Estimating capabilities with random scale models. Women’s freedom of movement

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Estimating capabilities with random scale models.

Women's freedom of movement.[1]

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Abstract

In Sen's capability approach well-being is evaluated not only in terms of functionings (what they do and who they are) but also in terms of capabilities (what people are free to do and to be). It implies that individuals with the same observed functionings may have different well-being because their choice sets (i.e. capabilities) are different. We utilise a Random Scale Model to measure the latent capability of Italian women to move based on observations of their realized choices. We demonstrate that such models can offer a suitable framework for measuring how individuals are restricted in their capabilities. Our estimations show that the percentage of women predicted to be restricted in their freedom of movement (have restricted capability sets) is 23-25 per cent. If all women were unconstrained, our model predicts that 15-17 per cent of them would choose to do more activities.

Keywords: measurement of capabilities, freedom of movement, random scale models, gender.

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1. Introduction

As an alternative to traditional welfare analysis, Amartya Sen has proposed what he calls the capability approach (Sen, 1985, 1992, 1999). Sen's capability approach distinguishes between what people are free to do and to be (their 'capabilities') and what they do and are (their 'functionings'). In the capability approach, individuals' well-being is evaluated not only in terms of achieved functionings (a vector of observed doings), but also in terms of the freedom to choose between different functionings. The notion of freedom enjoyed by the individual is represented by the individual's capability set (the set of all available vectors of functionings). The capability approach implies that individuals with the same observed functionings may have different well-being because their choice sets (i.e. capabilities) are different.

This paper presents a new approach to estimating capabilities and applies this approach to measuring the capability of freedom of movement. In the following, we only look at capabilities, without constructing an accompanying welfare measure. We use a limited definition of freedom of movement, defined on a set of 8 activities women participate in: going out in the evening, meeting friends, shopping, driving and participation in sports, cultural, political and social activities. The data only describe the women's functionings. In order to measure the capability "freedom of movement", and not just the functionings, we utilise a Random Scale Model, which is especially useful in situations where the individuals' capability sets are unobserved or only partially observed. Even if two women are observed doing the same activities, i.e. have the same functionings, they may have different capability sets. The model is used to estimate how many women are restricted in their capability to choose among different functionings related to freedom of movement.

Our approach is based on the Random Scale Modelling approach pioneered by Luce (1959) and McFadden (1973, 1984), extended to a setting with latent capability sets along the lines suggested in Dagsvik (2013). Based on the observed movements of women, we use this methodology to infer what their capability sets are and the degree to which they have freedom of movement. The probability that an individual is observed with a specific functioning is a combination of two types of probabilities: the probability of choosing this functioning given her capability set (choice probabilities) and the probability that she has a capability set which includes this functioning (restriction probabilities). For instance, the probability that a woman is doing 3 activities depends on the probability that she chooses to do 3 activities (the choice probability) and the probability that she has a capability set that includes doing 3 activities (the restriction probability). These probabilities are estimated parametrically on a set of exogenous variables that include individual characteristics, partner's characteristics, and social characteristics. The Random Scale Model, also known as the Random Utility Model, can be used to predict the number of women who are constrained in their capability sets, i.e. who cannot choose among all the possible functionings, and how many women would change their choices (their chosen functionings) if they were not restricted in their freedom of movement.

Measuring capabilities has usually been done either by directly measuring capabilities in surveys (Anand et al 2009) or by applying econometric techniques such as structural equation models (Krishnakumar 2007, Krishnakumar and Ballon 2008, Di Tommaso 2007, Di Tommaso et al 2009). Both these approaches have merits and limitations; on the one hand, direct questions about capabilities are a very straightforward methodology that does not require many assumptions, but on the other hand, they may lead to skewed answers because of the problem of adaptive preferences (Sen 1985, 1992, 2009, Elster 1983, Clark 2012). Structural equation models lead to a capability index that can be utilised to rank individuals, taking into account exogenous variables that influence either functionings or capabilities or both. In such models, the difference between functionings and capabilities relies only on a stochastic component since the capability index is derived through a factor analysis over the chosen indicators of functionings. The methodology of this paper, instead, provides an explicit representation of preference and choice constraints which is different from the structural equation models used in other capability studies. Although the choice sets are latent, our approach allows us to identify both the distributions of preferences and the choice constraints (probability distribution over the choice sets). It explicitly calculates the probability of each capability set being available to each individual, allowing one to distinguish between capabilities and preferences and to look at counterfactual scenarios (for example how many would change their chosen functioning if nobody was restricted in their capability sets). Both approaches (Structural equation models and random scale models) provide indirect measurements of something not observable, but the Random Scale Model makes it possible to distinguish between two groups of individuals who are observed in the same situation (same functioning) but who do not have the same capability set. Structural Equation models do not provide such an evaluation.

The data set is derived from a 2006 Italian survey of violence against women. It provides information about the activities women perform along with information about their partners. Our results show that one quarter of the women in the sample are constrained in their movements (have limited capability sets). If we remove the restrictions, around 15 per cent of the women would choose to exercise more freedom of movement (change their functioning).

Our paper is innovative in that it presents a new approach for estimating capabilities and provides an example of how to apply it. Section 2 gives a general description of random scale models applied to capabilities and gives a formal presentation of the model. Section 3 presents our definition of freedom of movement, while Section 4 describes the data and presents descriptive statistics. Section 5 includes the empirical specification and specifies the assumptions utilised on the empirical specification. Estimation results are given in Section 6 and simulations in Section 7. Identification issues are discussed in Appendix B.

2. A Random Scale Model

2.1 Our model and the capability approach

Sen (1985, 1992, 1999), argues for the importance of including capabilities when making welfare judgements. He also notes (Sen, 1991) that individual preferences are relevant, even if one rejects the welfarist approach. He says that "Preferences and freedom are very deeply interrelated and that an affirmation of the intrinsic importance of freedom must inter alia assign fundamental importance to preferences", (Sen 1991, p15). Building on Sen's concept of refined functionings, Fleurbaey (2006) argues persuasively that, in addition, information on the achieved functioning is also important when making welfare judgements. Unless one introduces a concept of responsibility, one must admit the possibility that individuals make mistakes. Fleurbaey notes that "One has to introduce the idea of responsibility and the view that responsible individuals, once they are given the freedom to achieve, lose any right to complain if they fail to achieve" (Fleurbaey 2006, pg 306). Only looking at capabilities can imply a loss of information that is relevant for those who think that achievements, and not only opportunities, matter. To summarize, a welfare function should both take into account preferences, observed functionings and capabilities.

Our paper analyses the determinants of welfare, by estimating capabilities and preferences based on observed functionings, but does not provide a welfare function. This is mainly because we are looking only at one capability, which is fairly easy to evaluate, and not at a combination of different capabilities. Dagsvik (2013) shows how, in more complicated situations, the random scale approach can be used to give money metric evaluations of different capability sets, but this requires information on income, which we do not have, and a willingness to evaluate the monetary worth of different capability sets.

A central feature of our approach is that utility is considered to have a probabilistic component. Quandt (1956) is an early example of a discussion of probabilistic consumer behaviour, while Koopmans (1962) and Kreps (1979) discuss how an increased opportunity set can increase utility by leading to increased flexibility when making future choices. In our approach, when considering choices made over a period of time or among a group of individuals, expected utility increases with increased opportunities[4].

Our paper does not directly make welfare judgements, but only limited social evaluations. To illustrate, consider three women with some unobserved characteristics (so that there is a random component to their choices): Mary, Julie and Anne. Mary is observed to only stay at home and her characteristics imply a low probability of having the full capability set. Julie also stays at home but has a high probability of having the full capability set. So, while they have the same achieved functioning (staying at home), Julie has greater expected freedom of movement than Mary, which in our approach implies a higher level of expected welfare/utility.

The third woman, Anne, goes out with her friends, but her characteristics imply a low probability of having the full capability set (she is restricted to only a few types of movement, one of which is going out with friends). So, Julie has a larger expected capability set than Anne, but Anne is observed using her freedom more. In this case our paper does not give any guidance to who is better off. One can argue that the person with the greatest expected freedom (Julie) is best off, or that the person (Anne) who is more active and uses her limited freedom is best off. The aim of our paper is therefore to describe (probabilistically) the capability sets of women such as these, but not to make general welfare judgements.

2.2 The random scale model

The paper assumes that the observed achieved functionings can be thought of as being choices made on the basis of a random scale model. The motivation of psychologists such as Thurstone (1927) for proposing a random scale framework was to deal with the observational fact

[4] Appendix A shows how this is the case for the model presented in section 2.

that individuals often violate transitivity when faced with replications of (seemingly) identical choice experiments. His explanation was that decision makers may be ambiguous about the precise value of the respective alternatives, in the sense that if the same choice setting is repeated they may choose a different alternative. This unpredictable temporal variation in tastes is represented by the stochastic error terms in the scale representation. The Random Scale Model is particularly designed to allow for this type of seemingly bounded rational behaviour. As an example, consider an agent who almost always prefers wine to beer with her meals. But once in a while, to her own surprise, she suddenly wants a little change and drinks beer. Even if we always observe her drinking wine (her functioning), this type of stochastic taste implies that she also prefers to have other elements in her capability set (e.g. beer) over time. In other words, assuming a Random Scale Model implies that the agents over time care both about their choices and about their opportunities (their capability sets). This framework allows one to relax the rather strong consistency assumptions central to the conventional deterministic utility theory. This goes some way towards meeting the objections of Sen (1985, 1992, 1999) which argue that the standard assumptions of utility theory, such as completeness and transitivity, do not hold, see Luce and Suppes (1965, p. 350).

The probability of making different choices is modelled using two different probabilities which we refer to as *choice probabilities* and *restriction probabilities*. *Choice probabilities* are the probabilities of choosing the different functionings available in the capability set, while *restriction probabilities* are the probabilities of having different capability sets. The choice probabilities are modelled using the Random Scale Modelling approach, leading to them having a multinomial distribution. The probabilities of having different choice sets (the restriction probabilities) are also assumed to have a multinomial distribution, but within our framework, it is possible for them to have other distributions. Both probabilities are estimated on exogenous variables (personal characteristics, partner’s characteristics, environment characteristics). Section 6.1 provides a discussion about the allocation of the exogenous variables in the two probabilities and a sensitivity analysis.

