., 2018).

Several experiments on cheap talk (admittedly, most of them in different game settings, i.e., sender-receiver games with one sided private information) have focused on the degree of truthfulness exhibited by informed players. These show that subjects tend to truthfully reveal more information than is predicted by standard equilibrium analysis (see for example, Cai and Wang 2006; Sanchez-Pages and Vorsatz 2007; Cabrales et al. 2018). Different theories have been suggested to rationalise this experimental evidence of excessive truth-telling such as, lying costs (Charness and Dufwenberg 2006; Sanchez-Pages and Vorsatz 2009; Hurkens and Kartik 2009), bounded rationality and level-*k* thinking (Cai and Wang 2006; Kawagoe and Takizawa 2009; Wang et al. 2010). In our paper, we have investigated the extent to which informative truth-telling can be sustained as an equilibrium behaviour in our cheap talk augmented incomplete information BoS.

# 2 Model

## 2.1 The game

We consider a version of BoS with incomplete information, as given below, in which each of the two players has a set of strategies, $S_i$, containing two pure strategies, $A$ and $B$, i.e., $S_i = \{A, B\}$, $i = 1, 2$. Let $S = S_1 \times S_2$ denote the set of strategy combinations of the two players. The payoffs are as in the following table, in which the value of $t_i \in T_i$ is the private information of player $i$, $i = 1, 2$, with $0 < t_i < 1$. We assume that $t_i$ is a discrete random variable that takes only two values $L$, $H$, where, $0 < L < H < 1$, whose realisation is observed only by player $i$, for $i = 1, 2$. We henceforth refer to the values of $t_i$ as player $i$'s type (Low, High), i.e., $T_i = \{L, H\}$, $i = 1, 2$. We further assume that each player's type is independently drawn from the set $\{L, H\}$, according to a probability distribution with $Prob(t_i = H) = p \in [0, 1]$. Also, the payoffs to both players from the miscoordinated outcome is normalised to 0, while the payoff to player 1 (player 2) from ($A$, $A$) (($B$, $B$)) is normalised to 1.

| | | Player 2 | |
|---|---|---|---|
| | | $A$ | $B$ |
| Player 1 | $A$ | $1, t_2$ | 0, 0 |
| | $B$ | 0, 0 | $t_1, 1$ |

These payoffs will also formally be denoted by the players' utility functions $u_i : S \times T_i \longrightarrow \mathbb{R}, i = 1, 2$. Note that player $i$'s utility depends here on own type $t_i$ only and not on the other player's private information, $t_j$. The unique symmetric Bayesian-Nash equilibrium of this game can be characterised by $\sigma_i(s_i|t_i)$, the probability that player $i$ of type $t_i$ plays the pure strategy $s_i$.[3]

**Proposition 1** *The unique symmetric Bayesian Nash equilibrium of the BoS with incomplete information is given by the following strategy for player 1 (player 2's strategy is symmetric and is given by* $\sigma_1(A|t) = \sigma_2(B|t)$, $t = H, L$*):*

$\sigma_1(A|H) = 0$ and $\sigma_1(A|L) = \frac{1}{(1-p)(1+L)}$, when $p < \frac{L}{1+L}$,
$\sigma_1(A|H) = 0$ and $\sigma_1(A|L) = 1$, when $\frac{L}{1+L} \leq p \leq \frac{H}{1+H}$,
$\sigma_1(A|H) = 1 - \frac{H}{p(1+H)}$ and $\sigma_1(A|L) = 1$, when $p > \frac{H}{1+H}$.

The proof is straightforward and hence has been omitted here.[4]

[3] The corresponding game with complete information with commonly known values $t_1$ and $t_2$, has two pure Nash equilibria, ($A$, $A$) and ($B$, $B$), and a mixed Nash equilibrium in which player 1 plays $A$ with probability $\frac{1}{1+t_2}$ and player 2 plays $B$ with probability $\frac{1}{1+t_1}$.

[4] In a similar game, Banks and Calvert (1992) also provided a similar characterisation.

## 2.2 Cheap talk

We study an extended game in which the players are first allowed to have a round of simultaneous canonical cheap talk intending to reveal their private information before they play the above BoS. In the first (cheap talk) stage of this extended game, each player $i$ simultaneously chooses a costless and nonbinding announcement $\tau_i$ from the set $\mathcal{T}_i = \{L, H\}$. Then, given a pair of announcements $(\tau_1, \tau_2)$, in the second (action) stage of this extended game, each player $i$ simultaneously chooses an action $s_i$ from the set $S_i$.

An announcement strategy in the first stage for player $i$ is a function $a_i : T_i \rightarrow \Delta(\mathcal{T}_i)$, where $\Delta(\mathcal{T}_i)$ is the set of probability distributions over $\mathcal{T}_i$. We write $a_i(H|t_i)$ for the probability that strategy $a_i(t_i)$ of player $i$ with type $t_i$ assigns to the announcement $H$. Thus, the announcement $\tau_i$ of player $i$ with type $t_i$ is a random variable drawn from $\mathcal{T}_i$ according to the probability distribution with $Prob(\tau_i = H) = a_i(H|t_i)$. Beliefs for player $i$ are given by $\widetilde{p}_i : \mathcal{T}_j \rightarrow \Delta(T_i), i,j = 1, 2$. We will denote $i$'s posterior belief by $\widetilde{p}_i(H\big|\tau_j) = Prob(t_j = H\big|\tau_j)$.

In the second (action) stage, a strategy for player $i$ is a function $\sigma_i : T_i \times \mathcal{T}_1 \times \mathcal{T}_2 \rightarrow \Delta(S_i)$, where $\Delta(S_i)$ is the set of probability distributions over $S_i$. We write $\sigma_i(A|t_i; \tau_1, \tau_2)$ for the probability that strategy $\sigma_i(t_i; \tau_1, \tau_2)$ of player $i$ with type $t_i$ assigns to the action $A$ when the first stage announcements are $(\tau_1, \tau_2)$. Thus, player $i$ with type $t_i$'s action choice $s_i$ is a random variable drawn from $\{A, B\}$ according to a probability distribution with $Prob(s_i = A) = \sigma_i(A|t_i; \tau_1, \tau_2)$. Given a pair of realised action choices $(s_1, s_2) \in S_1 \times S_2$, the corresponding outcome is generated. Thus, given a strategy profile $((a_1, \sigma_1), (a_2, \sigma_2))$, one can find the players' actual payoffs from the induced outcomes in the type-specific payoff matrix of the BoS and hence, the (ex-ante) expected payoffs. As the game is symmetric, in our analysis, we maintain the following notion of symmetry in the strategies, for the rest of the paper.

**Definition 1** A strategy profile $((a_1, \sigma_1), (a_2, \sigma_2))$ is called announcement-symmetric (in the announcement stage) if $a_i(H|t) = a_{-i}(H|t)$ for any $t \in \{L, H\}$. A strategy profile is called action-symmetric (in the action stage) if $\sigma_i(A|t; \tau_1, \tau_2) = \sigma_{-i}(B|t; \tau_2, \tau_1)$, for all $t, \tau_1, \tau_2$. A strategy profile is called symmetric if it is both announcement-symmetric and action-symmetric.

Note that Definition 1 preserves symmetry for both players and the types for each player. We consider the following standard notion of Perfect Bayesian Equilibrium (PBE) in this two-stage cheap talk game. Our definition follows the standard formulation of PBE in the literature (see for example, Fudenberg and Tirole, 1991 and Baliga and Morris, 2002).

**Definition 2** A symmetric strategy profile $((a_1, \sigma_1), (a_2, \sigma_2))$ together with beliefs $(\widetilde{p}_1, \widetilde{p}_2)$ is called a symmetric cheap talk equilibrium if it is a Perfect Bayesian Equilibrium (PBE) of the game with cheap talk, i.e., each player is playing optimally at all his information sets given the strategy of the other player and the beliefs are updated according to the Bayes rule whenever possible.

Formally, $((a_1, \sigma_1), (a_2, \sigma_2))$ and $(\widetilde{p}_1, \widetilde{p}_2)$ is a PBE of the game with cheap talk if

(1) $\forall t_i, \forall i \neq j, i,j = 1,2$

$$a_i(\tau_i|t_i) > 0 \Longrightarrow \tau_i \in \arg\max_{\tau_i' \in \mathcal{T}_i} \sum_{t_j} p(t_j) \sum_{\tau_j} a_j(\tau_j|t_j) \sum_{s \in S} \left[\sigma_i(s_i|t_i; \tau_i', \tau_j)\sigma_j(s_j|t_j; \tau_i', \tau_j)\right] u_i(s, t_i);$$

(2) $\forall t_i, \tau_i, \tau_j, \forall i \neq j, i,j = 1,2$

$$\sigma_i(s_i|t_i; \tau_i, \tau_j) > 0 \Longrightarrow s_i \in \arg\max_{s_i' \in \mathcal{S}_i} \sum \widetilde{p}_i(t_j|\tau_j) \sum_{s_j \in S_j} \sigma_j(s_j|t_j; \tau_i', \tau_j) u_i(s_i', s_j, t_i);$$

(3) $\forall i \neq j, i,j = 1,2$

$$\widetilde{p}_i(t_j|\tau_j) = \frac{a_j(\tau_j|t_j)p(t_j)}{\sum_{t_j' \in T_j} a_j(\tau_j|t_j')p(t_j')} \text{ if } \sum_{t_j' \in T_j} a_j(\tau_j|t_j')p(t_j') > 0$$

and $\widetilde{p}_i(.|\tau_j)$ is any probability distribution on $T_i$ if $\sum_{t_j' \in T_j} a_j(\tau_j|t_j')p(t_j') = 0$.

In Definition 2 above, Condition *(1)* ensures optimality at the cheap talk stage. For example, if the announcement strategy $a_i(.|t_i)$ is completely mixed, then condition *(1)* implies that both the messages ($\tau_i = H$ and $\tau_i = L$ ) should provide the same expected payoff to player $i$ with type $t_i$. If $a_i(.|t_i)$ is a pure strategy, then condition *(1)* will yield a weak inequality whereby the expected payoff from the chosen pure strategy announcement and then following the equilibrium strategy at the action stage is at least as high as the expected payoff from choosing the other message and subsequently using the optimal strategy at the action stage (which could possibly be a deviation from the prescribed equilibrium profile). Similarly, Condition *(2)* ensures optimality at the action stage whereby a completely mixed action strategy yields an equality constraint for expected payoffs (using suitable posterior beliefs) and a pure strategy yields an inequality constraint. Finally, Condition *(3)* ensures that posterior beliefs are derived using Bayes rule.

We do acknowledge that the restriction to symmetric equilibria may seem to be a strong one; however, we will present a result (Proposition 2) below that would justify our choice.

Definition 2 suggests that a symmetric cheap talk equilibrium can be characterised by a set of (symmetric) equilibrium constraints (2 for the announcement stage and another possible 8 for the action stage).

## 3 Main results

The main purpose of our paper is to find, if exists, an equilibrium with truthful talk. We thus first consider the possibility of full revelation of the types as a result of our canonical cheap talk. Subsequently, we present and analyse some other cheap talk equilibria in this section.

### 3.1 Fully revealing equilibrium

We consider a specific class of strategies in this subsection where we impose the property that the cheap talk announcement should be fully revealing.

**Definition 3** A symmetric strategy profile $((a_1, \sigma_1), (a_2, \sigma_2))$ is called fully revealing if the announcement strategy $a_i$ reveals the true types with certainty, i.e., $a_i(H|H) = 1$ and $a_i(H|L) = 0$.

