Full-fledged Real-Time Indexing for Constant Size Alphabets*

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Abstract

In this paper we describe a data structure that supports pattern matching queries on a dynamically arriving text over an alphabet of constant size. Each new symbol can be prepended to T in $O(1)$ worst-case time. At any moment, we can report all occurrences of a pattern P in the current text in $O(|P| + k)$ time, where $|P|$ is the length of P and k is the number of occurrences. This resolves, under assumption of constant size alphabet, a long-standing open problem of existence of a real-time indexing method for string matching (see [3]).

1 Introduction

Two main versions of the string matching problem differ in which of the two components – pattern P or text T – is provided first in the input (or is considered as fixed) and can then be preprocessed before processing the other component. The framework when the text has to be preprocessed is usually called *indexing*, as it can be viewed as constructing a text index supporting matching queries.

Real-time variants of the string matching problem are about as old as the string matching itself. In the 70s, existence of real-time string matching algorithms was first studied for Turing machines. For example, it has been shown that the language $\{P\#T|P \text{ occurs in } T\}$ can be recognized by a Turing machine, while the language $\{T\#P|P \text{ occurs in } T\}$ cannot [10]. In the realm of the RAM model, the real-time variant of pattern-preprocessing string matching has been extensively studied, leading to very efficient solutions (see e.g. [4] and references therein). The indexing variant, however, still has important unsolved questions.

Back in the 70s, Slisenko [18] claimed a real-time algorithm for recognizing the language $\{T\#P|P \text{ occurs in } T\}$ on the RAM model, but its complex and voluminous full description made it unacknowledged by the scientific community, and the problem remained to be considered open

*This version is an update of the paper published in ICALP'13 proceedings. We strengthen the main result (Theorem 2) by replacing the *expected* worst-case time by deterministic worst-case time. This is achieved by using a simpler suffix tree update construction in Section 2.2 (Theorem 1).

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for many years. In 1994, Kosaraju [14] reported another solution to this problem. In our present work, however, we are interested in a more general problem, when matching queries can be made at all moments, rather than after the entire text has been received. Specifically, in our problem, a streaming text should be processed in real time so that at each moment, a matching query P can be made to the portion of the text received so far. We call this the *real-time indexing problem*. This problem has been considered in 2008 by Amir and Nor [3], who extended Kosaraju's algorithm to deal with repetitive queries made at any moment of the text scan.

All the three existing real-time indexing solutions [18, 14, 3] support only existential queries asking whether the pattern occurs in the text, but are unable to report occurrences of the pattern. Designing a real-time text indexing algorithm that would support queries on all occurrences of a pattern is stated in [3] as the most important remaining open problem. The algorithms of [14, 3] assume a constant size alphabet and are both based on constructions of "incomplete" suffix trees which can be built real-time but can only answer existential queries. To output all occurrences of a pattern, a fully-featured suffix tree is needed, however a real-time suffix tree construction, first studied in [1], is in itself an open question. The best currently known algorithms spend on each character $O(\log\log n)$ worst-case time in the case of constant-size alphabets [5], or $O(\log\log n + \frac{\log^2\log|A|}{\log\log\log|A|})$ time for arbitrary alphabets A [7]. A truly real-time suffix tree construction seems unlikely to exist. Therefore, a suffix tree alone seems to be insufficient to solve the real-time indexing problem.

In this paper, we propose the first real-time text indexing solution that supports reporting all pattern occurrences, under the assumption of constant size alphabet. Similar to the previous works on real-time indexing, we assume that the text is read right-to-left, or otherwise the pattern needs to be reversed before executing the query. The general idea is to maintain several data structures, three in our case, each supporting queries for different pattern lengths. Each of these structures is based on a suffix tree (or suffix-tree-like structure) exteded by some auxiliary data structures. To update a suffix tree, we use an implementation of Weiner's algorithm which is somewhat similar to but simpler than that of [5]. The simplification is achieved by using some external algorithmic tools, such as colored predecessor queries [16]. As a result, we can update a suffix tree in $O(\log\log n)$ worst-case time per letter, under the assumption that alphabet size is bounded by $O(\log^{1/4} n)$ and without resorting to a deamortization as in [5]. This is an interesting result in itself.