2.3 Choice probabilities and restriction probabilities

The distribution of the choice probabilities follows from assumptions on the scale (utility) function of the agents. Let us assume that individuals have the possibility to choose among some or all alternatives in a universal set, *S* Let *H* denote the number of functionings in *S* and the functionings be numbered from 1 to *H*. The universal set, *S*, is the absolute maximal set of functionings that are relevant, regardless of whether or not they are available to everybody. The agent is assumed to have preferences over the functionings in *S*. Let *C* denote the choice set of a particular agent (for simplicity we drop the indexation of the agent). It consists of all the functionings available to the agent. For some agents *C* may be equal to *S*, but in many situations, the choice set will be a proper subset of *S*. In our context, *C* represents the agents capability set, and the elements of *C* are the functionings that are available to the agent. Furthermore, let $P_j(C)$ be the probability that the woman shall choose *j*, given the choice set C (this is the choice probability).

We assume that agents choose functionings from their capability sets in accordance with the Random Scale Model. Let U_j denote a scale function that represents the welfare of an agent observed utilizing functioning *j* (assuming functioning *j* is available to the agent). Following McFadden (1973, 1984), we assume that $U_j = v_j + \varepsilon_j$, where v_j is a deterministic term that depends on observed characteristics and ε_j is a random error term that is supposed to capture unobserved characteristics that affect the agent's welfare. The random error terms, ε_j, are assumed to be independently extreme value distributed.

Given this distribution and assuming that the agent chooses the alternative in *C* that maximizes the scale U_j, we get choice probabilities, $P_j(C)$, that are given by

$$P_j(C) = \frac{\exp(\nu_j)}{\sum_{k\in C} \exp(\nu_k)}, \qquad j \in C \subset S, \qquad (1)$$

which is the well-known Multinomial Logit Model, see (McFadden, 1984). Note that this transforms the stochastic nature of preferences, captured by the error term ε_j, into a stochastic choice framework represented by the choice probability $P_j(C)$. This is a different framework from expected utility, where the utility function is deterministic, while the state of nature is stochastic. In our approach, there is no uncertainty about the outcome arising from a choice. Since the choice sets form different combinations of the H available functionings, there can be many more possible capability sets than there are functionings (for example with two functionings, $j=1$ and $j=2$ there are four possible capability sets: ∅, {1}, {2} and {1,2}). In the following we let L denote the number of capability sets, letting them be numbered from 1 to L.

For individual i, the structural part of the scale function is given by,

$$v_{ij} = X_i\beta_j, \qquad (2)$$

for each of the functionings $j = 1, 2, \ldots, H$, with the structural term associated with the last functioning set to zero, $\nu_{iH} = 0$, and where X_i is a vector of characteristics which influences individual i's preferences (including 1 as one of the components) and $\{\beta_j\}$ are vectors of unknown parameters. The assumption that $\nu_{iH} = 0$ is simply a normalization and represents no loss of generality.

We denote the restriction probabilities by $r(C_j)$. They denote the probability that the capability set is equal to C_j, $r(C_j) = P(C = C_j)$ and they must satisfy the restriction $\sum_{j=1}^{L} r(C_j) = 1$.

Similarly to equation (1) above, we assume that the *restriction probabilities* have a multinomial logit form, given by

$$r(C_j) = \frac{\exp(Z_i\gamma_j)}{\sum_{k=1}^{L}\exp(Z_i\gamma_k)}, \tag{3}$$

for *j =1, 2, ... , L*, with γ_H normalized to zero, and where Z_i is a vector of covariates which influences the possibility that individual *i* will be restricted (including 1 as one of the components) and $\{\gamma_j\}$ are unknown parameter vectors[5]. The vector Z_i may include both environment, partner, and individual characteristics. The variables included in the X-vector should only be associated with preferences (on which choices are based), while other variables are included in the Z-vector. This distinction is not always easy to make. For example, in our analysis we choose to include work (working / not working) in the Z-vector because working requires a certain freedom of movement and thereby reduces the probability of being restricted. Also, work could decrease time available for social and cultural activities, increase resources or opportunity to meet other people. Instead of including the variable work in the *restriction probabilities*, one could argue for including this variable in the preference relationship by including it in the X-vector. This would imply an assumption that work influences the preferences, for example by increasing the desire for participating in many activities.

We now bring together the choice probabilities and the restriction probabilities that describe the opportunities available to the agents. These determine the probability of being observed in the different states. Let Q_j be the probability of being observed with functioning *j*. If an individual is observed choosing alternative *j*, this can only happen if her choice set includes this alternative.

[5] The multinomial logit is the most used distribution for modelling multinomial discrete variables because of its simple parametric structure. For example, the more general multivariate normal distribution with an arbitrary correlation matrix requires evaluating probabilities given by multidimensional normal integrals that restricts the application to only few alternatives. Ilmakunnas and Pudney (1990) model job opportunities in a similar manner to our modelling of opportunity sets. In a labour supply model, they model job opportunity using both a structural approach and the multinomial distribution approach. They consider both approaches valid, but conclude that "*on simple goodness-of-fit grounds ... the heavily-parameterised Multinomial Logit Model is clearly the preferred specification"* (p. 193).

Furthermore, we have that the joint probability of having choice set C_k and choosing alternative j, is equal to

$$P(J(C)=j, C=C_k)=P(J(C)=j \mid C=C_k)\cdot P(C=C_k)=P_j(C_k)\cdot r(C_k), \qquad (4)$$

where $J(C)$ denotes the choice of the agent when the choice set is equal to C. Hence, by summing over all possible choice sets it follows that we must have

$$Q_j=\sum_{k=1}^{L} P_j(C_k)\cdot r(C_k), \quad j\in\{1,\cdots,H\}. \qquad (5)$$

This specification can be traced back at least to the work of Manski (1977).

The model described above only uses individual specific variables, both in the utility function and in the restriction functions. This implies that we are only looking at how choices vary among women according to their characteristics and their situation. A more general model would also consider the characteristics of the choices. This would be difficult to do is our context, since we base our econometric model on an index of activities.

2.4 Defining the choice sets

Our notion of freedom is represented by the individual's capability set, defined as the set of all available vectors of functionings. We consider these capability sets to be discrete and that they therefore can be analysed using the Random Scale Model described in section 2.1 and 2.2 above. The question then becomes how to define these discrete capability sets based on the replies women give to eight questions about their movements. If we were to define the choice set directly using these eight questions, we would get 2^8=256 different capability sets, each containing a unique combination of the 8 functionings (activities). To get around this problem of dimensionality, we use an index of functionings based on the sum of activities a woman participates in. The activity index is equal to 1 if the woman participates in 0, 1 or 2 activities, it is equal to 2 if the woman participates in 3 activities, and it is equal to 3 if the woman participates in 4, 5, 6, 7 or 8 activities[6]. As with any type of aggregation, this means we lose information. Our aggregation into an index of activities is most suitable if the different activities can be substituted for each other.

Our approach considers the observed functionings of the individuals to be determined by a combination of preferences and the index of activities. Our concept of freedom of movement is therefore not solely based on counting the freedoms available to an individual, but also on how the individual evaluates the choices she faces. We would also like to point out that our approach does not in general require that the capability sets are defined using a counting measure. In a different setting, on might want to construct the possible capability sets in other ways. If we were, for example, looking at the labour market, the capability sets could be defined over the outcomes working / not working without resorting to a counting measure.

The random scale (utility) depends on both the number of activities (i.e. the level of the activity index) and on latent attributes of all the activities within each group. While it is true that the deterministic part v_j only depends on the activity index, the stochastic part ε_j depends on the latent attributes of all the activities. This because the utility U_j is the maximum over all the alternatives within each of the three groups of aggregate alternatives.

Our use of a movement index to describe the possible capability sets is based on the assumption that there is a link between how restricted a woman is in her movements and the number of activities she engages in. While this would seem intuitive, it can be useful to give a simple example where this is the case. Consider for example a utility function similar to the much used Stone-Geary utility function:

$$u = \sum_j \alpha_j \cdot \log(t_j + \theta_j) \qquad (6)$$

[6] See Section 3 for a detailed description of the index.

where u is utility, t_j is time used on activity j, α_j is a parameter indicating the preference for activity j, and θ_j is a parameter setting a utility threshold that must be surpassed if activity j is to be undertaken (in the usual Stone-Geary set-up it has the opposite sign and is considered a minimum subsistence quantity). Setting such a threshold ensures that some activities might not be undertaken. Furthermore we require $\sum_k t_k = T$ and $t_j \geq 0$, where T is total time spent on activities outside the home. Solving this, using the Kuhn-Tucker conditions, we get that we either have a positive amount of time used on activity l with

$$t_l = \frac{\alpha_l}{\sum_k \alpha_k + \sum_{k \neq l} \lambda_k \theta_k} \left(T + \sum_k \theta_k \right) - \theta_l \quad \text{and} \quad \lambda_l = 0 \qquad (7)$$

or the activity is not undertaken with

$$t_l = 0 \quad \text{and} \quad \lambda_l = \frac{\sum_{k \neq l} \alpha_k + \sum_{k \neq l} \lambda_k \theta_k}{T + \sum_{k \neq l} \theta_k} - \frac{\alpha_l}{\theta_l}, \qquad (8)$$

where the λ parameters are the Lagrange multipliers of the problem. The Lagrange multiplier connected to an activity is zero if the activity is undertaken and positive if it is not. Activity l will be undertaken only if

$$\frac{\alpha_l}{\sum_{k \neq l} \alpha_k + \sum_{k \neq l} \lambda_k \theta_k} \left(T + \sum_{k \neq l} \theta_k \right) > \theta_{l.} \qquad (9)$$

From this, we see that the greater a woman's preferences for activity l is (the larger α_l is) and the more time she has available for activities outside the home (the larger T is), the greater the chance that she engages in activity l (the greater the chance that the above condition will be met).

If we think of restrictions in a woman's freedom of movement as being a restriction on the time T she can spend outside the household (either because of social norms or due to restrictions imposed by the partner), then this model implies that as she becomes more restricted, the number of activities she engages in will fall. Those activities with a high threshold θ or for which the woman has low preferences, α, will be the first to be abandoned. The model implies a clear link between how restricted a woman is in her movements and the number of activities she engages in. The model is based on the assumption that the different activities are substitutable. If they are not, if some may even be disliked, and the restrictions are on each activity in itself, then it is more problematic to use the sum of activities as an indicator of freedom of movement. But even in such a case, there will probably be correlation between the number of activities a woman is restricted to and the number of activities she undertakes.[7] It is important to note that our econometric model is not directly based on this simple illustrative model, but is consistent with it. More general models will also be consistent with our approach.