We first consider a specific fully revealing (separating) strategy profile that we call $S_{separating}$, influenced by the equilibrium action profile in Farrell (1987) for the complete information version of this game. In this strategy profile, the players announce their types truthfully and then in the action stage, they play the mixed Nash equilibrium strategies of the complete information BoS when both players' types are identical and they play ($B$, $B$) (($A$, $A$)), when only player 1's type is $H$ ($L$). We now state our first result below, characterising the fully revealing symmetric cheap talk equilibrium.[5]

**Theorem 1** *$S_{separating}$ is the unique fully revealing symmetric cheap talk equilibrium and it exists only for* $\frac{L^2+L^2H}{1+L+L^2+L^2H} \le p \le \frac{LH+LH^2}{1+L+LH+LH^2}$.

Before proving Theorem 1, we first observe the following fact that follows from Definitions 2 and 3: in a fully revealing symmetric cheap talk equilibrium $((a_1, \sigma_1), (a_2, \sigma_2))$, the players' strategies in the action phase must constitute a (pure or mixed) Nash equilibrium of the corresponding complete information BoS, that is, $(\sigma_1(t_1, t_2), \sigma_2(t_1, t_2))$ is a (pure or mixed) Nash equilibrium of the BoS with values $t_1$ and $t_2$, $\forall t_1, t_2 \in \{H, L\}$. Thus, in a fully revealing symmetric cheap talk equilibrium $((a_1, \sigma_1), (a_2, \sigma_2))$, conditional on the announcement profile ($H$, $H$) or ($L$, $L$), the strategy profile in the action phase must be the mixed strategy Nash equilibrium of the corresponding complete information BoS, that is, whenever $t_1 = t_2$, $(\sigma_1(t_1, t_2), \sigma_2(t_1, t_2))$ is the mixed Nash equilibrium of the BoS with values $t_1 = t_2$.

Based on the above fact, one can easily identify all the candidate equilibrium strategy profiles of the extended game that are fully revealing and symmetric. It implies that these profiles are differentiated only by the actions played when $t_1 \neq t_2$, that is, when the players' types are ($H$, $L$) and ($L$, $H$). The proof of Theorem 1 may now be completed easily; we have postponed the details of the rest of the proof to the Appendix of this paper.

The following couple of claims illustrate some features of the equilibrium $S_{separating}$. The claims are easy to establish and hence, we have omitted the formal proofs for them.

[5] Ganguly and Ray (2009) have analysed this game to see if a truthful cheap talk equilibrium, in which the players reveal their types truthfully before playing, exists at all and compared it with the mediated equilibrium of Banks and Calvert (1992). Theorem 1 in this paper corrects and thus improves upon the main result presented in Ganguly and Ray (2009).

**Claim 1** *The ex-ante expected payoff for any player from* $S_{separating}$ *is given by* $EU_{separating} = p^2 \frac{H}{1+H} + p(1-p)(1+H) + (1-p)^2 \frac{L}{1+L}$, *which is increasing over the range of p where it exists.*

**Claim 2** *The upper bound for p in Theorem 1,* $\frac{LH+LH^2}{1+L+LH+LH^2}$ *is always* $< \frac{1}{2}$, *since* $\frac{1}{2} - \frac{HL+H^2L}{1+L+HL+H^2L} = \frac{(1+L-LH-LH^2)}{2(1+L+HL+H^2L)} > 0$, *as long as* $L < H < 1$.

To understand why $p$ must lie in such a low range for this equilibrium to exist (as stated in Theorem 1 and Claim 2), consider the incentives for deviations by player 1 at the announcement stage. By deviating and claiming to be an $L$-type, player 1($H$-type) gains $1 - \frac{H}{1+H} = \frac{1}{1+H}$ when player 2 is a $H$-type (with probability $p$) and loses $H - \frac{H}{1+L} = \frac{HL}{1+L}$ when player 2 is a $L$-type (with probability $1-p$).[6] Since $\frac{1}{1+H} - \frac{HL}{1+L} = \frac{(1+L-LH-LH^2)}{(1+H)(1+L)} > 0$, the gain from the deviation when playing against player 2 ($H$-type) is bigger than the loss when playing against player 2($L$ -type). If a $H$-type is equally or more likely than a $L$-type, then player 1($H$-type) will obviously deviate and truthful revelation will not be an equilibrium. So, $p$ must be $< \frac{1}{2}$. Indeed, $p$ needs to be small enough to make the above deviation unattractive and the precise value of $p$ for which this holds is $\frac{HL+H^2L}{1+L+HL+H^2L}$ or less. However, $p$ cannot be too close to 0 either. This is because of incentives for deviations by player 1($L$-type). By deviating and claiming to be an $H$ -type, player 1($L$-type) gains $L - \frac{L}{1+L} = \frac{L^2}{1+L}$ when player 2 is a $L$-type (with probability $1-p$) and loses $1 - \frac{H}{1+H} = \frac{1}{1+H}$ when player 2 is a $H$-type (with probability $p$). Since $\frac{1}{1+H} - \frac{L^2}{1+L} = \frac{(1+L-HL^2-L^2)}{(1+H)(1+L)} > 0$, the loss from the deviation when playing against player 2($H$-type) is bigger than the gain when playing against player 2($L$-type). The expected gain will outweigh the expected loss only if a $L$-type is much more likely than a $H$-type (and $L$ is bigger than 0). Hence, player 1($L$-type) would deviate at the cheap talk stage only if $p$ is too close to 0.

## 3.2 Symmetry

We have already argued at length why symmetry of behaviour is a natural assumption to make in the game being studied here. However, we realise that it is also natural to question the extent to which symmetry is driving our main result (Theorem 1) above.

We have characterised the unique fully revealing symmetric cheap talk equilibrium in the BoS with private information. We note that symmetry does indeed impose a serious restriction on the nature of equilibria that we study. If we remove this restriction, there is of course a multitude of asymmetric cheap talk equilibria of this game; for example, babbling equilibria exist in which the players ignore the communication (regardless whether it is truthfully revealing or not) and just play one of the pure strategy Nash equilibria of the complete information BoS for all type-profiles.

[6] Note that after deviating in the cheap talk stage, player 1 ($H$-type) may deviate at the action stage as well. In fact, the optimal deviation strategy for player 1 ($H$-type) is to play action $B$ when player 2 is a $L$ -type.

There are other asymmetric equilibria as well. The players could use the cheap talk communication not for the purpose of information revelation about their types but to just coordinate (for every type profile) on the correlated equilibrium of the complete information version that assigns probability $\frac{1}{2}$ to each pure strategy Nash equilibrium outcome. This could be achieved by having both players use two messages with equal probability and then if the messages match, they play ($A$, $A$); otherwise, ($B$, $B$) is played. Here, the cheap talk messages essentially enable the players to generate a "jointly controlled lottery" (Aumann et al. 1968) and thus, do the job of a public randomisation device. Such an equilibrium leads to higher payoffs than our symmetric fully revealing equilibrium. However, the strategy profiles used in this equilibrium do not satisfy our definition of symmetry (see Definition 1). It is announcement-symmetric but not action symmetric (because, for example, $\sigma_1(A|H;H,H) = 1$ and $\sigma_2(B|H;H,H) = 0$). Also, there is no revelation of information in this equilibrium. For these reasons, we did not consider such equilibria in our paper.

The question now arises: is it conceivable to have asymmetric equilibria with full revelation as well such that the actions in the second stage depend on the information revealed in the first stage? In particular, one may ask whether there are other asymmetric fully revealing equilibria which lead to higher expected payoffs for both players (than those from $S_{separating}$). The difficulty in answering these questions lies in the fact that the number of asymmetric strategy profiles is very large. We do not know of any easy procedure of searching for equilibria from this large set, other than the obviously tedious method of checking all potential profiles individually.

We first note that with truthful revelation of types in the cheap talk stage, consequent to each of the four possible message profiles (i.e., $(H,H)$, $(H,L)$, $(L,H)$, $(L,L)$), there are three strategy profiles in the action stage that can constitute an equilibrium (two pure Nash equilibria, one mixed Nash equilibrium of the corresponding complete information BoS) which gives rise to $3^4$ or 81 candidate profiles that could potentially be an equilibrium.

Additionally, we want to ensure that the equilibria that we consider within this search procedure are non-trivial and interesting in the following sense. First, we want to focus our attention only on equilibria where at least some of the actions in the second stage depend on the announcements from the first stage in a non-trivial manner. We define the concept of responsive strategies accordingly.

**Definition 4** A strategy profile in the game with simultaneous two-sided cheap talk is called responsive if at least one of the following holds:

(i) $\sigma_1(A|H,H)$, $\sigma_1(A|H,L)$, $\sigma_1(A|L,H)$, $\sigma_1(A|L,L)$ are not all equal;
(ii) $\sigma_2(A|H,H)$, $\sigma_2(A|H,L)$, $\sigma_2(A|L,H)$, $\sigma_2(A|L,L)$ are not all equal.

Secondly, we believe that knife-edge equilibria that exist only for a specific value of $p$ are of little interest as well. To formalise this notion, we define the concept of non-degenerate equilibria here.

**Definition 5** An equilibrium in the game with simultaneous two-sided cheap talk is called degenerate if it exists only for a specific value of $p$ and is called non-degenerate if it exists for a continuum of values of $p$.

We are now ready to state our findings. After checking each of the possible 81 strategy profiles, we arrive at the remarkable conclusion that our symmetric equilibrium, $S_{separating}$, is the only responsive, non-degenerate equilibrium in this entire set. This result is formalised below.

**Lemma 1** *The following two strategy profiles are the only non-responsive strategy profiles that are fully revealing cheap talk equilibria*:

(i) $\sigma_1(A|H,H) = \sigma_2(A|H,H) = \sigma_1(A|L,L) = \sigma_2(A|L,L) = \sigma_1(A|H,L) = \sigma_2(A|H,L) = \sigma_1(A|L,H) = \sigma_2(A|L,H) = 1$;

(ii) $\sigma_1(A|H,H) = \sigma_2(A|H,H) = \sigma_1(A|L,L) = \sigma_2(A|L,L) = \sigma_1(A|H,L) = \sigma_2(A|H,L) = \sigma_1(A|L,H) = \sigma_2(A|L,H) = 0$.

**Lemma 2** *The following two strategy profiles are the only degenerate fully revealing cheap talk equilibria*:

(i) $\sigma_1(A|H,H) = \sigma_2(A|H,H) = \sigma_1(A|L,L) = \sigma_2(A|L,L) = 1$ and $\sigma_1(A|H,L) = \sigma_2(A|H,L) = \sigma_1(A|L,H) = \sigma_2(A|L,H) = 0$; this exists only for $p = \frac{1}{2}$.

(ii) $\sigma_1(A|H,H) = \sigma_2(A|H,H) = \sigma_1(A|L,L) = \sigma_2(A|L,L) = 0$ and $\sigma_1(A|H,L) = \sigma_2(A|H,L) = \sigma_1(A|L,H) = \sigma_2(A|L,H) = 1$; this exists only for $p = \frac{1}{2}$.

**Proposition 2** *The only responsive and non-degenerate fully revealing cheap talk equilibrium of the BoS with private information is given by $S_{separating}$ which exists for* $\frac{L^2+L^2H}{1+L+L^2+L^2H} \leq p \leq \frac{LH+LH^2}{1+L+LH+LH^2}$.

The proof of Lemma 1 is obvious and thus has been omitted. Proofs of Lemma 2 and of Proposition 2 are in the Appendix. Proposition 2 proves that the only interesting equilibrium here is $S_{separating}$, indicating that our symmetry assumption is not very restrictive.

