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CORRECTION

Correction to: Every Cubic Bipartite Graph has a Prime Labeling Except $K_{3,3}$

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1 Explanation of the error

In the labeling algorithm used to prove Lemma 7 and Theorem 1 in [6], the required labels are placed in order based on smallest odd prime factor (i.e. first all the multiples of 3 are placed on open vertices, then the remaining multiples of 5, and so on). For a given prime p and label ℓ_1 such that $p|\ell_1$ but $p' \nmid \ell_1$ for all odd primes $p' < p$, ℓ_1 must be placed on an unlabeled vertex v such that for any neighbor of v labeled with ℓ_2, the greatest common divisor of ℓ_1 and ℓ_2 is 1. The algorithm given in [6] ensures that ℓ_1 and ℓ_2 have no common prime factors less than or equal to p, but it fails to exclude the possibility of a common prime factor greater than p. For example, when placing the even multiples of 5 in Step 3 of the proof of Lemma 7, the algorithm ensures that the vertices receiving these labels have no neighbors that were previously labeled with a multiple of 15, but it fails to account for the possibility that a new label ℓ_1 might share a factor larger than 7 with a previously placed label ℓ_2, such as $\ell_1 = 110$ and $\ell_2 = 33$.

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2 Restatement of the affected results

Lemma 1 (*Lemma* 7 *in* [6]) *Let* $12 \le n \le 32$. *Every cubic bipartite graph on* $2n$ *vertices has an agreeable* $\{3,5,7\}$*-partial prime labeling.*

Proof The original proof ensures that all the multiples of 3, 5, and 7 can be placed in such a way that the labels on any pair of adjacent vertices have no common prime factor less than or equal to 7. Since $n \le 32$, there are no even labels that have both an odd prime factor less than or equal to 7 and a prime factor greater than or equal to 11; thus, we can conclude that the labeling given in the original proof is indeed an agreeable $\{3,5,7\}$-partial prime labeling. □

Theorem 1 (*Theorem* 1 *in* [6]) *Let* $4 \le n \le 32$. *Every cubic bipartite graph on* $2n$ *vertices has a prime labeling.*

Proof For $4 \le n \le 11$, the desired result is given by [5, Theorem 2], so we assume $12 \le n \le 32$. The original proof starts with the agreeable $\{3,5,7\}$-partial prime labeling ensured by Lemma 1 and proceeds by induction to add the labels whose smallest odd prime factor is greater than or equal to 11. Since $n \le 32$, there are no labels to place that have two or more prime factors greater than or equal to 11, so the argument in the original proof still holds. □

3 Further impact

With this new amended result, two conjectures previously thought to be proved remain open; namely, the conjecture that the generalized Petersen graph $P(n, k)$ is prime precisely when it is bipartite (i.e., when n is even and k is odd) [3–5], and the conjecture that the Knödel graph $W_{3,n}$ is prime for all even $n \ge 4$ [2].

Moreover, in their recent paper [1] Bani Mostafa A. and Ghorbani proved Conjecture 2 from [6], which combined with the original statement of Theorem 1 would have implied that a 2-regular graph has a prime labeling if and only if it has at most one odd component. However, the amended version of Theorem 1 means that this long-standing conjecture of Tout, Dabboucy and Howalla from [7] remains open.

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Declarations

Conflict of interest The author has no relevant financial or non-financial interests to disclose.

References

1. Bani Mostafa A, M.H., Ghorbani, E.: Hamiltonicity of a coprime graph. Graphs Combin. **37**, 2387–2395 (2021)
2. Mominul Haque, Kh.Md., Haque, M., Xiaohui, L., Yuansheng, Y., Pingzhong, Z.: Prime labeling on Knödel graphs $W_{3,n}$. Ars Combin. **109**, 113–128 (2013)
3. Prajapati, U.M., Gajjar, S.J.: Prime labeling of generalized Petersen graph. Int. J. Math. Soft Comput. **5**, 65–71 (2015)
4. Schluchter, S.A., Schroeder, J.Z., et al.: Prime labelings of generalized Peterson graphs. Involve **10**, 109–124 (2017)
5. Schluchter, S.A., Wilson, T.W.: Prime labelings of bipartite generalized Petersen graphs and other prime cubic bipartite graphs. Congr. Numer. **226**, 227–241 (2016)
6. Schroeder, J.Z.: Every cubic bipartite graph has a prime labeling except $K_{3,3}$. Graphs Combin. **35**, 119–140 (2019)
7. Tout, A., Dabboucy, A.N., Howalla, K.: Prime labeling of graphs. Natl. Acad. Sci. Lett. **11**, 365–368 (1982)