3. An application to women's freedom of movement.

In the following we apply the Random Scale Model to an analysis of women's freedom of movement, which is an important aspect of gender inequality. Different aspects of gender inequalities have been considered by scholars working within the capability approach. Some authors have utilised indicators of functionings and capabilities similar to the ones utilised in this paper (Nussbaum 1999; Robeyns 2003, 2004; Anand et al. 2009). Robeyns (2003) provides a theoretical analysis for choosing relevant capabilities for measuring gender inequalities. Her list of capabilities includes among others: social interactions, leisure activities and mobility. Robeyns (2004) analyses gender inequalities in mobility, leisure activities, and social relations. Mobility is measured by having access to a car or a van, while leisure activities include indicators of social activities and sports attendance. For both functionings she finds that women have a disadvantage with respect to men, which increases with age. Social relations are measured by indicators like

[7] Ben-Akiva and Watanatada (1981) propose an interesting approach using a Random Scale Model when the choice alternatives can be classified into two levels: "main" observed alternatives and latent "elemental" alternatives. For each main alternative there would be a set of (different) latent elemental alternatives available. The main alternatives could for example be labor market sectors, while the set of latent elemental alternatives could be jobs within the sector. This approach is not suitable in our case because each aggregate state (each level of our index) can contain any of the underlying choice sets. There is not a unique elemental set that can be associated with each level of the index.

frequency of meeting friends and frequency of talking to neighbours. Her results show that women have a higher index of social relations than men. She measures functionings and not capabilities, because she only observes the realized choices and not the freedom space. Anand et al. (2009), in their survey of capabilities, include some questions related to the capability of enjoying recreational activities, the frequency of meeting friends, and feeling safe at night or during the day. Their main research goal is to find the correlation between a list of capabilities, including freedom of movement, and an index of life satisfaction. In another survey of capabilities in Italy, USA and UK, Anand et al. (2013) include questions about having opportunities to take part in local social events and the ability to walk safely in the neighbourhood at night. One finding is that Italians have on average more opportunities to take part in social events than British or Americans, but they are less able to walk safely. The papers by Anand et al (2009, 2013) try to infer capabilities by asking directly to the individuals their perception of the relevant capabilities (Questions like: "you have opportunities to take part to social events?" or "Do you feel safe to walk at night?"),

Our paper differs from the previous ones, both in the methodology utilised and in the use of an index for freedom of movement. This paper considers eight specific aspects of freedom of movement: going out in the evening, meeting friends, shopping, driving and participation in sports, cultural, political and social activities.

Table 1 shows the distribution of answers to the eight questions for women in our data set who are between 26 and 65 years of age, who are currently in a relationship (have a partner) and are not disabled[8].

[8] For a detailed analysis of the sample selection see next section

Table 1: Observed functionings among 17.350 Italian women*. Percent of women.

	How often do you meet friends?	How often do you go to the cinema, theatre, concerts?	How often do you practice sports?	Do you work as a volunteer or attend a club, an association or a political party?	Do you practice other activities?**
Often	25.9	8.3	18.3	8.2	6.63
Sometimes	33.5	26.0	11.7	5.8	22.7
Rarely	22.9	22.0	7.0	3.0	17.41
Never	17.7	43.7	62.9	83.0	53.25
No reply	0.01	0.01	0	0.01	0.01
	100.0	100.0	100.0	100.0	100.0

Table 1 continued: Observed functionings among 17.350 Italian women. Percent of women.

	How often do you go out in the evening?	How often do you go shopping?		Do you drive a car or motorcycle?
Once a week or more	51.0	96.3	Yes	70.06
Once a month or sometimes a month	27.0	2.8	No	29.94
Sometimes a year	13.4	0.3		
Never	8.5	0.5		
No reply	0.2	0.1		
	100.0	100.0		100.0

*The sample includes only women who are currently in a relationship (marriage, co-habitation or engagement), from 26 to 65 years old and excludes the disabled.

**The question about other activities performed outside the house includes for instance going to visit a museum or going dancing.

The variables in Table 1 are used to build an index of activities (functionings) based on the sum of activities a woman participates in, see Table 2. We consider that a woman participates in an activity if she answers “often or sometimes” to the first five questions, answers “once a week or more” to the questions on going out in the evening and going shopping or answers “yes” to the question on driving[9]. The activity index is equal to 1 if the woman participates in 0, 1 or 2 activities, it is equal to 2 if the woman participates in 3 activities, and it is equal to 3 if the woman participates in 4, 5, 6, 7 or 8 activities. Organising our data using such an index is a way of summarising the activities the women participate in, but is not essential for the use of our methodology. One could organise the data in other ways as long as the functionings in the end can be grouped into discrete categories.

Table 2: Activity index: the number of activities a woman participates in*.

Index	1	2	3	
Number of activities	0-2	3	4-8	Total
Number of women	4,548	3,423	9,379	17,350
Percentage	26.2	19.7	54.1	100

* We consider that a woman participates in an activity if she answers "often or sometimes" to the first five questions, answers "once a week or more" to the questions on going out in the evening and going shopping or answers "yes" to the question on driving. The sample includes only women who are currently in a relationship (marriage, co-habitation or engagement), from 26 to 65 years old and excludes the disabled

In devising the index, we chose to concentrate on extreme behaviour (participating in few activities) instead of distributing individuals evenly across the three values of the activity index. This builds on the implied assumption that it is more likely to find restricted women among those participating in few activities than among those participating in many[10].

We assume that the functionings are ranked, so that a capability set that includes the functioning "doing many activities" always includes the possibility of doing few activities.

[9] We have considered "Driving a car" as an activity in itself. It could also be seen as a resource/conversion factor, because it could be of help in practicing other activities. Nevertheless, in order to be taken into account as a conversion factor, we would have needed some other variables related to the availability of public transport in the area where the woman lives and we do not have such information.

[10] We have also estimated a model based on a four state activity index, but found that the increased number of values complicated our estimation due to empty cells, without changing the general results.

Individuals participating in many activities always have the option to participate in fewer activities, leading to there being three capability sets available to women. Women can be very restricted in their freedom of movement and have only the possibility to do 0, 1, 2 activities (activity index equal to 1). They can be less restricted and have the possibility to do 0, 1, 2, 3 activities (activity level equal 1 or 2). Finally, they can be completely free to do at least 4 activities or more (activity level equal 1 or 2 or 3), thereby having the full capability set. How the women are distributed across the number of activities (from 0 to 8) is reported in Appendix C table C2. To summarize, women can have one of the following three capability sets:

$C_1 = \{1\}$, (no freedom of choice, women can only choose activity index 1)

$C_2 = \{1, 2\}$, (women can choose activity index 1 or 2)

$C_3 = \{1, 2, 3\}$ (the full capability set: women can choose activity index 1, 2 or 3).

This brings out the difference between functionings and capabilities. For example, we could observe a woman doing 2 activities (activity level 1). This is her functioning, but we do not know if she has the capability set C_1 or C_2 or C_3. If she has capability set C_3 or C_2, she has chosen to do few activities, even though she has the freedom of doing more. If she has capability set C_1, she is restricted to doing no more than 3 activities. In other words, among the 26.2 percent of women with activity index equal 1 (see table 2), some of them may have the full capability set C_3, others could have capability set C_2, and others capability set C_1.

While we observe the activity index for all the women in our sample (i.e. their functionings) we cannot observe their capability sets. In the following, we use a Random Scale Model to estimate the percentages of women with capability set C_1 and C_2, i.e. women who are restricted in their capability of freedom of movement. This allows us to calculate how many women would like to have a functioning (a level of the activity index) that is currently not available to them. For instance, we could find that there are two women who both have a capability set equal to C_1. One would not like to change her activity level (her functionings), even if she had more freedom of choice, while the other would like to change her activity level if she had more freedom.

4. Data

The data set consists of a survey of 25,000 women living in Italy between 16 and 70 years old interviewed over the phone in 2006 (Indagine Multiscopo sulla Sicurezza delle donne, 2006; Istat 2006)[11]. The survey is designed to detect three types of violence against women: physical violence, sexual violence (ranging from harassment to rape), and psychological violence (your partner prevents you from working, from studying, from being in control of your money, from seeing your family, etc.). We chose to use this survey because it both provides information about the activities women perform and about the characteristics of their partners. In addition to information on social activities, the survey contains information on age, education, job qualification, full time/part time, and work at home or outside the home. The major limitations of this data set are that it does not contain information about income, children or disability. Education can be seen as a proxy for income, but we do not have any information on the other variables. Nevertheless, the survey utilised in this paper is the only Italian survey that contains information both about women's activities (the ones included in the index of freedom of movement) and their partners. For a methodological note about how the survey was conducted and how the problem of underreporting has been taken into account see Muratore and Sabbadini (2005) and Istat (2006)[12].

[11] The Italian Survey on Household Income and Wealth (SHIW) includes income, but does not have data on the activities women perform. Italian time use surveys have information on women's activities, but do not include income or information about the partner.

[12] For a detailed description of the survey and its results see the Istat report http://www.istat.it/it/files/2011/07/Full_text.pdf

The research team of the National Statistical Institute included sociologists, statisticians and psychologists. They designed the survey, selected and trained the interviewers and followed its implementation.

We select a sample of women who are currently in a relationship (marriage, co-habitation or engagement), from 26 to 65 years of age and exclude those who are unable to work. We select women who are currently in a relationship, because we are interested in analysing constraints due to their partners. Women under 26 are excluded because they could still be living with their parents and/or studying. We also exclude women who are unable to work because they are few and may have additional constraints that we do not wish to focus on in this paper[13]. See Table C1 in Appendix C for the sample selection. The resulting sample consists of 17,350 women.

Table 3 provides descriptive statistics of the exogenous variables for the three values of the activity index. The average age of the women in our sample is in line with other national data sets. As expected, younger women and educated women are involved in more activities than older women and those with a lower education. It is also the case that 66 per cent of women with the low activity level 1 have a partner with low education, against 35 per cent of women with the higher activity level 3. Working women are more active, while women with an older partner participate in fewer activities. The data does not contain information about income nor data about children. We have used education and the dummy "work" as proxies for income.

Muratore and Sabbadini (2005) describes the different phases for the implementation of the Italian Violence against Women Survey while Istat (2006) provides a description of the methodology

[13] It is not possible in our data to identify disability. There is only a question about the occupational status of the woman. The possible answers are: working, looking for a first job, looking for a job, student, housewife, unable to work, retired, missing. In the sample there were only 44 women who replied that their occupational status was being unable to work and were therefore excluded.