### 3.3 Partially revealing equilibrium

The fully revealing symmetric cheap talk equilibrium exists only for a moderately low range of the prior probability $p$. One may now ask what sort of equilibria, if any, exists for any given $p$ outside this range. Understandably, it is not easy to characterise all possible equilibria for this game. We thus find it natural to analyse the type of partial revelation in which only the $L$-type truthfully reveals while the $H$-type does not. Formally, we consider a symmetric announcement strategy profile in which the $H$-type of player $i$ announces $H$ with probability $r$ and $L$ with probability $(1-r)$ and the $L$-type of player $i$ announces $L$ with probability 1, i.e., $a_i(H|H) = r$ and $a_i(H|L) = 0$. Clearly, after the cheap talk phase, the possible message profiles $(\tau_1, \tau_2)$ that the $H$-type of player 1 may receive are ($H$, $H$), ($H$, $L$), ($L$, $H$) or ($L$, $L$)

while the $L$ -type of player 1 may receive either ($L$, $H$) or ($L$, $L$). Let us denote an action-strategy of player 1 by $\sigma_1(A|H;H,H) = q_0$, $\sigma_1(A|H;H,L) = q_1$, $\sigma_1(A|H;L,H) = q_2$, $\sigma_1(A|H;L,L) = q_3$, $\sigma_1(A|L;L,H) = q_4$ and $\sigma_1(A|L;L,L) = q_5$. By symmetry, a partially revealing symmetric strategy profile $((a_1,\sigma_1),(a_2,\sigma_2))$ in our set-up can thus be identified by $(r,q_0,q_1,q_2,q_3,q_4,q_5)$.

First note that, on receiving the message profile ($H$, $H$), the players know the true types and hence in any such partially revealing symmetric cheap talk equilibrium, $q_0$ has to correspond to the mixed Nash equilibrium of the complete information BoS with values $H$ and $H$. Thus, $q_0 = \frac{1}{1+H}$. One may indeed characterise the whole set of partially revealing symmetric cheap talk equilibria in this set up (in which only the $L$-type is truthful), by characterising the equilibrium values of $(r,q_1,q_2,q_3,q_4,q_5)$, using equilibrium conditions, as listed in the proposition below.

**Proposition 3** *The following profiles are the only partially revealing symmetric cheap talk equilibria in which only the L-type is truthful:*

(i) $q_1 = \frac{1}{1+H}$, $q_2 = q_3 = \frac{p+Hp-rp-H}{p+Hp-rp-Hrp}$, $q_4 = q_5 = 1$ with any $0 < r \leq 1 - \frac{H(1-p)}{p}$; exists when $p > \frac{H}{1+H}$,

(ii) $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$, $q_5 = \frac{1}{1+L+LH+LH^2-p-Lp-LHp-LH^2p}$ and $r = \frac{LH+LH^2}{p+Lp+LHp+LH^2p}$; exists when $\frac{LH+LH^2}{1+L+LH+LH^2} < p < \frac{L+LH+LH^2}{1+L+LH+LH^2}$,

(iii) $q_1 = 0$, $q_2 = 1$, $q_3 = \frac{p+Hp+H^2p+H^3p-H-H^2-H^3}{p+Hp+H^2p+H^3p-H^2-H^3}$, $q_4 = q_5 = 1$ and $r = \frac{H^2}{p+H^2p}$; exists when $p > \frac{H+H^2+H^3}{1+H+H^2+H^3}$,

(iv) $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$, $q_5 = 1$ and $r = \frac{H+H^2}{1+H+H^2}$; exists when $\frac{L+LH+LH^2}{1+L+LH+LH^2} < p < \frac{H+H^2+H^3}{1+H+H^2+H^3}$,

The proof of Proposition 3, that involves 15 candidate strategy profiles for possible equilibria, split into four cases, is in the Appendix. Further details can be found in a previous discussion paper version of our work (Ganguly and Ray 2013).

The strategy profile (*iv*) in Proposition 3 has some significance that we are going to note in the next subsection. Below, we describe the other three equilibrium profiles.

1. In the strategy profile (*i*), player 1($L$-type) always plays $A$ and player 2($L$-type) always plays $B$ because $q_4 = q_5 = 1$. Suppose player 1($H$-type) wants to play completely mixed strategies in the action stage after the message profiles ($H$, $L$) and ($L$, $L$). These message profiles could arise either because player 2 is truly an $L$-type or player 2 is a $H$-type who is not revealing her true type. Since player 2 ($L$-type) plays $B$ with probability 1 after both the message profiles ($H$, $L$) and ($L$, $L$), player 2($H$-type) will need to randomise between $A$ and $B$ in the same manner after both the message profiles ($H$, $L$) and ($L$, $L$). Otherwise, player 1($H$-type) won't be indifferent between $A$ and $B$ after these message profiles. This is why $q_2 = q_3$, where their value is derived from the indifference conditions. Also, note that if these message profiles arise because player 2 is an $L$-type, then player 1($H$ -type) gets a payoff of 0 by playing $A$. So, for player 1($H$-type) to be

indifferent between $A$ and $B$ after these message profiles, the likelihood of player 2 being an $L$-type has to be very low. In other words, $p$ has to be very large. Not only that, it must also be the case that the player 2($H$-type) misleads and sends the message $\tau_2 = L$ with a very high probability. So, $r$ needs to be low enough to sustain this profile as an equilibrium.

2. In the strategy profile (*ii*), player 1($H$-type) plays $B$ with probability 1 after the message profile ($H$, $L$) (this is incentive compatible because player 2($H$-type) as well as player 2($L$-type) are playing $B$ with probability 1 as $q_2 = q_4 = 1$). Player 1($H$-type) plays $A$ with probability 1 after the message profile ($L$, $H$) (this is incentive compatible because player 2($H$-type) is also playing $A$ with probability $1 - q_1 = 1$). Player 1($L$-type) plays $A$ after the message profile ($L$, $H$) (this is incentive compatible because player 2($H$-type) is also playing $A$ with probability $1 - q_1 = 1$). Player 1($L$-type) plays a completely mixed strategy in the action stage after the message profile ($L$, $L$) and player 1($H$-type) plays a completely mixed strategy in the cheap talk phase. The two corresponding indifference conditions determine the specific values of $q_5$ and $r$. Player 1($H$-type) plays $B$ with probability 1 after the message profile ($L$, $L$) (this is incentive compatible only if $p$ is not very large so that the probability of this message coming from a player 2($H$-type) is low because player 2( $H$-type) will be playing $B$ with probability $q_3 = 0$; however, $p$ can't be too small either because the probability of player 2($L$-type) also playing $B$ after this message profile, i.e. $q_5$, needs to be sufficiently high and $q_5$ is an increasing function of $p$). This explains why $p$ has to lie within the range specified in this strategy profile.
3. In the strategy profile (*iii*), as in profile (*ii*), player 1($H$ -type) plays $B$ with probability 1 after the message profile ($H$, $L$) (this is incentive compatible because player 2($H$-type) as well as player 2($L$-type) are playing $B$ with probability 1 as $q_2 = q_4 = 1$). Player 1($H$-type) plays $A$ with probability 1 after the message profile ($L$, $H$) (this is incentive compatible because player 2($H$-type) is also playing $A$ with probability $1 - q_1 = 1$). In addition, as in profile (*i*) and (*ii*), player 1($L$-type) plays $A$ after the message profile ($L$, $H$) (this is incentive compatible because player 2($H$-type) is also playing $A$ with probability $1 - q_1 = 1$). Player 1($H$-type) plays a completely mixed strategy in the action stage after the message profile ($L$, $L$) as well as in the cheap talk phase. The two corresponding indifference conditions determine the specific values of $q_3$ and $r$. Player 1($L$-type) plays $A$ with probability 1 after the message profile ($L$, $L$) (this is incentive compatible only for large enough $p$ so that the probability of this message coming from a player 2($L$-type) is low because player 2($L$-type) will be playing $B$ with probability $q_5 = 1$ and the probability of player 2($H$-type) also playing $A$ after this message profile is higher because $q_3$ is a decreasing function of $p$). Finally, given this strategy profile, it can be checked that player 1 ($L$-type) receives a greater expected payoff by revealing his type truthfully.

### 3.4 Coordination

$S_{separating}$ features a specific form of coordination: when the players' types are different, they fully coordinate on the ex-post efficient outcome. We call this property "type-coordination".

**Definition 6** A strategy profile is said to have the type-coordination property if the induced outcome is ($A$,$A$) and ($B$,$B$), when the players' true type profile is ($L$,$H$) and ($H$,$L$), respectively.

Clearly, the type-coordination property can be achieved in other kinds of equilibria. Indeed, the Bayesian Nash equilibrium of the game without the cheap talk (as mentioned in Proposition 1) also satisfies type-coordination property when the prior $p$ is between $\frac{L}{1+L}$ and $\frac{H}{1+H}$. One may wonder whether at all type-coordination can be obtained in any of the partially revealing equilibria, as described in Proposition 2. Following Proposition 2, note that for the type-coordination property to hold, we need profiles satisfying $q_1 = 0$, $q_3 = 0$ and $q_5 = 1$. Using symmetry, for player 2($H$-type), we then must have $\sigma_2(A|H; L, H) = 1 - q_1 = 1$. This implies that in any such profile, $q_2 = 1$ and $q_4 = 1$. Thus, a candidate partially revealing equilibrium profile with the type-coordination property must have $q_0 = \frac{1}{1+H}$, $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$ and $q_5 = 1$. We now state our second main result. The proof has been postponed to the Appendix.

**Theorem 2** *In a partially revealing symmetric cheap talk equilibrium, in which only the L-type is truthful, that satisfies the type-coordination property, r must be* $\frac{H+H^2}{1+H+H^2}$; *this equilibrium exists only when* $\frac{L+LH+LH^2}{1+L+LH+LH^2} < p < \frac{H+H^2+H^3}{1+H+H^2+H^3}$.

Since $\frac{H+H^2+H^3}{1+H+H^2+H^3} - \frac{L+LH+LH^2}{1+L+LH+LH^2} > 0$, the above gives us a meaningful range for $p$.

Note that the profile in Theorem 2 above is the same as that given in (*iv*) in Proposition 3. Let the equilibrium profile stated in Theorem 2 be called $S_{pooling}$.

To understand why $p$ must lie within such a range for $S_{pooling}$ to be an equilibrium (as stated in Theorem 2), consider the incentives for deviations by player 1 at the action stage. According to the above strategy profile, on receiving the message profile ($L$, $L$), player 1($H$ -type) needs to play $B$. Given that player 2($H$-type) plays $A$ and player 2($L$-type) plays $B$, player 1($H$-type) will indeed play $B$ only if he believes that player 2 is more likely to be an $L$-type than an $H$-type. This means that player 1 ($H$-type)'s posterior belief about player 2 being an $H$-type should not be too high. If we denote this posterior belief by $p\prime$, then $p\prime = P(t_i = H|\tau_i = L) = \frac{p-rp}{1-rp}$. Since this posterior $p\prime$ is an increasing function of the prior $p$ ($\frac{\partial}{\partial p}(\frac{p-rp}{1-rp}) = \frac{(1-r)}{(1-rp)^2} > 0$), the constraint that $p\prime$ should not be too high implies that the prior $p$ cannot be very high either.[7] Hence, there is an upper bound for $p$ that is strictly less than 1. Similarly, according to the above strategy profile, after receiving the message profile ($L$, $L$), player 1($L$-type) needs to play $A$. Again, given that player 2($H$-type) plays $A$ and

[7] If $p$ were to be equal to 1, i.e., player 2 were certainly an $H$-type, player 1($H$-type) would then definitely have preferred playing $A$, not $B$.

player 2($L$-type) plays $B$, player 1($L$-type) will play $A$ only if the posterior $p\prime$ is not too small which explains the lower bound on $p$.

The following couple of claims illustrate some features of the equilibrium $S_{pooling}$. These two claims are easy to establish and hence we have omitted the formal proofs for them.

**Claim 3** *The ex-ante expected payoff for any player from the equilibrium $S_{pooling}$ is given by $EU_{pooling} = \frac{p(1+H)(1+H+H^2-p-H^2p)}{1+H+H^2}$.*

**Claim 4** *The upper bound of the range for p in Theorem2 does not involve L, is increasing in H and is bounded by $\frac{3}{4}$.*

Our cheap talk equilibria, $S_{separating}$ and $S_{pooling}$, both satisfy the type-coordination property. We end this subsection with the following observation.