The paper is organized as follows. In Section 2, we describe auxiliary data structures and present our method for online update of suffix trees. In Section 3, we describe the three data structures for different pattern lengths that constitute a basis of our solution. These data structures, however, do not provide a fully real-time algorithm. Then in Section 4, we show how to "fix" the solution of Section 3 in order to obtain a fully real-time algorithm.

Throughout the paper, Σ is an alphabet of constant size σ. Since the text T is read right-to-left, it will be convenient for us to enumerate symbols of T from the end, i.e. $T = t_n \dots t_1$ and substring $t_{i+\ell}t_{i+\ell-1}\dots t_i$ will be denoted $T[i+\ell..i]$. $T[i..]$ denotes suffix $T[i..1]$. Throughout this paper, we reserve k to denote the number of objects (occurrences of a pattern, elements in a list, etc) in the query answer.

2 Preliminaries

In this Section, we describe main algorithmic tools used by our algorithms.

2.1 Range Reporting and Predecessor Queries on Colored Lists

We use data structures from [16] for searching in dynamic colored lists.

Colored Range Reporting in a List. Let elements of a dynamic linked list $\mathcal{L}$ be assigned positive integer values called *colors*. A colored range reporting query on a list $\mathcal{L}$ consists of two integers $col_1 < col_2$ and two pointers ptr_1 and ptr_2 that point to elements e_1 and e_2 of $\mathcal{L}$. An answer to a colored range reporting query consists of all elements $e \in \mathcal{L}$ occurring between e_1 and e_2 (including e_1 and e_2) such that $col_1 \le \mathtt{col}(e) \le col_2$, where $\mathtt{col}(e)$ is the color of e. The following result on colored range reporting has been proved by Mortensen [16].

Lemma 1 ([16]) *Suppose that* $\mathtt{col}(e) \le \log^f n$ *for all* $e \in \mathcal{L}$ *and some constant* $f \le 1/4$. *We can answer color range reporting queries on* $\mathcal{L}$ *in* $O(\log\log m + k)$ *time using an* $O(m)$*-space data structure, where* m *is the number of elements in* $\mathcal{L}$. *Insertion of a new element into* $\mathcal{L}$ *is supported in* $O(\log\log m)$ *time.*

Note that the bound $f \le 1/4$ follows from the description in [12]: the data structure in [16] uses Q-heaps [9] to answer certain queries on the set of colors in constant time.

Colored Predecessor Problem. The colored predecessor query on a list $\mathcal{L}$ consists of an element $e \in \mathcal{L}$ and a color col. The answer to a query (e, col) is the closest element $e' \in \mathcal{L}$ which precedes e such that $\mathtt{col}(e) = col$. The following Lemma is also proved in [16]; we also refer to [11], where a similar problem is solved.

Lemma 2 ([16]) *Suppose that* $\mathtt{col}(e) \le \log^f n$ *for all* $e \in \mathcal{L}$ *and some constant* $f \le 1/4$. *There exists an* $O(m)$ *space data structure that answers colored predecessor queries on* $\mathcal{L}$ *in* $O(\log\log m)$ *time and supports insertions in* $O(\log\log m)$ *time, where* m *is the number of elements in* $\mathcal{L}$.

2.2 Online Update of Suffix Trees for Small Alphabets

Classical suffix tree construction algorithms read the input text online and spend an amortized constant time on each text letter, however in the worst-case, they can spend as much as a linear time on an individual letter. Several papers studied the question of reducing the worst-case time spent on a letter, trying to approach the real-time update [2, 5, 12, 7]. All of them follow Weiner's algorithm and process the text right-to-left, as only one new suffix has to be added when a new letter is prepended from left, resulting in a constant amount of modifications. Breslauer and Italiano [5] showed how to deamortize Weiner's algorithm *in the case of constant-size alphabets* in order to obtain $O(\log\log n)$ worst-case time on each new letter. Kopelowitz [12] proposed a solution for an arbitrary alphabet A spending $O(\log\log n + \log\log|A|)$ worst-case *expected* time on each prepended letter. Very recently, Fischer and Gawrychowski [7] showed how to obtain a (deterministic) worst-case time $O(\log\log n + \frac{\log^2\log|A|}{\log\log\log|A|})$ for arbitrary alphabets.