Table 3: Descriptive statistics: 17,350 Italian women

	Mean		
	Activity index=1	Activity index=2	Activity index= 3
Woman's age in years	50.062	46.459	44.742
Woman degree= 1 if the woman has a university degree;=0 otherwise	0.053	0.088	0.205
Woman high school=1 if the woman has a high school diploma; =0 otherwise	0.269	0.394	0.491
Woman low education=1 if the woman has no high school diploma; =0 otherwise	0.679	0.519	0.304
Woman healthy= 1 if does not have any health problems based on a list of 10 questions*	0.305	0.362	0.416
Psychological violence by partner=number of positive responses to questions about psychological violence**	1.103	0.994	0.925
Physical or sexual violence by partner=1 if such violence is flagged in survey	0.062	0.064	0.068
Woman Works = 1 if the woman works;=0 otherwise	0.375	0.470	0.603
Age difference = age of partner minus age of woman	3.771	3.499	3.261
Northern Italy = 1 if the woman and partner live in northern Italy; =0 otherwise	0.381	0.426	0.468
Central Italy = 1 if the woman and partner live in central Italy; =0 otherwise	0.176	0.183	0.206
Southern Italy = 1 if the woman and partner live in southern Italy; =0 otherwise	0.443	0.391	0.326
Partner low education=1 if the partner has no high school diploma ;=0 otherwise	0.660	0.526	0.349

*See table C3in the appendix for questions on health.

**See table C4 in the appendix for questions on psychological violence.

As expected, the health of women doing few activities is worse than the health of those doing many, with 31 per cent of women who are involved in 0, 1 or 2 activities (activity index 1) being healthy, while 42 per cent of women who are involved in 4,5,6,7 or 8 activities (activity index 3) are healthy. The health variable is a dummy variable taking the value of one if the woman replies that she had never had any of a set of ten health problems. The list of questions and their descriptive statistics are reported in table C3 in Appendix C.

In addition to the above variables, we utilize two variables for domestic violence: a dummy variable equal to one if a woman has been subjected to either physical or sexual violence and a variable for psychological violence that is equal to the sum of positive responses to questions about psychological violence. Table C4 in Appendix C contains the list of questions that were asked about psychological violence. Table 3 shows that psychological violence decreases when going from activity level 1 to 3. On average women with activity level 1 have been subjected to 1.1 different types of psychological violence while women with activity level 3 have been subjected to 0.92 types of psychological violence. Table 3 also shows that physical and sexual violence increases slightly with increased activity. This counter-intuitive result is debated in the literature, where the correlation between domestic violence and different indicators of freedom of movement, autonomy or income is unclear.The relationship between an index such as our index of "freedom of movement" and violence is complex with the direction of causality being difficult to identify. On the one hand, domestic violence can induce women to curtail their autonomy to avoid the pain and humiliation of being beaten. On the other hand, women with greater autonomy may elicit greater violence from their husbands.

Previous studies have not looked at a link between an index of activities and violence, but there have been studies on the link between autonomy and violence. A positive correlation between domestic violence and different forms of autonomy has been found by Menon and Johnson (2007). Eswaran and Malhotra (2011) have gotten mixed results according to the methodology used. They find a negative correlation between domestic violence and autonomy only when they use an instrumental variable as a proxy for autonomy (height of the woman); otherwise they find a positive correlation. Bloch and Rao (2002) find a positive correlation between income of the spouse's family and domestic violence.

Our focus on individual specific variables has led us to drop a variable indicating whether a woman has experienced violence from a non-partner. Including this variable did not significantly affect the estimates[14] we will report later on, but had a significant negative effect on the probability of a woman being restricted in her freedom of movement. The only plausible explanation for this result is that the greater freedom a woman has to go out, the greater is her risk in encountering violence from non-partners (outside the home). In other words, the variable is mainly a characteristic of the choice and not the individual. Experiencing violence from a non-partner probably also decreases the willingness of the woman to go out, but this is overwhelmed by the effect of encountering violence when going out. We have therefore not included this variable in our model. On the other hand, we include violence from the partner, because this can be seen as being an individual specific variable (connected to the individual and the not the activity).

Among women doing few activities (activity level 1) there is a higher percentage of southern women (44 per cent), while among women doing many activities (activity level 3) 47 per cent are from the north. Unfortunately, because of the privacy policies of the Italian Institute of Statistics, variables regarding the type of town or village the woman lives in are unavailable to researchers.

5. The empirical specification

In the following, we estimate a model for freedom of movement based on the activity index defined in Section 2 where we have three possible functionings (activity level 1 or 2 or 3) and three possible capability sets.

The theoretical model outlined in section 3 includes 2 main assumptions: 1) choices are made according to the Random Scale Model and 2) the restriction probabilities are distributed according to the multinomial logit model. The empirical specification involves making additional

[14] Estimates are available from the authors upon request.

assumptions. We also assume for simplicity that 1) the explanatory variables enter the probability function linearly and 2) that the capability sets are strictly ranked from smallest to largest. This follows from the fact that we described the capability sets solely by the number of available activities. Since the index is strictly increasing, so are the capability sets. This assumption is based on the way we organise our data, but is not a necessary assumption for using our theoretical model.

In the case of ranked capability sets, the special case of *j=1* can be written

$$Q_1 = r(C_1) + \sum_{k=2}^{L} P_1(C_k) \cdot r(C_k), \qquad (10)$$

and the special case of *j=H*, it can be written

$$Q_H = P_H(C_H) \cdot r(C_H). \qquad (11)$$

For the capability sets we discussed in section 5, the probability of being observed in activity level 1 is given by the following equation:

$$Q_1 = P_1(C_3) \cdot r(C_3) + P_1(C_2) \cdot r(C_2) + r(C_1), \qquad (12a)$$

the probability of being observed in activity level 2 is given by:

$$Q_2 = P_2(C_3) \cdot r(C_3) + P_2(C_2) \cdot r(C_2) \qquad (12b)$$

and, finally, the probability of being observed in activity level 3

$$Q_3 = P_3(C_3) \cdot r(C_3). \qquad (12c)$$

The identification of the model relies on excluding variables from the choice probabilities and the restriction probabilities. The model is not identified if one does not either assume that an identifiable group of individuals, some choosing many activities and some choosing few, always have the full capability set (are never restricted in their opportunities) or include identifying explanatory variables in the restriction probabilities. The identifying explanatory variables cannot be included in the choice probabilities, but it is possible to have some variables in both the preference and the restriction probabilities (in both the X-vector and the Z-vector). Identification using explanatory variables is analogous to the exclusion restrictions used to identify supply and demand in the econometric analysis of markets. Including different variables in the restriction probabilities can lead to different estimates of the number of individuals who have restricted opportunities. It is therefore important to discuss carefully which variables should be included and to check the sensitivity of the results to how this is done[15]. See appendix B for a further discussion of identification.

6. Results

Tables 4 and 5 report the parameter estimates and marginal effects for two specifications of the model. The first specification is our preferred specification. It includes among the X variables only the personal characteristics of the woman (age and education), and among the Z variables the dummies for woman's health and work, a variable for psychological violence by the partner, a dummy for sexual or physical violence by the partner, a dummy for a partner with a low education, a variable denoting the age difference between the partners and regional dummies. We prefer this specification because we consider all these variables to be related to whether a woman is constrained in her freedom of movement or not.

Even so, it is open to discussion whether other variables should be included in the preference probabilities (among the X variables) instead of being included in the restriction probabilities (among the Z variables). To see how much of a difference this makes, we have estimated an alternative specification, specification 2, where the dummies for woman's health and whether she works are included in the preference probability instead of in the restriction probability.

[15] Our data lack information regarding income and children. We acknowledge that this is an important limitation, because children could restrict women's activities outside the house (or increase her activities) and income can only be imperfectly proxied by education.

Specification 2 shows that our results seem fairly robust to our choice of which variables to include in the preference probability and which to include in the restriction probability. It should also be noted that the log-likelihood is larger (less negative) for specification 1 than for specification 2.

In discussing the estimation results, we focus on the marginal effect of each variable on the probability of being observed with activity level 3, doing many activities[16] (for the choice probabilities) and on the marginal effect of each variable on the probability of having the full capability set, C_3, (for the restriction probabilities). In assessing the marginal effects on the observed probability, Q_j, it is important to remember that some variables work through the preference probability, P_j, and some through the restriction probability $r(C_j)$. A positive marginal effect can be due to an increase in the desire to do many activities if the variable affects the preference relationship (is included among the *X*-variables). Or, it can be because of a decrease in the probability of being restricted in one's freedom of movement (less chance of being restricted to doing few activities) if the variable affects the restriction probability (is included among the *Z*-variables).

The probability of Italian women wishing to do many activities decreases with age and increases with education in both the specifications we look at. Including health and work in the preference probability, as is done in specification 2, increases the marginal effects of the other preference variables compared to specification 1. The marginal effect (on having activity level 3) of age is negative, decreasing the probability of doing 4 or more activities by 0.7 percentage points both in specification 1 and 2. Having a university degree increases the probability of doing many

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[16] Marginal effects for continuous variables are the derivatives of the Q probabilities (the probability of being observed in one of the states) with respect to a change in the variable. Marginal effects for dummy variables are the changes in the Q probabilities when the dummy goes from 0 to 1. The marginal effect for psychological violence is also for a change from 0 to 1, even though it is continuous. Since most women answer no to all the questions on psychological violence, the median size of this variable is 0 and it is natural to look at the change from 0 to answering yes to one question. Note that the marginal effects for each variable sums to zero across states. The base category for the choice probabilities is activity level 3: doing 4 or more activities, and the base category for the restriction probabilities is the full capability set C_3

activities by 29 percentage points in specification 1 and by 36 points in specification 2. A high school degree increases the probability by 19 percentage points in specification 1 and by 22 percentage points in specification 2. Education might be a proxy for income, with educated women being more involved in many activities not only for socio-cultural reasons, but also because activities are costly.

Table 4. Parameter estimates and marginal effects, 17,350 observations. Specification 1.