**Claim 5** *For a fixed value of H and L, the lower bound for p for $S_{pooling}$ to exist is bigger than the upper bound for p for $S_{separating}$ to exist.*

Two different equilibria, $S_{separating}$ and $S_{pooling}$, both with the type-coordination property, exist for distinct values of $p$. For $\frac{L+LH+LH^2}{1+L+LH+LH^2} < p < \frac{H+H^2+H^3}{1+H+H^2+H^3}$, when $S_{pooling}$ exists as an equilibrium, $S_{separating}$ is not an equilibrium because the $H$-type does not want to truthfully reveal his information. Allowing the $H$-type to reveal his information partially in the cheap talk stage helps sustain the partially revealing equilibrium.

# 4 Further results

## 4.1 Cheap talk vs. Bayesian Nash equilibria

As noted earlier, it is possible to achieve type-coordination in the unique symmetric Bayesian Nash equilibrium (BNE) of the BoS (without the cheap talk stage) itself when $\frac{L}{1+L} \leq p \leq \frac{H}{1+H}$ (see Proposition 1). It is also conceivable that for some parameter values of $L$ and $H$, the ranges of $p$ where $S_{separating}$ and $S_{pooling}$ respectively exist, do separately overlap with the interval $[\frac{L}{1+L}, \frac{H}{1+H}]$. The range of $p$ where $S_{separating}$ exists, can only overlap with the low and middle ranges of $p$ in the description of the BNE (as in Proposition 1), i.e., the intervals $[0, \frac{L}{1+L})$ and $[\frac{L}{1+L}, \frac{H}{1+H}]$. This is because $\frac{H}{1+H} - \frac{LH+LH^2}{1+L+LH+LH^2} = \frac{H(1-HL)}{(1+H)(1+L+LH+LH^2)} > 0$ and $\frac{L}{1+L} - \frac{L^2+L^2H}{1+L+L^2+L^2H} = \frac{L(1-HL)}{(1+L)(1+L+L^2+L^2H)} > 0$. Next, we note that the range of $p$ where $S_{pooling}$ exists, can only overlap with the middle and high ranges of $p$ in the description of the BNE (as in Proposition 1), i.e., the intervals $[\frac{L}{1+L}, \frac{H}{1+H}]$ and $(\frac{H}{1+H}, 1]$. This is because $\frac{L+LH+LH^2}{1+L+LH+LH^2} - \frac{L}{1+L} = \frac{HL(1+H)}{(1+L)(1+L+LH+LH^2)} > 0$ and $\frac{H+H^2+H^3}{1+H+H^2+H^3} - \frac{H}{1+H} = \frac{H^2}{1+H+H^2+H^3} > 0$.

This raises the obvious question of how these different equilibria compare in terms of expected utilities for the players. Do the equilibria from the game with cheap talk, i.e., $S_{separating}$ and $S_{pooling}$, always outperform the BNE without cheap talk when they simultaneously exist? The answer is yes!

**Proposition 4** *The expected payoff for each player from $S_{separating}$ ($S_{pooling}$) is greater than that from the Bayesian Nash equilibrium of the game without cheap talk when the Bayesian Nash equilibrium and $S_{separating}$ ($S_{pooling}$) simultaneously exist.*

The proof is in the Appendix.

### 4.2 One-sided cheap talk

One-sided cheap talk with two-sided private information has also been studied in the literature (see, for example, Seidmann (1990) and more recently, Moreno de Barreda (2012)). One thus may be interested to know whether the properties of truthfulness and type-coordination of the two-sided cheap talk equilibria can be achieved with one-sided cheap talk in our game, when only one player (say, player 1) talks. To do so, we assume that player 1 chooses a costless and nonbinding announcement $\tau_1$ from the set $\mathcal{T}_1 = \{L, H\}$.We now write $\sigma_i(A|t_i;\tau_1)$ for the probability that strategy $\sigma_i(t_i;\tau_1)$ of player $i$ with type $t_i$ assigns to the action $A$ when the first stage announcement by player 1 is $\tau_1$.

We consider two specific strategy profiles which we believe are closest to the two equilibrium strategy profiles studied earlier (in this paper) with two-sided cheap talk and we show that these strategy profiles are no longer equilibrium profiles. The first strategy profile we analyse concerns the situation where player 1 reveals his information truthfully. Consider the following strategy profile: in the cheap talk stage, player 1 reports his type truthfully, i.e., $a_1(H|H) = 1$ and $a_1(H|L) = 0$; in the action stage, player 1's strategy consists of any $0 \leq \sigma_1(A|H;H) \leq 1$ and $0 \leq \sigma_1(A|L;L) \leq 1$ and player 2's strategy is given by $\sigma_2(A|H;H) = \frac{H}{1+H}$, $\sigma_2(A|H;L) = 1$, $\sigma_2(A|L;H) = 0$ and $\sigma_2(A|L;L) = \frac{L}{1+L}$. Call this strategy $S^{onesided}_{separating}$. It is easy to prove that $S^{onesided}_{separating}$ is not an equilibrium in the game with one-sided cheap talk where only player 1 talks. To see this note that, in the action stage, player 1($H$-type)'s expected payoff from playing $A$ is $p\frac{H}{1+H}$ whereas his expected payoff from playing $B$ is $p\frac{H}{1+H} + (1-p)H$; hence, $\sigma_1(A|H;H) = 0$. But this implies that player 2($H$-type) should play the pure strategy $B$ after player 1($H$ -type) talks, i.e., $\sigma_2(A|H;H) = 0$; therefore, $S^{onesided}_{separating}$ cannot be an equilibrium.

The second strategy profile concerns the situation where player 1($H$ -type) partially reveals his information, while player 1($L$-type) announces $L$ truthfully. Formally, consider the following strategy profile: in the cheap talk stage, player 1 reveals his type partially, i.e., $a_1(H|H) = r$ and $a_1(H|L) = 0$; in the action stage, player 1's strategy consists of any $0 \leq \sigma_1(A|H;H) \leq 1$, $0 \leq \sigma_1(A|H;L) \leq 1$ and $0 \leq \sigma_1(A|L;L) \leq 1$ and player 2's strategy is given by $\sigma_2(A|H;H) = \frac{H}{1+H}$, $\sigma_2(A|H;L) = 1$, $\sigma_2(A|L;H) = 0$, $\sigma_2(A|L;L) = \frac{L}{1+L}$. Call this strategy $S^{onesided}_{pooling}$. Following the same logic as in the case of $S^{onesided}_{separating}$, one can also show that $S^{onesided}_{pooling}$ is not an equilibrium.

We are now going to show a more general result, namely, that any strategy profile involving truthful revelation in the cheap talk stage does not lead to any meaningful equilibrium. To show this, we thus focus our attention only on equilibria where at least some of the actions in the second stage depend on the announcement from the first stage in a non-trivial manner.

**Definition 7** A strategy profile in the game with one-sided cheap talk (by player 1) is called responsive if at least one of the following holds:

(i) $\sigma_1(A|H;H) \neq \sigma_1(A|L;L)$;
(ii) $\sigma_2(A|H;H) \neq \sigma_2(A|H;L)$;
(iii) $\sigma_2(A|L;H) \neq \sigma_2(A|L;L)$.

The following theorem confirms that truthfully revealed messages followed by actions that depend meaningfully on the messages are no longer equilibrium profiles when only one player (player 1) talks.

**Theorem 3** *With one-sided cheap talk where only player* 1 *talks*, *there does not exist an equilibrium with a responsive strategy profile where player* 1 *reports his type truthfully in the cheap talk stage*, *i.e.*, $a_1(H|H) = 1$ *and* $a_1(H|L) = 0$.

The proof of Theorem 3 has been postponed to the Appendix.

### 4.3 Sequential cheap talk

Critics may suggest that simultaneous cheap talk is meaningless unless there is a mediator. We, however, disagree; as evidence, we would like to mention that a significant part of related cheap talk literature studies unmediated simultaneous cheap talk (e.g. Matthews and Postlewaite 1989; Gilligan and Krehbiel 1989; Baliga and Sjöström 2004). Of course, it is an interesting issue to check whether full revelation of information is possible when the players use cheap talk messages sequentially.

Suppose that in the announcement stage, player 1 talks first and then, having heard player 1's message, player 2 makes an announcement; after these cheap talk announcements from the two players, they choose an action from the set $S_i$ in the BoS as before. In the first (cheap talk) stage, each player $i$ again chooses an announcement $\tau_i$ from the set $\mathcal{T}_i = \{L, H\}$, taking into account the sequential nature of the cheap talk: an announcement strategy for player 1 is a function $a_1 : T_1 \rightarrow \Delta(\mathcal{T}_1)$, where $\Delta(\mathcal{T}_1)$ is the set of probability distributions over $\mathcal{T}_1$, however, an announcement strategy for player 2 is a function $a_2 : T_2 \times \mathcal{T}_1 \rightarrow \Delta(\mathcal{T}_2)$, where $\Delta(\mathcal{T}_2)$ is the set of probability distributions over $\mathcal{T}_2$. So, for example, $a_2(H|\tau_1, t_2)$ stands for the probability that strategy $a_2(\tau_1, t_2)$ of player 2 with type $t_2$, after hearing message $\tau_1$ from player 1, assigns to the announcement $H$. After the two announcements from the two players, the action stage is played in the same manner as with simultaneous cheap talk.

We consider a specific class of strategies in which the cheap talk announcements are indeed fully revealing. Note that we cannot impose any symmetry assumption anyway in this case because the sequential announcements make the two players' circumstances inherently asymmetric.

**Definition 8** A strategy profile $((a_1, \sigma_1), (a_2, \sigma_2))$ is called fully revealing if the announcement strategies $a_i$ reveal the true types with certainty, i.e., $a_1(H|H) = 1$ and $a_1(H|L) = 0$; $a_2(H|H,H) = a_2(H|L,H) = 1$ and $a_2(H|H,L) = a_2(H|L,L) = 0$.

We again (as in the previous subsection related to one-sided talk) focus our attention only on equilibria where at least some of the actions in the second (action) stage depend on the announcements from the first stage in a non-trivial manner.

**Definition 9** A strategy profile in the game with sequential cheap talk is called responsive if at least one of the following holds:

(i) $\sigma_1(A|H,H), \sigma_1(A|H,L), \sigma_1(A|L,H), \sigma_1(A|L;L)$ are not all equal;
(ii) $\sigma_2(A|H,H), \sigma_2(A|H,L), \sigma_2(A|L,H), \sigma_2(A|L;L)$ are not all equal.

The following result (Theorem 4) shows that, similar to the one-sided cheap talk scenario, the players will never want to truthfully reveal their information if they talk sequentially.

**Theorem 4** *With sequential cheap talk about their types, where player* 1 *talks first and then, having heard player* 1*'s message, player* 2 *talks, there does not exist an equilibrium with a responsive strategy profile where both players reveal their types truthfully in the cheap talk stage.*

The proof of Theorem 4 is in the Appendix.

This result shows that, similar to one-sided cheap talk, the players will never want to truthfully reveal their information if they talk sequentially. Although Theorem 4 appears to be similar to Theorem 3, the reasoning behind it is quite different: with sequential cheap talk, firstly, player 2 has potential deviations available in the cheap talk stage after player 1 's message and secondly, in the action stage, both players can condition their actions on a pair of messages instead of one message; these give rise to strategies and deviations which are impossible with one-sided cheap talk.

In the context of our model of the BoS with incomplete information, Theorem 4 can be interpreted as indicating why full revelation of information might be difficult in the real world because of the sequential nature of most communication. Theorem 1, on the other hand, shows the theoretical possibility of overcoming this problem to some extent if the players are willing to commit to simultaneous exchange of messages.