In this Section, we show a simple implementation of Weiner's algorithm that achieves a worst-case $O(\log\log n)$ time per letter in the case when the alphabet size is bounded by $\log^{1/4} n$. Our solution uses Lemma 2 as well as a constant-time solution to dynamic lowest common ancestor (lca) problem [6]. Thus, the solution below can be viewed as a simpler and slightly more general version of the result of [5], extending it from constant-size alphabets to alphabets of size $\log^{1/4} n$.

We first briefly recall the main idea of Weiner's algorithm using a description similar to [5]. Updating a suffix tree when a new letter a is prepended to the current text T is done through maintaining W-links defined as follows. For a suffix tree node labeled u and a letter $a \in A$, W-link $W_a(u)$ points to the locus of string au in the suffix tree, provided that au is a substring of T (i.e. exists in the current suffix tree). Note that the locus of au can be an explicit or an implicit node, and $W_a(u)$ is called a *hard* or *soft* W-link respectively. The following properties of W-links will be useful in the sequel.

Lemma 3 ([7]) *(i) If for some letter a, a node has a defined W-link W_a, then any its ancestor node has a defined W-link W_a too.*

(ii) If two nodes u and v have defined hard W-links W_a, then $lca(u, v)$ has a defined hard W-link W_a too.

When a is prepended to a current text T, a new leaf labeled aT must be created and attached to either an existing node or a new node created by splitting an existing edge. To find the attachment node, the algorithm finds the lowest ancestor u of the leaf labeled T for which a (possibly soft) W-link $W_a(u)$ is defined. Then the target node $W_a(u)$ is the branching node. The main difficulty of Weiner's approach is to find the lowest ancestor of a leaf with a defined W-link $W_a(u)$. Another difficulty is to update (soft) W-links when the attachment node results from an edge split (see [5]).

In our solution, we store only hard W-links and do not store soft W-links at all. Note that a hard W-link, once installed, does not need to be updated for the rest of the algorithm [7]. Information about soft W-links is computed "on the fly" using the following Lemma.

Lemma 4 ([7]) *Assume that for a node u, $W_a(u)$ is defined and is a soft link pointing to an implicit node located on an edge (v, w). Then there exists a unique highest descendant u' of u for which $W_a(u')$ is a hard link, and, moreover, $W_a(u') = w$.*

To find the lowest ancestor u of a given node t with a defined (possibly soft) W-link $W_a(u)$, consider the Euler tour of the current suffix tree in which each internal node occurs two times corresponding to its first and last visits. Then the following Lemma holds.

Lemma 5 *Consider a node t. Assume that $W_a(t)$ is not defined and u is the lowest ancestor of t for which a (possibly soft) link $W_a(u)$ is defined. Let v_1 be the closest node preceding t in the Euler tour of the suffix tree such that $W_a(v_1)$ is a hard link. Let v_2 be the closest node following u in Euler tour of the suffix tree such that $W_a(v_2)$ is a hard link. Then u is the lowest node between $lca(t, v_1)$ and $lca(t, v_2)$. Moreover, if $lca(t, v_1)$ is the lowest, then v_1 is the highest descendant of u with a defined hard W-link W_a, otherwise v_2 is such a descendant.*

Proof: By Lemma 3(i), if $W_a(t)$ is not defined, then W_a is not defined for any descendant of t. Thus, no node occurring between the first and the second occurrences of t in the Euler tour has a defined link W_a. Consequently, definitions of nodes v_1 and v_2 are unambiguous.

By Lemma 4, u has a unique closest descendant, say v, with a defined hard link $W_a(v)$. If v occurs before t in the Euler tour, then v is the closest node preceding t in the Euler tour with defined $W_a(v)$. To show this, assume there is a closer such node v'. Observe that v' is also a descendant of u and v' is not a descendant of v. By Lemma 3(ii), $lca(v, v')$ is a node with a defined hard link W_a. On the other hand, $lca(v, v')$ is a proper ancestor of v which is a contradiction. Therefore, v is node v_1 from the Lemma.