	Parameter	**Marginal effect**	**Parameter**	**Marginal effect**	**Parameter**	**Marginal effect**
Preference variables, *X*	**Activity index=1 0-2 activities**		**Activity index= 2 3 activities**		**Activity index= 3 4-8 activities**	
Woman's Age	0.0630*	0.0087*	0.0112*	-0.0017*	-	-0.0071*
	(0.0056)	*(0.0005)*	*(0.0034)*	*(0.0005)*		*(0.0006)*
Woman Degree[1]	-4.2028*	-0.1865*	-1.6899*	-0.1054*	-	0.2919*
	(1.2684)	*(0.0108)*	*(0.3336)*	*(0.0114)*		*(0.0134)*
Woman High School[1]	-1.7174*	-0.1359*	-0.7930*	-0.0551*	-	0.1910*
	(0.1697)	*(0.0084)*	*(0.1002)*	*(0.0082)*		*(0.0102)*
Constant	-4.0309*		-1.4398*		-	
	(0.3546)		*(0.2145)*			
Restriction variables, *Z*	**Capability set C_1** $C_1 = \{1\}$		**Capability set C_2** $C_2 = \{1, 2\}$		**Capability set C_3** $C_3 = \{1, 2, 3\}$	
Woman is Healthy[1]	-0.5048*	-0.0305*	-0.3303*	0.0016	-	0.0289*
	(0.0826)	*(0.0047)*	*(0.1312)*	*(0.0031)*		*(0.0050)*
Woman Works[1]	-0.2057*	-0.0173*	-0.4561*	-0.0051	-	0.0225*
	(0.0729)	*(0.0048)*	*(0.1324)*	*(0.0035)*		*(0.0057)*
Psychological violence by partner.[1]	0.0744*	0.0050*	0.0102	-0.0013	-	-0.0036*
	(0.0202)	*(0.0014)*	*(0.0310)*	*(0.0009)*		*(0.0014)*
Physical or sexual violence by partner.[1]	-0.3474*	-0.0199*	-0.0554	0.0049		0.0149
	(0.1506)	*(0.0079)*	*(0.2104)*	*(0.0055)*		*(0.0093)*
Age Difference	0.0208*	0.0779*	0.0137	0.0031	-	-0.0810*
	(0.0086)	*(0.0088)*	*(0.0129)*	*(0.0090)*		*(0.0141)*
Central Italy[1]	0.1111	0.0082	0.0658	-0.0008	-	-0.0074
	(0.0978)	*(0.0067)*	*(0.1668)*	*(0.0047)*		*(0.0072)*
Southern Italy[1]	0.5733*	0.0545*	0.7134*	0.0077	-	-0.0622*
	(0.0803)	*(0.0061)*	*(0.1794)*	*(0.0047)*		*(0.0067)*
Partner Low Education[1]	0.8334*	0.0849*	1.0573*	0.0143*	-	-0.0992*
	(0.0814)	*(0.0074)*	*(0.2311)*	*(0.0065)*		*(0.0085)*
Constant	-2.3185*		-2.4345*		-	
	(0.1689)		*(0.4090)*			

The base category for the choice probabilities is activity level 3 (doing 4-8 activities) and the base category for the restriction probabilities is the full capability set C_3. The base category (in terms of dummies) is a woman who is not healthy, does not work and lives in northern Italy with a low education level and a non-violent partner with a high school degree or higher.

The marginal effects are evaluated for the base category at the average age of the woman (46.48 years of age) and at the average age difference of the couple (3.44 years).

1) The marginal effect $\partial Q^R/\partial x$ is for discrete change of dummy variables (and psychological violence variable) from 0 to 1.

*p<0.05, log likelihood = -15980.5, standard errors in parenthesis

Table 5. Parameter estimates and marginal effects, 17,350 observations. Specification 2.

	Parameter	**Marginal effect**	**Parameter**	**Marginal effect**	**Parameter**	**Marginal effect**
Preference variables, *X*	**Activity index=1 0-2 activities**		**Activity index= 2 3 activities**		**Activity index= 3 4-8 activities**	
Woman's Age	0.0447*	0.0084*	0.0119*	-0.0017*	-	-0.0068*
	(0.0042)	*(0.0005)*	*(0.0042)*	*(0.0005)*		*(0.0007)*
Woman Degree[1]	-2.4978*	-0.2319*	-2.0155*	-0.1267*	-	0.3586*
	(0.4829)	*(0.0117)*	*(0.5058)*	*(0.0121)*		*(0.0179)*
Woman High School[1]	-1.2956*	-0.1631*	-0.8720*	-0.0563*	-	0.2195*
	(0.1169)	*(0.0090)*	*(0.1251)*	*(0.0085)*		*(0.0115)*
Woman is healthy[1]	-0.4615*	-0.0709*	-0.2565*	-0.0094	-	0.0803*
	(0.0661)	*(0.0096)*	*(0.0662)*	*(0.0084)*		*(0.0118)*
Woman Works[1]	-0.2740*	-0.0359*	-0.2539*	-0.0219*	-	0.0579*
	(0.0657)	*(0.0105)*	*(0.0655)*	*(0.0085)*		*(0.0123)*
Constant	-2.6076*		-1.3976*		-	
	(0.2728)		*(0.2771)*			
Restriction variables, *Z*	**Capability set C_1** $C_1 = \{1\}$		**Capability set C_2** $C_2 = \{1, 2\}$		**Capability set C_3** $C_3 = \{1, 2, 3\}$	
Psychological violence by partner.[1]	0.1485*	0.0012	0.0306	0.0006		-0.0018
	(0.0508)	*(0.0008)*	*(0.0245)*	*(0.0007)*		*(0.0014)*
Physical or sexual violence by partner.[1]	-0.5604	-0.0062	-0.1968	-0.0039	-	0.0101
	(0.4130)	*(0.0044)*	*(0.1623)*	*(0.0035)*		*(0.0078)*
Age Difference	0.0799*	0.0321*	0.0050	0.0219*	-	-0.0540*
	(0.0254)	*(0.0084)*	*(0.0091)*	*(0.0068)*		*(0.0150)*
Central Italy[1]	0.5864	0.0014	0.0061	-0.0003	-	-0.0011
	(0.4961)	*(0.0033)*	*(0.1093)*	*(0.0027)*		*(0.0055)*
Southern Italy[1]	1.6967*	0.0248*	0.5142*	0.0116*	-	-0.0364*
	(0.5188)	*(0.0061)*	*(0.1176)*	*(0.0044)*		*(0.0057)*
Partner Low Education[1]	2.5723*	0.0457*	0.7702*	0.0167*	-	-0.0624*
	(1.1991)	*(0.0072)*	*(0.1552)*	*(0.0050)*		*(0.0070)*
Constant	-6.2498*		-1.9932*		-	
	(1.7125)		*(0.3371)*			

The base category for the choice probabilities is activity level 3 (doing 4-8 activities) and the base category for the restriction probabilities is the full capability set C_3; the base category (in terms of dummies) is a woman who is not healthy, does not work and lives in northern Italy with a low education level and a non-violent partner with a high school degree or higher.

The marginal effects are evaluated for the base category at the average age of the woman (46.48 years of age) and at the average age difference of the couple (3.44 years).

1) The marginal effect $\partial Q^R/\partial x$ is for discrete change of dummy variables (and psychological violence variable) from 0 to 1.

$^*p<0.05$, log likelihood = -15987.04, standard errors in parenthesis

In both specifications, increased age difference, living in southern Italy or having a partner with a low education increase the probability of being restricted in one's ability to be active, decreasing the probability of having a full capability set C_3 . Living in the south of Italy decreases the probability of having a full capability set by 6 percentage points in specification 1 and by 4 percentage points in specification 2. This could be due to differences in cultural norms for the behaviour of women, since the south has more gender inequality than the north. The largest marginal effect (among the variables in the restriction probabilities) is found for women with a low educated partner. Having a partner with a low education decreases the probability of having a full capability set by 10 and 6 percentage points in specification 1 and 2 respectively. As with the education of the women, this variable may be a proxy for income.

The difference between our two specifications is in whether health and work are included in the preference relationship or in the restriction relationship. For this reason, the size of the marginal effects in the two cases cannot be directly compared, but one would expect their signs to be the same (which is the case in our estimations). In specification 1, being healthy decreases the probability of being restricted to capability set C_1 and C_2 thereby increasing the probability of having the full capability set C_3 by 3 percentage points. In specification 2, being healthy increases the desire for being active, thereby increasing the probability of having activity level 3 by 8 percentage points. Work has a similar positive (significant) effect as health, but the effect is weaker. As with education, work can be considered a proxy for income.

Of the variables considered in our two specifications, the violence variables seem to have the least stable (and least significant) relationship to freedom of movement (aside from living in central Italy). Even when significant, the effects of violence are small. The marginal effects for psychological and physical violence are insignificant in specification 2, while they are larger and significant in specification 1. In specification 1, being exposed to psychological violence (answering positively to one of the questions about psychological violence) increases the probability of being constrained and thereby decreases the probability of having the full capability set C_3 by 0.4 percentage points, while being exposed to physical or sexual violence is associated with a lower probability of having a restricted capability set C_1 by 2 percentage points. In specification 2 only the parameter estimate of psychological violence for activity level 1 is significant, while the marginal effects are all insignificant (some could of course be significant if we choose a different base category when calculating the marginal effects). The sign of this parameter is the same for both specifications. These results conform to the descriptive statistics of Table 3, where those doing 4-8 activities experience a greater prevalence of such violence than those doing fewer activities. It is not obvious why psychological and sexual/physical violence should have opposite effects on the probability of being restricted in one's freedom of movement. Psychological violence might be considered a controlling behaviour like behaviour that limits freedom of movement. Anand and Santos 2007 find similar results: fears and vulnerability have a negative impact on freedom of movement, There is not sufficient evidence in the literature, either for developing countries or for industrialised countries, to establish the direction of causation between women's activities (or autonomy or income) and violence[17]. Physical violence may restrain women's freedom of movement, but, on the other hand, it can also be considered a reactive behaviour, increasing as the control of the women decreases (for example, when she engages in more activities outside the home). In any case, our results do not give a clear answer and must be left to further inquiry.

In order to interpret the results, we include 2 graphical illustrations of the probabilities described by the parameters shown in tables 4 and 5. Figure 1 reports 3 different probabilities and how they change with the age of the woman. P_3 is the choice probability, i.e. the probability of preferring activity level 3. This would be the chosen functioning if no women were restricted. Q_3 is

[17] See literature discussion in section 4

the probability of being observed with activity level 3 (i.e. with functioning equal to activity level 3), taking into account that the women are restricted. Both probabilities are calculated for women who are healthy, do not work, live in the North of Italy, have a low educational level and a non-violent partner with a high education. Q_3^* is the same as probability Q_3, except that it applies to a woman from the South of Italy with a low educated partner.