### 4.4 Cheap talk with more messages

Finally, we consider the implications of the players using more messages at the cheap talk stage. Let each player $i$ choose an announcement $\tau_i'$ from the set $\mathcal{T}_i' = \{L, H\} \times \{X, Y\}$. An announcement strategy in the first (cheap talk) stage for player $i$ now is a function $a_i : T_i \to \Delta(\mathcal{T}_i')$, where $\Delta(\mathcal{T}_i')$ is the set of probability distributions over $\mathcal{T}_i'$. So, for example, $a_i(HX|t_i)$ stands for the probability that strategy $a_i(t_i)$ of player $i$ with type $t_i$ assigns to the announcement $HX$. Beliefs for player $i$ are now based on the new expanded message spaces. In the second (action) stage, a strategy for player $i$ is a function $\sigma_i : T_i \times \mathcal{T}'_1 \times \mathcal{T}'_2 \to \Delta(S_i)$, where $\Delta(S_i)$ is the set of probability distributions over $S_i$. We again restrict our attention to symmetric strategy profiles, with the message spaces appropriately adjusted.

**Definition 10** A symmetric strategy profile $((a_1, \sigma_1), (a_2, \sigma_2))$ is called fully revealing if the announcement strategy $a_i$ reveals the true types with certainty, i.e., $a_i(HX|H) + a_i(HY|H) = 1$ and $a_i(HX|L) = a_i(HY|L) = 0$.

We consider a symmetric fully revealing announcement strategy profile in which the $H$-type of player $i$ announces $HX$ with probability $r_1$ and $HY$ with probability $(1 - r_1)$ and the $L$-type of player $i$ announces $LX$ with probability $r_2$ and $LY$ with probability $(1 - r_2)$, i.e., $a_i(HX|H) = r_1$ and $a_i(LX|L) = r_2$.

The aim in studying such profiles clearly should be to improve upon $S_{separating}$; we thus would like to keep the desirable type-coordination property when the types are different. For the type-coordination property to hold, we therefore restrict our attention to action-strategies where $\sigma_1(A|H; HX, LX) = \sigma_1(A|H; HX, LY) = \sigma_1(A|H; HY, LX) = \sigma_1(A|H; HY, LY) = 0$. This implies, again by symmetry, that $\sigma_1(A|L; LX, HX) = \sigma_1(A|L; LX, HY) = \sigma_1(A|L; LY, HX) = \sigma_1(A|L; LY, HY) = 1$. Also, note that in equilibrium, in the second (action) stage, by symmetry, an action-strategy of player 1 must have: $\sigma_1(A|H; HX, HX) = \sigma_1(A|H; HY, HY) = \frac{1}{1+H}$, $\sigma_1(A|L; LX, LX) = \sigma_1(A|L; LY, LY) = \frac{1}{1+L}$.

It is worth noting that the above partial specification of part of the players' strategies mimics $S_{separating}$. Let the class of strategy profiles satisfying the above partial specification be denoted by $S'_{separating}$. The following result shows that new and distinct equilibria emerge by virtue of the richer message spaces used by the players.

**Theorem 5** *Within the class of strategy profiles described by $S'_{separating}$, the following four profiles constitute fully revealing symmetric cheap talk equilibria*:

(i) $\sigma_1(A|H; HX, HY) = \sigma_1(A|L; LX, LY) = 0$, $r_1 = \frac{H^2}{1+H^2}$, $r_2 = \frac{L^2}{1+L^2}$;
(ii) $\sigma_1(A|H; HX, HY) = \sigma_1(A|L; LX, LY) = 1$, $r_1 = \frac{1}{1+H^2}$, $r_2 = \frac{1}{1+L^2}$;
(iii) $\sigma_1(A|H; HX, HY) = 0$, $\sigma_1(A|L; LX, LY) = 1$, $r_1 = \frac{H^2}{1+H^2}$, $r_2 = \frac{1}{1+L^2}$;
(iv) $\sigma_1(A|H; HX, HY) = 1$, $\sigma_1(A|L; LX, LY) = 0$, $r_1 = \frac{1}{1+H^2}$, $r_2 = \frac{L^2}{1+L^2}$.

All of the above equilibria exist when

$$\begin{aligned} \frac{L^4(1 + H + H^2 + H^3)}{1 + L + L^2 + L^3 + L^4 + HL^4 + H^2L^4 + H^3L^4} &\le p \\ &\le \frac{HL^3(1 + H + H^2 + H^3)}{1 + L + L^2 + L^3 + HL^3 + H^2L^3 + H^3L^3 + H^4L^3}. \end{aligned}$$

In the Appendix, we provide the proof of $(i)$. The other three cases are very similar and the proofs are therefore omitted. Note that in $(iv)$, we have a legitimate interval for $p$, as

$$\begin{aligned}&\frac{HL^3(1+H+H^2+H^3)}{1+L+L^2+L^3+HL^3+H^2L^3+H^3L^3+H^4L^3}\\&-\frac{L^4(1+H+H^2+H^3)}{1+L+L^2+L^3+L^4+HL^4+H^2L^4+H^3L^4}\\&=\frac{L^3(H-L)(1+H+H^2+H^3)(1+L+L^2+L^3)}{(1+L+L^2+L^3+HL^3+H^2L^3+H^3L^3+H^4L^3)(1+L+L^2+L^3+L^4+HL^4+H^2L^4+H^3L^4)}\\&>0.\end{aligned}$$

Note that in each of the four profiles in Theorem 5, players are able to coordinate in the action stage on one of the pure equilibrium outcomes of the BoS, even when their types are the same by using different messages ($X$ or $Y$).

We now compare the upper and lower bounds for $p$ for the above equilibrium ( $S'_{separating}$) and that of $S_{separating}$. Observe that:

$$\begin{aligned}&\frac{HL^3(1+H+H^2+H^3)}{1+L+L^2+L^3+HL^3+H^2L^3+H^3L^3+H^4L^3}-\frac{L^2+L^2H}{1+L+L^2+L^2H}\\&\quad=\frac{L^2(H+1)(L+1)(H^3L+HL-L^2-1)}{(1+L+L^2+L^3+HL^3+H^2L^3+H^3L^3+H^4L^3)(1+L+L^2+L^2H)}<0,\end{aligned}$$

because, $H^3L+HL-L^2-1<L+L-L^2-1=-(1-L)^2<0$. This shows that the two intervals of $p$ for the two equilibria ($S_{separating}$ and $S'_{separating}$) to exist are non-overlapping and the range for $p$ for $S'_{separating}$ is strictly lower than the range for $p$ for $S_{separating}$.

## 5 Conclusion

In this paper, we have analysed a simple 2 x 2 x 2 Bayesian game and studied the possibility of information revelation and desirable coordination using one round of direct cheap talk. The main takeaway of our paper is that the desirable type-coordination is achieved at the unique fully revealing equilibrium (when it exists); moreover, such a coordination may also be achievable with partial revelation when fully revealing cheap talk equilibrium does not exist.

Our paper demonstrates that there are equilibria where information revelation aids coordination to a certain extent and hence improves on the symmetric Bayes-Nash equilibrium of the BoS game without cheap talk. However, in each state of nature, perfect coordination on the pure strategy Nash equilibria of the corresponding complete information BoS is not possible in our set up. The reason is that some degree of miscoordination is needed to provide the appropriate incentives for truthful revelation in the cheap talk phase. Coordination can be further improved if information revelation and symmetry are not required in the cheap talk stage.

Banks and Calvert (1992) characterised the (ex-ante) efficient symmetric incentive compatible direct mechanism for a similar game so that the players are truthful and obedient to the mechanism (mediator). Following Banks and Calvert (1992), Ganguly and Ray (2009) analysed a (direct) symmetric mediated equilibrium that provides the players with incentives (i) to truthfully reveal their types to the mediator and (ii) to follow the mediator's recommendations following their type-announcements. Clearly, our cheap talk equilibria can be achieved as outcomes of such mediated equilibria using incentive compatible mechanisms (see, Ganguly and Ray (2009) for formal comparisons).

One criticism of our model of two-sided cheap talk is that most of our main results are based on the assumption that the communication of messages by the players is simultaneous whereas most real life conversations tend to be sequential. However, we would like to point out that most of the literature on two-sided cheap talk have modelled communication as simultaneous. This includes Farrell (1987), Farrell and Gibbons (1989), Matthews and Postlewaite (1989), Baliga and Sjöström (2004) and Ben–Porath (2003). In our set-up, fully revealing equilibria do not exist under sequential communication; a cheap talk game with sequential communication is incapable of yielding any information about types. We have also proved that truthfully revealed messages followed by actions that depend meaningfully on these messages are not equilibrium profiles in the one-sided cheap talk game.

One may raise concerns about the robustness of the existence of type-coordinating fully revealing equilibria when richer type spaces are considered. We have shown (Theorem 5) that enriching the message space enables fully revealing equilibria to exist for values of $p$ that were not possible otherwise; however, we do agree that extending our results to even richer type spaces is indeed an important issue which we intend to address in the future.

It seems challenging to fully characterise the set of symmetric equilibria and to assess whether a trade-off between information revelation and payoffs exists in our set-up. Our results could have been much more compelling had we provided a full characterisation of the set of symmetric equilibria and selected an equilibrium among them using some reasonable criterion (e.g. Pareto dominance). One could have asked whether we lose something by selecting fully revealing equilibria out of the set of all symmetric equilibria. We intend to study these questions in future.

Following Banks and Calvert (1992), one may also be interested in characterising the ex-ante efficient cheap talk equilibrium in our set up. Many other interesting open questions come out of our analysis. For example, one may ask whether non-babbling or responsive cheap talk equilibria always exist in our game for any given $p$ or not. We also do not characterise the general case where neither type reveals truthfully in the cheap talk phase. We postpone all these issues for future research.

## Appendix

We collect the proofs of our results in this section.

***Proof of Theorem 1*** As the strategies are symmetric, it is sufficient to characterise these candidate profiles only by $\sigma_1[A|H,L]$. There are only three possible candidates for $\sigma_1[A|H,L]$ as the complete information BoS with values $H$ and $L$ has three (two pure and one mixed) Nash equilibria. These profiles are (i) $\sigma_1[A|H,L] = \sigma^1(HL)$ where $\sigma^1(HL)$ is the probability of playing $A$ in the mixed Nash equilibrium strategy of player 1 of the complete information BoS with values $t_1 = H$ and $t_2 = L$, that we call $S_m$; (ii) $\sigma_1[A|H,L] = 1$, that we call $S_{ineff}$ and (iii)$\sigma_1[A|H,L] = 0$, which indeed is $S_{separating}$.
We first show that $S_m$ is not an equilibrium. Under $S_m$, $H$-type will announce his type truthfully only if $p(\frac{H}{1+H}) + (1-p)(\frac{H}{1+H}) \geq p(\frac{H}{1+L}) + (1-p)(\frac{H}{1+L})$, where the LHS is the expected payoff from truthfully announcing $H$ and the RHS is the expected payoff from announcing $L$ and choosing the corresponding optimal action strategy. This inequality implies $\frac{1}{1+H} \geq \frac{1}{1+L}$ which can never be satisfied as $H > L$.
The second candidate strategy profile, $S_{ineff}$ is an equilibrium only when $\frac{1+H}{1+L+HL^2+L^2} \leq p$ and $p \leq \frac{1+L+HL-H^2}{1+L+HL+H^2L}$. To see this, note that under $S_{ineff}$, $H$-type will announce his type truthfully only if $p(\frac{H}{1+H}) + (1-p) \geq pH + (1-p)(\frac{H}{1+L})$ which implies $p \leq \frac{1+L+HL-H^2}{1+L+HL+H^2L}$. Similarly, $L$-type will announce his type truthfully only if $pL + (1-p)(\frac{L}{1+L}) \geq p(\frac{H}{1+H}) + (1-p)$ which implies $\frac{1+H}{1+L+HL^2+L^2} \leq p$. However, it can be shown that $\frac{1+H}{1+L+HL^2+L^2} > \frac{1+L+HL-H^2}{1+L+HL+H^2L}$. Hence, $S_{ineff}$ cannot be an equilibrium.
Finally, we prove that $S_{separating}$ is an equilibrium only when $\frac{HL^2+L^2}{1+L+HL^2+L^2} \leq p \leq \frac{HL+H^2L}{1+L+HL+H^2L}$. Under $S_{separating}$, $H$-type will announce his type truthfully only if $p(\frac{H}{1+H}) + (1-p)H \geq p + (1-p)(\frac{H}{1+L})$ which implies $p \leq \frac{HL+H^2L}{1+L+HL+H^2L}$. Similarly, $L$-type will announce his type truthfully only if $p + (1-p)(\frac{L}{1+L}) \geq p(\frac{H}{1+H}) + (1-p)L$ which implies $\frac{HL^2+L^2}{1+L+HL^2+L^2} \leq p$. □