The gap between P_3 and Q_3 is the difference between the percentage of women wishing to do 4-8 activities and the percentage of these women who are actually observed doing this many activities. The difference therefore illustrates the effect of women being restricted in their freedom of movement. Note that it takes into account that some women who are restricted do not wish to do many activities. The gap decreases as women age, due to women preferring to do less activities as they grow older. The probability Q_3^* is lower than Q_3 because women from the South of Italy with a low educated partner have a higher probability of being restricted in their freedom of movement than women from the North with a high educated partner.

Figure 2 is similar to figure 1 but it represents the results from specification 2 given in table 5. It shows that moving the health and work variables from the restriction probabilities to the preference probability does not greatly affect the underlying probabilities, P_3, Q_3, and Q_3^*.

Figure 1 The probability of preferring to do 4-8 activities, P_3 and the probability of being observed doing so many activities, Q_3 and Q_3^* (specification 1).

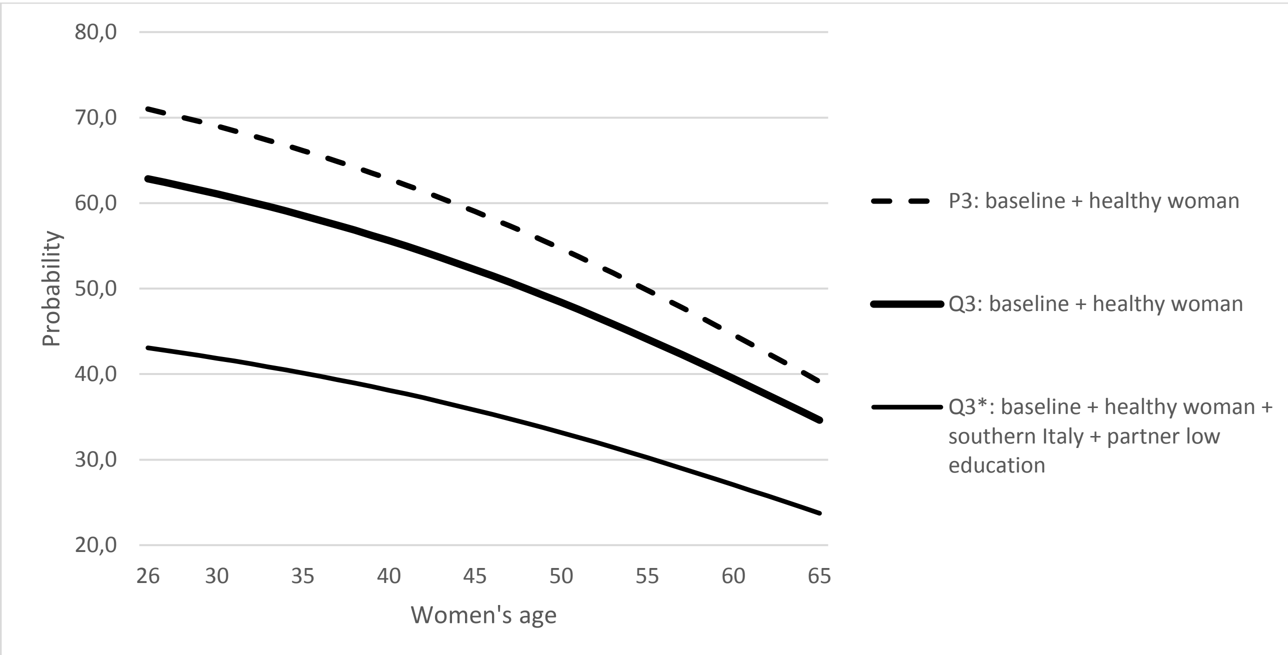
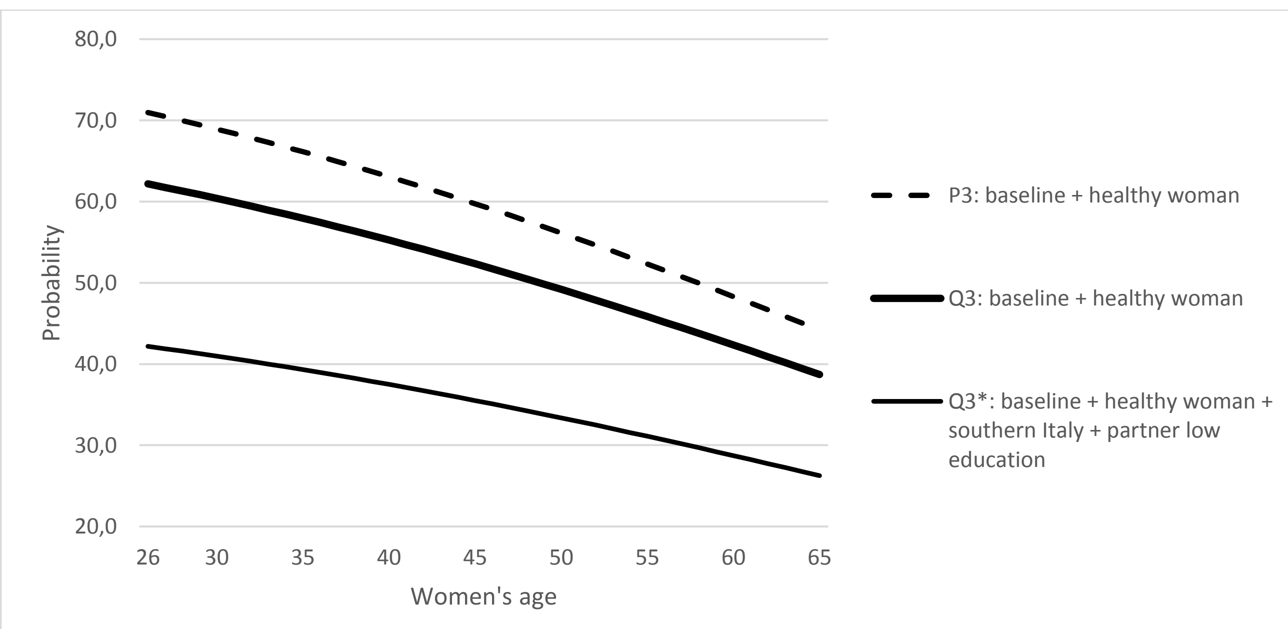


Figure 2 The probability of preferring to do 4-8 activities, P_3 and the probability of being observed doing so many activities, Q_3 and Q_3^* (specification 2).



6.1 Alternative specifications – sensitivity analysis

We have also estimated the model with health in both the choice and the restriction probabilities[18]. In this case, the parameters for the health dummy are not significant at the 95% level in the choice probability (but are at the 90% level), while in the restriction probability the health parameter for activity level equal 1 is significant, but not the one for activity level 2. The in-sample predictions are close to those of specification 1. This might indicate that specification 1 is to be preferred, but we believe the ultimate choice of specification must rest on information or assumptions outside the data (it is in general the case that latent variables can only be identified using outside restricting assumptions of either a stochastic or functional nature, otherwise they would not be considered latent). To us it seems more intuitive to model health and work (and the other variables in the restriction probability) as influencing the restrictions women face than modelling them as determining preferences.

It is a possibility that the effect of the violence variables is reduced due to multicollinearity with the partner variables (age difference, partner's education, and where they live). The discussion above indicates that there might be simultaneity between the number of activities a woman pursues and her experience of violence by her partner. To check whether multicollinearity is a problem, we have re-estimated specification 2 with only the violence variables in the restriction probabilities. In this case, we get that the parameters for both the violence variables are significant for having capability set C_1 and not significant for capability set C_2, as in specification 1, and the signs of the parameters are the same as in specification 1 and 2. It would therefore seem that multicollinearity is not a significant problem for the significance of the violence variables.

Furthermore, we have checked for any simultaneity bias by re-estimating both specifications 1 and 2 without the violence variables. Compared to specifications 1 and 2, we find only minor

[18] Tables with results are available from the authors upon request

changes in the parameter estimates and in the marginal effects. The number of women restricted in their freedom of movement declines by 0.9 per cent in specification 1, while it declines by 5.5 per cent in specification 2. The better robustness of specification 1 in this context is another reason to prefer it to specification 2. So it seems that violence is not a main determinant (if a determinant at all) of whether a woman is constrained in her freedom of movement and thereby cannot lead to strong multicollinearity or simultaneity problems.

As mentioned in section 4, we have not included violence by non-partner in our estimations because this variable is not a characteristic of the woman or her situation in the home, but is a characteristic of the varying activities (our model does not take into account alternative specific variables).

7. Counterfactual predictions

Above we discussed the effect of the different variables separately. In this section we will look more closely at the aggregate behaviour of our model, looking at in-sample predictions of the number of women who are constrained and how the restriction probabilities vary for different groups of women

A measure of the capability of having freedom of movement can be found by simulating the number of women who do not have the full capability set, i.e. are constrained to capability sets C_2 or C_1 (the expected number of women with restricted capability sets is found by summing the individual restriction probabilities $r_i(C_j)$ across all i individuals in our sample). In the following tables, we report results for both specifications 1 and 2, but mainly limit our comments in the text to our preferred specification, specification 1.

Table 6. Number of women predicted to be constrained. Number and per cent of all women, 17,350 observations.

	Constrained to capability set $C_1 = \{1\}$	Constrained to capability set $C_2 = \{1,2\}$	Total constrained
Specification 1			
Expected number of constrained women	2,201	2,117	4,317
- Percent constrained	12.7	12.2	24.9
Specification 2			
Expected number of constrained women	662	3,390	4,051
- Percent constrained	3.8	19.5	23.4

Table 6 shows the expected number of women to be constrained. The expected number of women constrained to capability set C_1 (with a capability set, $C_1 = \{1\}$, consisting only of activity level 1) is 2,201, consisting of 12.7 per cent of the women in the sample. These women cannot choose activity level equal 2 or 3. There are 2,117 women constrained to capability set, $C_2 = \{1,2\}$, which is 12.2 per cent of the women in the sample. Women with choice set $C_2 = \{1,2\}$ are prevented from choosing activity level equal 3 (doing 4 or more activities). The total number of women who are constrained, those with either capability set $C_1 = \{1\}$ or $C_2 = \{1,2\}$, is thereby equal to 4,317, which is 24.9 per cent of the sample.

Using the estimated parameters, we also simulate how many women would change their level of activities if no one is restricted. This implies that all women are given the capability set $C_3 = \{1,2,3\}$, so their choices are solely determined by their preference probabilities $P_j(C)$. Some of the women constrained to capability set $C_1 = \{1\}$ will now choose to become more active, ending up with activity level 2 or 3. Some of the women constrained to capability set $C_2 = \{1,2\}$ will now choose to have activity level equal 3. Note that our model specification implies that some women who are constrained to capability set C_2, having chosen activity level 1, now move to activity level 3.

Table 7. Change in the number of women in each activity level if there are no restrictions. Number and per cent of all women, 17,350 observations.

	Activity index=1 0-2 activities	Activity index=2 3 activities	Activity index= 3 4-8 activities
Specification 1			
Net change if all women are simulated to be unconstrained	-2,336	-547	+2,883
- Percent change	-13.5	-3.2	+16.6
Specification 2			
Net change if all women are simulated to be unconstrained	-1,600	-1,071	+2,671
- Percent change	-9.2	-6.2	+15.4

Table 7 shows that 2,336 women would leave activity level 1 if they had the full capability set and could choose to do more activities. Of these 2,336 women, 1,874 were constrained to capability set C_1 and 462 were constrained to capability set C_2.[19] [20] Those leaving activity level equal to 1 is 50.6 per cent of the women originally in this state (which is 13.5 per cent of all women in our sample). The table also shows that the net change for activity level 2 is a loss of 547 women, consisting partly of women entering activity level 2 from activity level 1 and partly of women leaving activity level 2 for activity level 3. Finally, 16.6 per cent of the population of women go from being restricted to either activity level 1 or 2 to choosing activity level 3 (doing 4 or more activities).

[19] In the first case, there are 1,874 women who leave activity level 1 from among those who were previously constrained to this level. It is calculated as the expected value of $(1-\hat{P}_1)\cdot\hat{r}(C_1)$, where $\hat{P}$ and $\hat{r}$ are predicted probabilities based on our estimates.

[20] In the second case, there are 462 women who leave activity level 1 from among those who were previously constrained to activity level 2 or lower. It is calculated as the expected value of $((\hat{P}_1|C_2-P_1)\cdot\hat{r}(C_2)$, where $\hat{P}_1|C_2$ is the predicted condtional probability of choosing level 1 conditional on being restricted to level 2 or lower.

Table 8. Predicted probability of being constrained to either capability set $C_1 = \{1\}$ or $C_2 = \{1,2\}$ for some categories of individuals. Per cent and 95% confidence interval.

	Predicted probability of being constrained	95 % confidence interval	
Base category[1]			
Specification 1	16.5	10.7	22.3
Specification 2	12.4	5.6	19.1
Minimum probability category[2]			
Specification 1	6.6	2.6	10.7
Specification 2	10.1	3.7	16.6
Maximum probability category[3]			
Specification 1	53.7	49.9	57.5
Specification 2	48.1	42.6	53.7

[1] The base category is a woman with a non-violent partner who is 3.44 years older (the average age difference) and where the partner has at least a high school education. The woman is not healthy, does not work and lives in northern Italy.

[2] The minimum probability category is a woman with a physically or sexually violent partner (we do not imply any causality from violence to freedom of movement) who is 0.60 years younger (the average age difference minus one standard deviation) and where the partner has at least a high school education. The woman is healthy, works and lives in northern Italy.

[3]The maximum probability category is a woman with a psychologically violent partner, who is 7.49 years older (the average age difference plus one standard deviation). The partner has low level of education. The woman is not healthy, does not works and lives in southern Italy.

Table 8 shows the variability in the probabilities resulting from our estimated model. It illustrates how the probabilities vary according to changes in the explanatory variables, showing maximum and minimum probabilities along with their 95% confidence intervals. The table reports the predicted probability of being constrained to capabilities set C_1 or C_2 for three types of individuals. The base category is a woman with a non-violent partner who is 3.44 years older (the average age difference) and where the partner has at least a high school education. The woman is not healthy, does not work and lives in northern Italy. For specification 1, the base category has a 16.5 per cent probability of being constrained, which is lower than the 24.9 per cent we find over the whole sample population (see Table 6). Note that Table 8 only shows the restriction probabilities. The woman's education does not enter these probabilities, but only the preference probabilities.

The minimum probability category is a woman with a physically or sexually violent partner (this does not imply any causality between violence and freedom of movement, it just reflects the positive correlation that we find in our data between violence and participation in activities) who is 0.60 years younger (the average age difference minus one standard deviation) and where the partner has at least a high school education. The woman is healthy, works and lives in northern Italy. For this category, the predicted probability of being constrained is equal to 6.6 per cent.

The maximum probability category is a woman with a psychologically violent partner, who is 7.49 years older (the average age difference plus one standard deviation). The partner has a low level of education. The woman is not healthy, does not work and lives in southern Italy. For this category, the predicted probability of being constrained is equal to 53.7 per cent.

To illustrate the accuracy of our estimation method we have also reported in Table 8 the confidence intervals for the predicted probabilities. We see that the parameter estimates give a variation in predicted probability from 6.6 to 53.7 per cent, while the 95 per cent confidence intervals give a range from 2.6 to 57.5 per cent. For the three categories shown in table 8, the 95 confidence interval is plus minus 4 to 7 percentage points.

8. Conclusions

Our paper is innovative and important for showing a new approach to measuring capabilities. It also provide an application of the methodology to an aspect of gender inequality, women's freedom of movement, which is potentially very interesting to measure in many other cultural, social and religious contexts.

We have used a Random Scale Model to measure the capability of freedom of movement for Italian women. Our estimates imply that between 23 and 25 per cent of women in our sample are constrained in their movements (have limited capability sets). If we remove their constraints, between 15.4 and 16.6 per cent of the population of women would choose to exert more freedom of movement, changing their functionings (doing 4 or more activities). Not all constrained women would change because some will prefer doing few activities even when unconstrained.

We find that the probability of women being constrained in their freedom of movement increases with increased age difference between the partners, with living in southern Italy or having a partner with a low education. Being healthy and being exposed to physical violence are both positively correlated to freedom of movement, while being exposed to psychological violence is negatively related to freedom of movement. This result does not imply any causation from violence to freedom of movement, it just reflects the correlation that we find in our data. Further investigations are needed on this issue because the relation between violence and activities performed is complex and the direction of causality is very difficult to identify. A panel data set would be very helpful in order to explore the dynamic relation between violence and movement over the life cycle. Our paper also has some limitations due to lack of information on income and children.

The methodology proposed in this paper can be extended to measure other capabilities and to more complex problems. The index for functionings could have more than 3 values and the capability sets do not need to be ranked. It is also possible to consider more than one capability. The functional form of the structural part of the scale function can be more complex, not linear, for instance a Box-Cox, and the restriction probabilities can have different functional forms. Further development of this approach could include looking at the capability of men to work and to provide child care or the capability of women to actively participate into politics.

Within the capability approach, it is important to develop methods to input the restrictions in freedom faced by individuals when their restrictions are unobserved. Having such methods increases the applicability of the capability approach and opens up many interesting research questions that would otherwise be difficult to analyse. Even so, it is important to acknowledge that the inference measures presented in our paper are less precise than what we would get if we could measure the restrictions directly.

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Appendix A: Expected utility is increasing in opportunities

In the following we discuss in more detail our assertion in section 2 that expected utility is increasing in opportunities. It is based on a discussion of the indirect random scale. This is analogous to indirect utility, which gives the maximal attainable utility when faced with given choice set. It reflects both preferences and the choice set. Towards the end, we also discuss how one might analyse how welfare varies across households, though we do not do this in the present paper.

The conditional indirect random scale $V_C(\varepsilon_1,..., \varepsilon_H)$ will under our distributional assumptions be extreme value distributed. Let $\bar{V}(C_s)$ be the deterministic part (representative part) of the conditional indirect scale, conditional on choice set C_s being available, defined as $\bar{V}(C_s) = \mathrm{E} \max_{k\in C_s} U_k$. Due to the distributional assumptions about U_k, it is well known that one obtains

$$\bar{V}(C_s) = \log\left(\sum_{k\in C_s} \exp(v_k)\right), \qquad \text{(a1)}$$

where it should be noted that the evaluation $\exp(v_k)$ is the same across choice sets.

From equation (C.1) it follows that in our case, with ranked latent capability sets, we have $\bar{V}(C_1) < \bar{V}(C_2) < \cdots < \bar{V}(C_H)$. In other words, the conditional indirect scale is increasing in the size of the opportunity set.

As a measure of the well-being of individuals, it thereby has the desired property of valuing opportunities instead of only choices. In the following analysis of freedom of movement, we will not be using this measure, since we only consider a one-dimensional concept of freedom and thereby can directly say that it is better to have an unconstrained freedom of movement than a constrained one. If we were trying to evaluate different combinations of freedoms, then having a measure of the above type would be valuable. The unconditional representative indirect scale function is defined by

$$\mathrm{E}\bar{V}(C) = \sum_{s=1}^{H} \bar{V}(C_s) \cdot r(C_s) = \sum_{s=1}^{H} r(C_s) \cdot \log\left(\sum_{k \in C_s} \exp(v_k)\right). \qquad \text{(a2)}$$

Thus the conditional indirect scale function is the mean value of the chosen functioning restricted to a given capability set C_s, whereas the unconditional indirect scale function is the mean value of the conditional indirect scale where the mean is taken over the possible capability sets. By means of $\mathrm{E}\bar{V}(C)$ one may analyse how welfare (in an ordinal sense) varies across households (identified by covariate values) for given selected capability sets. See Dagsvik (2013) for more details on this and for a discussion of how to develop a welfare function and a capability adjusted income distribution based on the indirect random scale function.

Appendix B: Identification

In the following we illustrate how identification can be achieved by introducing observed discrete covariates into the preference terms $\{v_j\}$ and the restriction probabilities. To see that the model can be identified in this case, we show that the unrestricted *choice probabilities* and the *restriction probabilities* can be expressed as functions of the observable probabilities, Q_j. By this we mean that, within subsamples of observationally identical households, all the probabilities $r(C_k)$ and $P_j(C_k)$, $j \in C_k$, $k = 1, 2, \ldots, L$, can in principle be estimated by replacing the respective observable probabilities by their empirical counterparts, provided the subsamples are sufficiently large.