***Proof of Lemma 2*** *(i)* Given these strategies at the action stage of the game, we check that truthful revelation is incentive compatible at the cheap talk phase for each type of each player. This gives us the following four constraints:

(a) For player 1($H$-type), the expected payoff from announcing $H$ must be greater than or equal to the expected payoff from announcing $L$, which implies: $p + (1-p)H \geq pH + (1-p) \implies p \geq \frac{1}{2}$.
(b) For player 1($L$-type), the expected payoff from announcing $L$ must be greater than or equal to the expected payoff from announcing $H$, which implies: $pL + (1-p) \geq p + (1-p)L \implies p \leq \frac{1}{2}$.
(c) For player 2($H$-type), the expected payoff from announcing $H$ must be greater than or equal to the expected payoff from announcing $L$, which implies: $pH + (1-p) \geq p + (1-p)H \implies p \leq \frac{1}{2}$.

(d) For player 2($L$-type), the expected payoff from announcing $L$ must be greater than or equal to the expected payoff from announcing $H$, which implies: $p + (1-p)L \geq pL + (1-p) \implies p \geq \frac{1}{2}$.

We then note that all the above four constraints are satisfied only when $p = \frac{1}{2}$.

The proof of part *(ii)* is very similar and hence, has been omitted here. □

***Proof of Proposition 2*** We divide the whole set of 81 candidate strategy profiles into the following five subsets according to which of the action profiles among ($A$, $A$), ($B$, $B$) and the mixed strategy equilibrium (of the corresponding complete information BoS) is used in the action stage after each of the four possible message profiles from the cheap talk stage ($(H,H)$, $(H,L)$, $(L,H)$ and $(L,L)$):

(a) All four action profiles are pure (($A$, $A$) or ($B$, $B$)); there are $2^4 = 16$ possible elements in this subset.
(b) One action profile is mixed and the remaining three action profiles are pure (($A$, $A$) or ($B$, $B$)); there are $4 \times 2^3 = 32$ possible elements in this subset.
(c) Two of the action profiles are mixed and the remaining two action profiles are pure (($A$, $A$) or ($B$, $B$)); there are $6 \times 2^2 = 24$ possible elements in this subset.
(d) Three of the action profiles are mixed and the remaining one action profile is pure (($A$, $A$) or ($B$, $B$)); there are $4 \times 2 = 8$ possible elements in this subset.
(e) All four action profiles are mixed; there is only 1 element in this subset.

We systematically check every element of each of these subsets for incentive compatibility at the cheap talk phase (similar to the proof of the Lemma 2 ) and we find deviations for at least one type of one of the players in all these cases except the ones listed in Lemmata 1 and 2 and this proposition. The details of the proof are omitted here and is available on request from the authors. □

***Proof of Proposition 3*** If $q_1, q_2, q_3, q_4$ and $q_5$ correspond to completely mixed strategies in the action stage, then we must have the following five conditions for player 1 to be indifferent between playing $A$ and $B$ (where the LHS in each equation is the expected payoff from $A$ and the RHS in each equation is the expected payoff from $B$ ):

Player 1($H$-type) receiving the message profile $(L,H)$:

$$1 - q_1 = q_1 H, \tag{1}$$

Player 1($H$-type) receiving the message profile $(H,L)$:

$$\begin{aligned} &\left(\frac{p-rp}{1-rp}\right)(1-q_2) + \left(1 - \frac{p-rp}{1-rp}\right)(1-q_4) \\ &= \left(\frac{p-rp}{1-rp}\right)q_2 H + \left(1 - \frac{p-rp}{1-rp}\right)q_4 H, \end{aligned} \tag{2}$$

Player 1($H$-type) receiving the message profile $(L,L)$:

$$\left(\frac{p-rp}{1-rp}\right)(1-q_3)+\left(1-\frac{p-rp}{1-rp}\right)(1-q_5)$$
$$=\left(\frac{p-rp}{1-rp}\right)q_3H+\left(1-\frac{p-rp}{1-rp}\right)q_5H, \tag{3}$$

Player 1($L$-type) receiving the message profile $(L, L)$:

$$\left(\frac{p-rp}{1-rp}\right)(1-q_3)+\left(1-\frac{p-rp}{1-rp}\right)(1-q_5)$$
$$=\left(\frac{p-rp}{1-rp}\right)q_3L+\left(1-\frac{p-rp}{1-rp}\right)q_5L, \tag{4}$$

Player 1($L$-type) receiving the message profile $(L, H)$:

$$1-q_1=q_1L. \tag{5}$$

Also, in the cheap talk phase, player 1($H$-type) should be indifferent between announcing $H$ and $L$, which implies

$$\begin{aligned}&(1-p)(q_1(1-q_4)+(1-q_1)q_4H)\\&\quad+p\left(r\frac{H}{1+H}+(1-r)(q_1(1-q_2)+(1-q_1)q_2H)\right)\\&=p(r(q_2(1-q_1)+(1-q_2)q_1H)+(1-r)(q_3(1-q_3)+(1-q_3)q_3H))\\&\quad+(1-p)(q_3(1-q_5)+(1-q_3)q_5H).\end{aligned} \tag{6}$$

Finally, in the cheap talk phase, it should be incentive compatible for player 1($L$-type) to announce $L$, which implies

$$\begin{aligned}&(1-p)(q_5(1-q_5)(1+L))+p(r(q_4(1-q_1)\\&\quad+(1-q_4)q_1L)+(1-r)(q_5(1-q_3)+(1-q_5)q_3L))\\&\geq \underset{x}{Max}(1-p)(x(1-q_4)+(1-x)q_4L)\\&\quad+p\left(r\frac{H}{1+H}+(1-r)(x(1-q_2)+(1-x)q_2L)\right)\end{aligned} \tag{7}$$

where $x$ is the optimal probability of playing $F$ in the action phase if player 1($L$-type) deviates and announces $H$ and receives the message profile $(H, L)$.

By virtue of symmetry, the conditions for player 2 are identical to the above.

It is obvious that only one of the equations between (1 and 5 can be satisfied. Similarly, both (3) and (4) cannot hold simultaneously. So, at least one of $q_2$ or $q_4$ and at least one of $q_3$ or $q_5$ must correspond to a pure strategy.

Considering Eqs. (1) and (5), one can see that if $0 < q_4 < 1$ and hence (5) holds, then $q_2 = 0$. Similarly, from (1) and (5), if $0 < q_2 < 1$ and hence (1) holds, then $q_4 = 1$. Also, from (3) and (4), if $0 < q_5 < 1$ and hence (4) holds, then $q_3 = 0$ and if $0 < q_3 < 1$ and hence (3) holds, then $q_5 = 1$.

This gives us the following 15 candidate strategy profiles for possible equilibria, split into four cases, as listed below.

*Case 1. One of* $q_2$ *or* $q_4$ *and one of* $q_3$ *or* $q_5$ *correspond to a pure strategy.*

Profile (i): $0<q_1<1$, $q_2=0$, $q_3=0$, $0<q_4<1$ and $0<q_5<1$. This profile is not an equilibrium.

Profile (ii): $0<q_1<1$, $q_2=0$, $0<q_3<1$, $0<q_4<1$ and $q_5=1$. This cannot be an equilibrium.

Profile (iii): $0<q_1<1$, $0<q_2<1$, $q_3=0$, $q_4=1$ and $0<q_5<1$. This profile is not an equilibrium.

Profile (iv): $0<q_1<1$, $0<q_2<1$, $0<q_3<1$, $q_4=1$ and $q_5=1$. This profile constitutes an equilibrium if $p>\frac{H}{1+H}$. The expected utility from this equilibrium (to each player)$=EU_{(iv)}=\frac{H}{1+H}$.

*Case 2. One of* $q_2$ *or* $q_4$ *corresponds to a pure strategy and both* $q_3$ *and* $q_5$ *correspond to pure strategies.*

If $q_3=1$ and $q_5=1$, then player 1($H$-type), after receiving the message profile $(L,L)$, obtains a higher expected payoff from playing $B$ which is a contradiction.

If $q_3=0$ and $q_5=0$, then player 1($H$-type), after receiving the message profile $(L,L)$, obtains a higher expected payoff from playing $A$ which is a contradiction.

If $q_3=1$ and $q_5=0$, then we must have $(1-\frac{p-rp}{1-rp})>(\frac{p-rp}{1-rp})H$ and $(1-\frac{p-rp}{1-rp})<(\frac{p-rp}{1-rp})L$, which is not possible.

So, the only possible candidate is $q_3=0$ and $q_5=1$ which implies that $(\frac{p-rp}{1-rp})<(1-\frac{p-rp}{1-rp})H$ and $(\frac{p-rp}{1-rp})>(1-\frac{p-rp}{1-rp})L$.

Profile (v): $0<q_1<1$, $0<q_2<1$, $q_3=0$, $q_4=1$ and $q_5=1$. This profile cannot be an equilibrium.

Profile (vi): $0<q_1<1$, $q_2=0$, $q_3=0$, $0<q_4<1$ and $q_5=1$. This profile cannot constitute an equilibrium.

*Case 3. Both* $q_2$ *and* $q_4$ *correspond to pure strategies and one of* $q_3$ *or* $q_5$ *corresponds to a pure strategy.*

*Subcase 3a. Suppose* $q_2=q_4=1$.

Under this subcase, $q_1$ must be 0. Hence, the possible candidate profiles under this subcase are as follows.

Profile (vii): $q_1=0$, $q_2=1$, $q_3=0$, $q_4=1$ and $0<q_5<1$. This constitutes an equilibrium if $\frac{LH+LH^2}{1+L+LH+LH^2}<p<\frac{L+LH+LH^2}{1+L+LH+LH^2}$. The expected utility from this equilibrium (to each player)$=EU_{(vii)}=\frac{L+LH+LH^2+Hp-Lp}{1+L+LH+LH^2}$.

Profile (viii): $q_1=0$, $q_2=1$, $0<q_3<1$, $q_4=1$ and $q_5=1$. This profile constitutes an equilibrium if $p>\frac{H+H^2+H^3}{1+H+H^2+H^3}$. The expected utility from this equilibrium (to each player)$=EU_{(viii)}=\frac{H+H^2+H^3}{1+H+H^2+H^3}$.

*Subcase 3b. Suppose* $q_2=q_4=0$.

Under this subcase, $q_1$ must be 1. Hence, the possible candidate profiles under this subcase are as follows.

Profile (ix): $q_1=1$, $q_2=0$, $q_3=0$, $q_4=0$ and $0<q_5<1$. This profile cannot constitute an equilibrium.