To see that introducing discrete covariates can identify our model, consider a two state model. From equations (1) we have

$$P_1(C_2) = \frac{\exp(X_i\beta)}{1+\exp(X_i\beta)}, \qquad \text{(b1)}$$

$$P_2(C_2) = \frac{1}{1+\exp(X_i\beta)}. \tag{b2}$$

From this, together with equations (3) and (5) we get

$$Q_1 = r(C_1) + P_1(C_2)\cdot r(C_2) = \frac{\exp(X_i\beta)+\exp(Z_i\gamma)+\exp(X_i\beta+Z_i\gamma)}{1+\exp(X_i\beta)+\exp(Z_i\gamma)+\exp(X_i\beta+Z_i\gamma)} \tag{b3}$$

$$Q_2 = P_2(C_2)\cdot r(C_2) = \frac{1}{1+\exp(X_i\beta)+\exp(Z_i\gamma)+\exp(X_i\beta+Z_i\gamma)}. \tag{b4}$$

Rewriting equations (b3 and b4) as odds-ratios we get

$$\frac{Q_1}{Q_2} = \exp(X_i\beta)+\exp(Z_i\gamma)+\exp(X_i\beta+Z_i\gamma). \tag{b5}$$

Assume there is one dichotomous explanatory variable in each of the vectors so that $X_i=\{1,x_i\}$ and $Z_i=\{1,z_i\}$, with $x_i \in \{0,1\}$ and $z_i \in \{0,1\}$. This means that we can view women as belonging to one of four groups composed of the four different possible combinations of x_i and z_i (note that as the number of variables increases linearly, the number of possible combinations increases geometrically). We therefore get the following four equations for the four different subgroups among those who might be restricted in their choices:

$$\frac{Q_1}{Q_2} = \exp(\beta_0)+\exp(\gamma_0)+\exp(\beta_0+\gamma_0), \qquad \text{if } x_i = 0,\ z_i = 0 \tag{b6}$$

$$\frac{Q_1}{Q_2} = \exp(\beta_0+\beta_1)+\exp(\gamma_0)+\exp(\beta_0+\beta_1+\gamma_0), \qquad \text{if } x_i = 1,\ z_i = 0 \tag{b7}$$

$$\frac{Q_1}{Q_2} = \exp(\beta_0)+\exp(\gamma_0+\gamma_1)+\exp(\beta_0+\gamma_0+\gamma_1), \qquad \text{if } x_i = 0,\ z_i = 1 \tag{b8}$$

$$\frac{Q_1}{Q_2} = \exp(\beta_0+\beta_1)+\exp(\gamma_0+\gamma_1)+\exp(\beta_0+\beta_1+\gamma_0+\gamma_1), \qquad \text{if } x_i = 1,\ z_i = 1 \tag{b9}$$

where the parameter vectors are given as $\beta = \{\beta_0, \beta_1\}'$ and $\gamma = \{\gamma_0, \gamma_1\}'$. This is four equations in four parameters, so there is now a possibility of the model being identified. Since these equations are non-linear, one cannot generally use a simple counting rule to generally establish identifiability, but the above indicates that a fairly small set of discrete explanatory variable should in practice lead to identification without requiring assumptions about who might be at risk of being restricted.

In general, the above model is only identified if we exogenously decide that a subgroup is never restricted, but in our case we have enough discrete explanatory variables to identify the model in the manner described above without needing to specify an unrestricted subgroup.

For continuous variables identification is readily established. Let

$$R(X,Z) = Q_1(X,Z)/Q_2(X,Z). \tag{b10}$$

Assume that $R(X,Z)$ is not constant, as a function of *X* for given *Z* and as a function of *Z* for given *X*. From (b5) we have that

$$R(X,Z) = \frac{Q_1}{Q_2} = \exp(X_i\beta) + \exp(Z_i\gamma) + \exp(X_i\beta + Z_i\gamma). \tag{b11}$$

Assume that the function $R(X,Z)$ is known for all vectors (*X*, *Z*) belonging to some set *A*. If (b11) holds for all vectors in *A* we shall show that the vectors of coefficients β and γ are uniquely determined. From (b11) it follows that

$$\frac{\partial R(X,Z)}{\partial X_k} = \exp(X\beta)\beta_k(1+\exp(Z\gamma)), \quad \frac{\partial^2 R(X,Z)}{\partial X_k^2} = \exp(X\beta)\beta_k^2(1+\exp(Z\gamma)) \tag{b12}$$

$$\frac{\partial R(X,Z)}{\partial Z_k} = \exp(Z\gamma)\gamma_k(1+\exp(X\beta)) \quad \text{and} \quad \frac{\partial^2 R(X,Z)}{\partial Z_k^2} = \exp(Z\gamma)\gamma_k^2(1+\exp(X\beta)). \tag{b13}$$

From (b12) and (b13) it follows that

$$\frac{\partial^2 R(X,Z)}{\partial X_k^2} / \frac{\partial R(X,Z)}{\partial X_k} = \beta_k \tag{b14}$$

and

$$\frac{\partial^2 R(X,Z)}{\partial Z_k^2} / \frac{\partial R(X,Z)}{\partial Z_k} = \gamma_k . \qquad \text{(b10)}$$

Since the partial derivatives of $R(X,Z)$ are known, the relations above demonstrate that β_k and γ_k are identified for $k > 0$. It remains to show that the constant terms β_0 and γ_0 are identified. Note that we can write (b11) as

$$R(X,Z)+1 = (1+\exp(X\beta))(1+\exp(Z\gamma)) = (1+g_1(X)\exp\beta_0)(1+g_2(Z)\exp\gamma_0) \qquad \text{(b11)}$$

where g_1 and g_2 are *known* functions, due to the identification results above. Let X' be in *A* and be different from *X*. Hence, we obtain that

$$\frac{R(X,Z)+1}{R(X',Z)+1} = \frac{1+g_1(X)\exp\beta_0}{1+g_1(X')\exp\beta_0} \qquad \text{(b12)}$$

which implies that

$$\left(g_1(X')\cdot\frac{R(X,Z)+1}{R(X',Z)+1} - g_1(X)\right)\exp\beta_0 = 1 - \frac{R(X,Z)+1}{R(X',Z)+1}. \qquad \text{(b13)}$$

Similarly, it follows that

$$\left(g_2(Z')\cdot\frac{R(X,Z)+1}{R(X,Z')+1} - g_2(Z)\right)\exp\gamma_0 = 1 - \frac{R(X,Z)+1}{R(X,Z')+1} \qquad \text{(b14)}$$

where Z' is in *A* and is different from *Z*. Since by assumption $\partial R(X,Z)/\partial X \neq 0$ and $\partial R(X,Z)/\partial Z \neq 0$ the last two equations show that β_0 and γ_0 are identified (the equations only consist of known fuctions in the variables *X* and *Z)*. Note that establishing that identification is possible in theory does not necessarily mean that it is always achieved in practice.

Appendix C: Sample selection, Cumulative distribution of activities, Health and Psychological violence variables

Table C1: Sample selection.

	Change	Number women
Total sample		25,065
Un- married and not partnered women	-4,182	20,883
Age <26 and age>65 women	-2,942	17,941
Unable to work	-44	17,897
Missing age of the partner	-547	17,350
Our sample		17,350

Table C2: Cumulative distribution of the number of activities a woman participates in*.

Number of activities	Number of women	Per cent of total	Cumulative distribution
0	169	1.0%	1.0%
1	1 361	7.8%	8.8%
2	3 018	17.4%	26.2%
3	3 423	19.7%	45.9%
4	3 131	18.0%	64.0%
5	2 735	15.8%	79.8%
6	2 050	11.8%	91.6%
7	1 172	6.8%	98.3%
8	291	1.7%	100.0%

* We consider that a woman participates in an activity if she answers "often or sometimes" to the first five questions, answers "once a week or more" to the questions on going out in the evening and going shopping or answers "yes" to the question on driving.

Table C3: Women who are healthy; not having health problems*, 17,350 women

	Mean		
	Activity index=1	Activity index=2	Activity index=3
Does not have headache=1; 0 otherwise.	0.660	0.687	0.726
Does not have toothache	0.888	0.913	0.938
Does not have a disturbed stomach, nausea or vomit	0.850	0.876	0.886
Does not have an irregular heartbeat	0.832	0.879	0.900
Does not experience weakness and fatigue	0.701	0.772	0.799
Does not suffer from insomnia	0.785	0.829	0.860
Does not suffer from depression	0.893	0.934	0.958
Does not suffer from a weakening of memory or of the capacity to concentrate	0.879	0.916	0.935
Does not have recurrent pain in other parts of the body	0.711	0.765	0.801
Does not have other health problems	0.911	0.928	0.940

* Possible answers to the health questions are “often”, “sometimes”, “rarely”, “never”, “no response” and “do not know”. We only consider those who answer “never” to not have the health problem in question.

Table C4: Women who have been subjected to psychological violence by partner, 17,350 women

	Mean		
	Activity index=1	Activity index=2	Activity index=3
Partner is angry if you talk to another man*	0.146	0.140	0.127
Partner humiliates you in front of others*	0.092	0.082	0.074
Partner criticizes appearance*	0.099	0.111	0.096
Partner criticizes housework*	0.107	0.111	0.109
Partner ignores you*	0.182	0.164	0.173
Partner insults or verbally abuses you*	0.111	0.105	0.090
Partner hinders contact with friends or family**	0.059	0.057	0.046
Partner hinders work**	0.056	0.036	0.028
Partner hinders studying**	0.051	0.036	0.038
Partner controls appearance**	0.019	0.014	0.012
Partner doubts faithfulness**	0.044	0.038	0.038
Partner controls the woman's movements**	0.011	0.011	0.008
Partner controls the woman's spending**	0.068	0.053	0.052
Partner hinders the women in having knowledge of family income**	0.022	0.015	0.015
Partner hinders use of his or the family's money**	0.015	0.010	0.007
Partner ruins or destroys your personal things**	0.008	0.005	0.005
Partner harms or threatens to harm his children**	0.005	0.003	0.002
Partner harms or threatens to harm those close to you**	0.003	0.001	0.001
Partner harms or threatens to harm his animals**	0.004	0.002	0.003

* Possible answers to this question is "often", "sometimes", "rarely", "never", "no response" and "do not know". We consider all those who do not answer "never" to have been subjected to the psychological violence in question.

** Possible answers to this question is "yes", "no", "no response" and "do not know". We only consider those who answer "yes" to have been subjected to the psychological violence problem in question.