Profile (x): $q_1 = 1$, $q_2 = 0$, $0 < q_3 < 1$, $q_4 = 0$ and $q_5 = 1$. This profile cannot constitute an equilibrium.

*Subcase 3c. Suppose* $q_2 = 1$, $q_4 = 0$.

This requires $1 - q_1 < q_1 L$ and $1 - q_1 > q_1 H$, which is not possible.

*Subcase 3d. Suppose* $q_2 = 0$, $q_4 = 1$.

This requires $1 - q_1 < q_1 H$ and $1 - q_1 > q_1 L$, implying the possible candidate profiles under this subcase are as follows:

Profile (xi): $0 < q_1 < 1$, $q_2 = 0$, $q_3 = 0$, $q_4 = 1$ and $0 < q_5 < 1$. This cannot be an equilibrium.

Profile (xii): $0 < q_1 < 1$, $q_2 = 0$, $0 < q_3 < 1$, $q_4 = 1$ and $q_5 = 1$. This cannot be an equilibrium.

*Case 4. Both* $q_2$ *and* $q_4$ *correspond to pure strategies and both* $q_3$ *and* $q_5$ correspond to pure strategies.

Using previous arguments, we must have $q_3 = 0$ and $q_5 = 1$ and so, the following are the only potential candidates for equilibrium.

Profile (xiii): $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$ and $q_5 = 1$.

The above constitutes an equilibrium if $\frac{L+LH+LH^2}{1+L+LH+LH^2} < p < \frac{H+H^2+H^3}{1+H+H^2+H^3}$. The expected utility from this equilibrium (to each player)$= EU_{(xiii)} = \frac{p(1+H)(1+H+H^2-p-H^2p)}{1+H+H^2}$.

Profile (xiv): $q_1 = 1$, $q_2 = 0$, $q_3 = 0$, $q_4 = 0$ and $q_5 = 1$. This cannot constitute an equilibrium.

Profile (xv): $0 < q_1 < 1$, $q_2 = 0$, $q_3 = 0$, $q_4 = 1$ and $q_5 = 1$.

The above constitutes an equilibrium if $\frac{H}{1+H} < p < 1$. The expected utility from this equilibrium (to each player)$= EU_{(xv)} = \frac{H}{1+H}$.

***Proof of Theorem 2*** For the strategy profile given in Theorem 2 (denoted by $S_{pooling}$) to be an equilibrium, we first need to check that $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$ and $q_5 = 1$ are indeed consistent with the equilibrium conditions mentioned in the paper (in Sect. 3.2).

We first note that in the cheap talk phase, player 1($H$-type) needs to be indifferent between announcing $H$ and $L$. With the given values of $q_0 = \frac{1}{1+H}$, $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$ and $q_5 = 1$, the expected payoff from announcing $H$ is $H(1-p) + p\left(H(1-r) + H\frac{r}{H+1}\right)$ while the expected payoff from announcing $L$ is $pr + H(1-p)$ which will be equal when $r = \frac{H+H^2}{1+H+H^2}$.

Now, we observe the following:

if player 1($H$-type) receives the message profile $(H, L)$, then the expected payoff from playing $A$ $(=0)$ is less than the expected payoff from playing $B$ $(=H)$, implying $q_1 = 0$;

if player 1($H$-type) receives the message profile $(L, H)$, then the expected payoff from playing $A$ $(=1)$ is greater than the expected payoff from playing $B$ $(=0)$, implying $q_2 = 1$;

if player 1($H$-type) receives the message profile $(L, L)$, then the expected payoff from playing $A$ $(=\frac{p-rp}{1-rp})$ is less than the expected payoff from playing $B$

$((1-\frac{p-rp}{1-rp})H)$, implying $q_3=0$, only when $p<\frac{H+H^2+H^3}{1+H+H^2+H^3}$, using $r=\frac{H+H^2}{1+H+H^2}$;

if player 1($L$-type) receives the message profile $(L,H)$, then the expected payoff from playing $A$ $(=1)$ is greater than the expected payoff from playing $B$ $(=0)$, implying $q_4=1$;

if player 1($L$-type) receives the message profile $(L,L)$, then the expected payoff from playing $A$ $(=\frac{p-rp}{1-rp})$ is greater than the expected payoff from playing $B$ $((1-\frac{p-rp}{1-rp})L)$, implying $q_5=1$, only when $\frac{L+LH+LH^2}{1+L+LH+LH^2}<p$, using $r=\frac{H+H^2}{1+H+H^2}$.

Finally, in the cheap talk phase, it should be incentive compatible for player 1($L$-type) to announce $L$. This requires $p\geq \underset{x}{Max}\ (1-p)((1-x)L)+p(r\frac{H}{1+H}+(1-r)((1-x)L))$, where $x$ is the optimal probability of playing $A$ in the action phase if player 1($L$-type) deviates and announces $H$ and receives the message profile $(H,L)$. Again, note that the RHS of this last inequality constraint allows player 1($L$-type) to deviate in both stages of the game. The derivative of the RHS of this inequality with respect to $x$ is $L(p-1)+Lp(\frac{H+H^2}{1+H+H^2}-1)<0$, which implies $x=0$ and in turn shows that this condition is satisfied (LHS $=p\geq$ RHS $=\frac{(L+LH+LH^2+H^2p-LHp-LH^2p)}{1+H+H^2})$ only when $p\geq\frac{L+LH+LH^2}{1+H+LH+LH^2}$. Hence, the proof. □

***Proof of Proposition 4*** First, consider the case $p<\frac{L}{1+L}$. In this case, when the players' type profiles are ($H$, $H$), ($H$, $L$) or ($L$, $H$), the payoff to each player from $S_{separating}$ is obviously greater than that from the BNE because there is more miscoordination in the BNE in each of these states. When the type profile is ($L$, $L$), the payoff to each player from $S_{separating}$ is $\frac{L}{1+L}$ whereas the payoff from the BNE is $\left(\frac{1}{(1-p)(1+L)}\right)\left(1-\frac{1}{(1-p)(1+L)}\right)(1+L)$. Since $\frac{L}{1+L}-\left(\frac{1}{(1-p)(1+L)}\right)\left(1-\frac{1}{(1-p)(1+L)}\right)(1+L)=\frac{p(Lp-L+1)}{(1+L)(1-p)^2}>0$, this proves that the expected utility from $S_{separating}$ ($EU_{separating}$) is greater than the expected utility from the BNE ($EU_{BNE}$).

Now, let $\frac{L}{1+L}\leq p\leq\frac{H}{1+H}$. Here, the structure of the BNE is such that the players fully miscoordinate when their true type profile is ($H$, $H$) or ($L$, $L$). This contrasts with $S_{separating}$ where the players achieve some degree of coordination in both the states ($H$, $H$) and ($L$, $L$). Since both $S_{separating}$ and the BNE achieve type-coordination, clearly $EU_{separating}$ is greater than $EU_{BNE}$.

Let us now consider $S_{pooling}$ and the BNE. In $S_{pooling}$, the players manage to coordinate with positive probability when the state is ($H$, $H$) but there is complete miscoordination when the true type profile is ($L$, $L$). Also, although there is type-coordination when the type profiles are ($H$, $L$) and ($L$, $H$), the probabilities of these are different from that of the BNE because of partial revelation at the cheap talk stage.

When $\frac{L}{1+L}\leq p\leq\frac{H}{1+H}$, $EU_{BNE}=p(1-p)(1+H)$. Since $EU_{pooling}=\frac{p(1+H)(1+H+H^2-p-H^2p)}{1+H+H^2}$ and $\frac{p(1+H)(1+H+H^2-p-H^2p)}{1+H+H^2}-p(1-p)(1+H)=\frac{p^2H(1+H)}{1+H+H^2}>0$,

$EU_{pooling}$ is greater than $EU_{BNE}$.

Finally, assume that $p > \frac{H}{1+H}$. Then,

$$\begin{aligned} EU_{BNE} =& p^2\left(\left(\frac{H}{p(1+H)}\right)\left(1-\frac{H}{p(1+H)}\right)(1+H)\right) \\ &+p(1-p)\left(\frac{H}{p(1+H)}\right)(1+H)=\frac{H}{1+H}. \end{aligned}$$

So, in this case,

$$\begin{aligned} &EU_{pooling} - EU_{BNE} = \\ =&\frac{(p+3Hp+4H^2p+3H^3p+H^4p-H^3-H^2-H-p^2-2Hp^2-2H^2p^2-2H^3p^2-H^4p^2)}{(1+H+H^2)(1+H)}. \end{aligned}$$

At $p=\frac{H}{1+H}$, it is easy to check that $EU_{pooling} - EU_{BNE} = H^3 > 0$.

Note that,

$$\frac{\partial}{\partial p}\left(EU_{pooling} - EU_{BNE}\right) = \frac{(1+H)(1+H+H^2-2p-2H^2p)}{1+H+H^2}$$

and

$$\frac{\partial^2}{\partial p^2}\left(EU_{pooling} - EU_{BNE}\right) = \frac{(1+H)(-2-2H^2)}{1+H+H^2} < 0.$$

So, $\left(EU_{pooling} - EU_{BNE}\right)$ is strictly concave in $p$ with a maximum value at $p=\frac{1+H+H^2}{2+2H^2}$,which is greater than the upper bound of the range of $p$ for which $S_{pooling}$ exists (because $\frac{1+H+H^2}{2+2H^2}-\frac{H+H^2+H^3}{1+H+H^2+H^3}=\frac{1}{2}\frac{1-H^3}{1+H+H^2+H^3}>0$). This shows that $\left(EU_{pooling} - EU_{BNE}\right)$ is strictly increasing in the interval $(\frac{H}{1+H}, \frac{H+H^2+H^3}{1+H+H^2+H^3})$. Given that $EU_{pooling} - EU_{BNE} > 0$ at $p=\frac{H}{1+H}$, this proves that $EU_{pooling} > EU_{BNE}$ in the interval $(\frac{H}{1+H}, \frac{H+H^2+H^3}{1+H+H^2+H^3})$. Hence, both $EU_{separating}$ and $EU_{pooling}$ are strictly greater than $EU_{BNE}$, whenever these simultaneously exist, confirming that cheap talk is strictly beneficial to both players. □

***Proof of Theorem 3*** Let us first denote the action stage strategies by $\sigma_1(A|H;H)=s_1$, $\sigma_1(A|L;L)=s_2$, $\sigma_2(A|H;H)=s_3$, $\sigma_2(A|H;L)=s_4$, $\sigma_2(A|L;H)=s_5$, $\sigma_2(A|L;L)=s_6$. We divide all potential action strategy profiles into the following subcategories and prove that none of these can be an equilibrium.

*Case (i)*: Both $s_1$ and $s_2$ are pure strategies:

If $s_1 = s_2$, then this would imply that $s_3 = s_4 = s_5 = s_6$. This would then be a babbling strategy profile.

If $s_1 \neq s_2$, then it must be that $s_1 = s_3 = s_5$ and $s_2 = s_4 = s_6$. It is easy to check that in the first stage, one of the types for player 1 will deviate. For example, if $s_1 = 1$ and $s_2 = 0$, then player 1 ($L$-type) will want to announce that he is a $H$-type.

*Case (ii)*: Both $s_1$ and $s_2$ are completely mixed strategies:

For player 1($H$-type) to be indifferent between playing $A$ and $B$ (where the LHS of the equation is the expected payoff from $A$ and the RHS of the equation is the expected payoff from $B$), we must have:

$$ps_3 + (1-p)s_5 = p(1-s_3)H + (1-p)(1-s_5)H \tag{8}$$

For player 1($L$-type) to be indifferent between playing $A$ and $B$, we must have:

$$ps_4 + (1-p)s_6 = p(1-s_4)L + (1-p)(1-s_6)L \tag{9}$$

Also, in the cheap talk phase, it should be incentive compatible for player 1($H$-type) to announce $H$, which implies

$$\begin{aligned} &p[s_1s_3 + (1-s_1)(1-s_3)H] + (1-p)[s_1s_5 + (1-s_1)(1-s_5)H] \\ &\quad \geq \underset{s}{Max}\ p[ss_4 + (1-s)(1-s_4)H] + (1-p)[ss_6 + (1-s)(1-s_6)H] \end{aligned} \tag{10}$$

where $s$ is the optimal probability of playing $A$ in the action phase if player 1($H$-type) deviates and announces $L$.

Using (8), the LHS of (10) $= p(1-s_3)H + (1-p)(1-s_5)H$. Note that the derivative of the RHS of (10) with respect to $s$ is $ps_4 + (1-p)s_6 - p(1-s_4)H - (1-p)(1-s_6)H < 0$, which implies that $s = 0$. So, the RHS of 10 $= p(1-s_4)H + (1-p)(1-s_6)H$.

(10) $\Longrightarrow p(1-s_3) + (1-p)(1-s_5) \geq p(1-s_4) + (1-p)(1-s_6)$ which implies that $p(1-s_3)H + (1-p)(1-s_5)H > p(1-s_4)L + (1-p)(1-s_6)L$ and $ps_3 + (1-p)s_5 \leq ps_4 + (1-p)s_6$. This contradicts (8) and (9).

*Case (iii)*: $s_1$ is a completely mixed strategy and $s_2$ is a pure strategy:

If $s_2 = 1$, then $s_4 = s_6 = 1$. Then,(10) $\Longrightarrow ps_3 + (1-p)s_5 \geq 1$ which can be satisfied only if $s_3 = s_5 = 1$ but that would mean that $s_1 = 1$ which is a contradiction.

If $s_2 = 0$, then $s_4 = s_6 = 0$. Then,(10) $\Longrightarrow p(1-s_3)H + (1-p)(1-s_5)H \geq H$ which can be satisfied only if $s_3 = s_5 = 0$ but that would mean that $s_1 = 0$ which is a contradiction.

*Case (iv)*: $s_1$ is a pure strategy and $s_2$is a completely mixed strategy

In the cheap talk phase, it should be incentive compatible for player 1($L$ -type) to announce $L$, which implies

$$\begin{aligned} &p[s_2s_4 + (1-s_2)(1-s_4)L] + (1-p)[s_2s_6 + (1-s_2)(1-s_6)L] \\ &\quad \geq \underset{s}{Max}\ p[ss_3 + (1-s)(1-s_3)L] \\ &\qquad + (1-p)[ss_5 + (1-s)(1-s_5)L] \end{aligned} \tag{11}$$

If $s_1 = 1$, then $s_3 = s_5 = 1$. Using (9), the LHS of (11) $= p(1-s_4)L + (1-p)(1-s_6)L$. Then (11) $\Longrightarrow ps_4 + (1-p)s_6 \geq 1$ which can be satisfied only if $s_4 = s_6 = 1$ but that would mean that $s_2 = 1$ which is a contradiction.

If $s_1 = 0$, then $s_3 = s_5 = 0$. Then (11) $\Longrightarrow p(1-s_4)L + (1-p)(1-s_6)L \geq L$ which can be satisfied only if $s_4 = s_6 = 0$ but that would mean that $s_2 = 0$ which is a contradiction. □

***Proof of Theorem 4*** Given truthful revelation in the cheap talk stage, the actions in the second stage must constitute a (pure or mixed) Nash equilibrium of the corresponding complete information BoS, that is, $(\sigma_1(t_1,t_2),\sigma_2(t_1,t_2))$ is a (pure or mixed) Nash equilibrium of the BoS with values $t_1$ and $t_2$, $\forall t_1,t_2 \in \{H,L\}$. Since asymmetric profiles are also being considered, there are $3^4$ $(=81)$ strategy profiles that could potentially form an equilibrium. We categorise these under the following three cases, each of which will have 27 possible combinations.

*Case (i)*: $\sigma_1(A|H,L) = \sigma_2(A|H,L) = 0$, i.e., the players play (*B*, *B*) after the message profile $(\tau_1,\tau_2) = (H,L)$;

*Case (ii)*: $\sigma_1(A|H,L) = \sigma_2(A|H,L) = 1$, i.e., the players play (*A*, *A*) after the message profile $(\tau_1,\tau_2) = (H,L)$;

*Case (iii)*: $\sigma_1(A|H,L) = \frac{1}{1+L}$ and $\sigma_2(A|H,L) = \frac{H}{1+H}$, i.e., the players play the mixed strategy equilibrium after the message profile $(\tau_1,\tau_2) = (H,L)$.

We just provide the proof for *Case (i)* here; the proofs of the other cases are very similar and thus omitted.

*Case (i)*: If the players play (*A*, *A*) or the mixed strategy equilibrium $(\frac{1}{1+H},\frac{H}{1+H})$ after the message profile $(\tau_1,\tau_2) = (H,H)$, then player 2(*H*-type) will deviate in the cheap talk stage after player 1(*H*-type)'s message.

If the players play (*B*, *B*) after the message profile $(\tau_1,\tau_2) = (H,H)$, then to be responsive, after at least one of the message profiles $(L,H)$ or $(L,L)$, a Nash equilibrium other than (*B*, *B*) has to be played. We now look at strategy profiles for these subcases (i.e., in which the players play (*B*, *B*) after both message profiles $(H,H)$ and $(H,L)$):

if the players play (*A*, *A*) after one of the message profiles $(L,H)$ or $(L,L)$, and a pure strategy Nash equilibrium in the other, then player 1(*H*-type) will deviate in the cheap talk stage;

if the players play a pure strategy Nash equilibrium after the message profile $(L,H)$, and the mixed strategy Nash equilibrium after the message profile $(L,L)$, then player 2(*L*-type) will deviate in the cheap talk stage after player 1(*L*-type)'s message;

if the players play the mixed strategy Nash equilibrium after the message profile $(L,H)$, and a pure strategy Nash equilibrium after the message profile $(L,L)$, then player 2(*H*-type) will deviate in the cheap talk stage after player 1(*L*-type)'s message;

if the players play the mixed strategy Nash equilibrium after the message profile $(L,H)$, and the mixed strategy Nash equilibrium after the message profile $(L,L)$, then player 2(*H*-type) will again deviate in the cheap talk stage after player 1(*L*-type)'s message. This is because the expected payoff from this profile $\left(=\frac{H}{1+H}\right)$ is less than the expected payoff from deviating and announcing $L$ and then playing $A$ $\left(=\frac{H}{1+L}\right)$.

This rules out all possible profiles under *Case (i)* as potential equilibria. □

***Proof of Theorem 5*** For (i) to be an equilibrium, in the cheap talk phase, player 1($H$-type) needs to be indifferent between announcing $HX$ and $HY$. The expected payoff from announcing $HX$ is $p\left(r_1 \frac{H}{1+H} + (1-r_1)H\right) + (1-p)H$ while the expected payoff from announcing $HY$ is $p\left(r_1 + (1-r_1)\frac{H}{1+H}\right) + (1-p)H$ which will be equal when $r_1 = \frac{H^2}{1+H^2}$.

Similarly, player 1($L$-type) needs to be indifferent between announcing $LX$ and $LY$. The expected payoff from announcing $LX$ is $p + (1-p)\left(r_2 \frac{L}{1+L} + (1-r_2)L\right)$ while the expected payoff from announcing $LY$ is $p + (1-p)\left(r_2 + (1-r_2)\frac{L}{1+L}\right)$ which will be equal when $r_2 = \frac{L^2}{1+L^2}$.

For player 1($H$-type), the expected payoff from announcing $HX$ must be greater than or equal to the expected payoff from announcing $LX$, which implies

$$\begin{aligned} &p\left(\left(\frac{H^2}{H^2+1}\right)\left(\frac{H}{1+H}\right) + \left(1 - \frac{H^2}{H^2+1}\right)H\right) + (1-p)H \\ &\geq p + (1-p)\left(\left(\frac{L^2}{L^2+1}\right)\left(\frac{H}{1+L}\right) + \left(1 - \frac{L^2}{L^2+1}\right)H\right) \end{aligned} \tag{12}$$

This will be satisfied if $p \leq \frac{HL^3\left(1+H+H^2+H^3\right)}{1+L+L^2+L^3+HL^3+H^2L^3+H^3L^3+H^4L^3}$.

For player 1($H$-type), the expected payoff from announcing $HX$ must also be greater than or equal to the expected payoff from announcing $LY$, which implies

$$\begin{aligned} &p\left(\left(\frac{H^2}{H^2+1}\right)\left(\frac{H}{1+H}\right) + \left(1 - \frac{H^2}{H^2+1}\right)H\right) + (1-p)H \\ &\geq p + (1-p)\left(\left(\frac{L^2}{L^2+1}\right)(1) + \left(1 - \frac{L^2}{L^2+1}\right)\left(\frac{H}{1+L}\right)\right) \end{aligned} \tag{13}$$

Note that $\left(\left(\frac{L^2}{L^2+1}\right)\left(\frac{H}{1+L}\right) + \left(1 - \frac{L^2}{L^2+1}\right)H\right) - \left(\left(\frac{L^2}{L^2+1}\right)(1) + \left(1 - \frac{L^2}{L^2+1}\right)\left(\frac{H}{1+L}\right)\right) = L\frac{H-L}{L^2+1} > 0$. So, if the constraint (12) is satisfied, then this constraint (13) will also be satisfied.

For player 1($L$-type), the expected payoff from announcing $LX$ must be greater than or equal to the expected payoff from announcing $HY$, which implies

$$\begin{aligned} &p + (1-p)\left(\left(\frac{L^2}{L^2+1}\right)\left(\frac{L}{1+L}\right) + \left(1 - \frac{L^2}{L^2+1}\right)L\right) \\ &\geq p\left(\left(\frac{H^2}{H^2+1}\right)(1) + \left(1 - \frac{H^2}{H^2+1}\right)\left(\frac{H}{1+H}\right)\right) + (1-p)L \end{aligned} \tag{14}$$

This will be satisfied if $p \geq \frac{L^4\left(1+H+H^2+H^3\right)}{1+L+L^2+L^3+L^4+HL^4+H^2L^4+H^3L^4}$.

For player 1($L$-type), the expected payoff from announcing $LX$ must be greater than or equal to the expected payoff from announcing $HX$, which implies

$$
\begin{aligned}
& p + (1-p)\left(\left(\frac{L^2}{L^2+1}\right)\left(\frac{L}{1+L}\right) + \left(1 - \frac{L^2}{L^2+1}\right)L\right) \\
& \quad \geq p\left(\left(\frac{H^2}{H^2+1}\right)\left(\frac{H}{1+H}\right) + \left(1 - \frac{H^2}{H^2+1}\right)L\right) + (1-p)L
\end{aligned} \tag{15}
$$

Note that $\left(\left(\frac{H^2}{H^2+1}\right)\left(\frac{H}{1+H}\right) + \left(1 - \frac{H^2}{H^2+1}\right)L\right) - \left(\left(\frac{H^2}{H^2+1}\right)(1) + \left(1 - \frac{H^2}{H^2+1}\right)\left(\frac{H}{1+H}\right)\right) = -\frac{H-L}{H^2+1} < 0$. So, if the constraint (14) is satisfied, then this constraint (15) will also be satisfied.

This implies that the strategy profile given by (i) is an equilibrium if

$$
\begin{aligned}
& \frac{L^4(1+H+H^2+H^3)}{1+L+L^2+L^3+L^4+HL^4+H^2L^4+H^3L^4} \leq p \\
& \quad \leq \frac{HL^3(1+H+H^2+H^3)}{1+L+L^2+L^3+HL^3+H^2L^3+H^3L^3+H^4L^3}.
\end{aligned}
$$

□

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