Symmetrically, if v occurs after t in the Euler tour, then v is node v_2 from the Lemma. Clearly, to compute u, it is sufficient to pick the lowest between $lca(t, v_1)$ and $lca(t, v_2)$. $\square$

Based on the above, we implement Weiner's algorithm by maintaining the Euler tour of the current suffix tree in a colored list $\mathcal{L}_W$. If a node u has a defined hard W-link $W_a(u)$, then both occurrences of u in $\mathcal{L}_W$ are colored with a. Note that a node can have up to $|A|$ hard W-links and therefore have up to $2|A|$ occurrences in $\mathcal{L}_W$. However, the total number of hard W-links is limited by the number of tree nodes, as a node has at most one *incoming* hard W-link.

By Lemma 2, we can answer colored predecessor and successor queries on $\mathcal{L}_W$ in $O(\log\log|\mathcal{L}_W|)$ time. Therefore, nodes v_1 and v_2 defined in Lemma 5 can be found in $O(\log\log|\mathcal{L}_W|)$ time. Using lowest common ancestor queries on a dynamic tree [6], $lca(t, v_1)$ and $lca(t, v_2)$ can be computed in $O(1)$ time. Therefore, updating the suffix tree after prepending a new symbol is done is $O(\lg\lg|\mathcal{L}_W|) = O(\lg\lg|T|)$ time. As an update can introduce two new hard W-links, we also need to update the colored list $\mathcal{L}_w$. This is easily done in $O(1)$ time. (Details are left out and can be found e.g. in [15].)

We conclude with the final result of this Section.

Theorem 1 *Consider a text over an alphabet A, $|A| \leq \log^{1/4} n$, arriving online right-to-left. After prepending a new letter to the current text T, the suffix tree of T can be updated in time $O(\log\log|T|)$ using an auxiliary data structure of size $O(|T|)$.*

3 Fast Off-Line Solution

In this section we describe the main part of our algorithm of real-time text indexing. Based on the suffix tree construction from the previous Section, the algorithm updates the index by reading the text in the right-to-left order. However, the algorithm we describe in this Section will not be on-line, as it will have to access symbols to the left of the currently processed symbol. Another "flaw" of the algorithm is that it will support pattern matching queries only with an additional exception: we will be able to report all occurrences of a pattern except for those with start positions among a small number of most recently processed symbols of T. In the next section we will show how to fix these issues and turn our algorithm into a fully real-time indexing solution that reports all occurrences of a pattern.

The algorithm distinguishes between three types of query patterns depending on their length: *long patterns* contain at least $(\log\log n)^2$ symbols, *medium-size patterns* contain between $(\log^{(3)} n)^2$ and $(\log\log n)^2$ symbols, and *short patterns* contain less than $(\log^{(3)} n)^2$ symbols[1]. For each of the three types of patterns, the algorithm will maintain a separate data structure supporting queries in $O(|P| + k)$ time for matching patterns of the corresponding type.

3.1 Long Patterns

To match long patterns, we maintain a *sparse suffix tree* $\mathcal{T}_L$ storing only suffixes that start at positions $q \cdot d$ for $q \geq 1$ and $d = \log\log n/(4\log\sigma)$. Suffixes stored in $\mathcal{T}_L$ are regarded as strings over a meta-alphabet of size $\sigma^d = \log^{1/4} n$. This allows us to use the method of Section 2.2 to update $\mathcal{T}_L$, spending $O(\log\log n)$ time on each each meta-character encoding $O(\log\log n)$ regular characters. (We recall that $\sigma = O(1)$.)

[1] Henceforth, $\log^{(3)} n = \log\log\log n$.

Using $\mathcal{T}_L$ we can find occurrences of a pattern P that start at positions qd for $q \geq 1$, but not occurrences starting at positions $qd + \delta$ for $1 \leq \delta < d$. To be able to find all occurrences, we maintain a list $\mathcal{L}_E$ defined similarly to list $\mathcal{L}_W$ from Section 2.2.

The list $\mathcal{L}_E$ contains copies of all nodes of $\mathcal{T}_L$ as they occur during the Euler tour of $\mathcal{T}_L$. $\mathcal{L}_E$ contains one element for each leaf and two elements for each internal node of $\mathcal{T}_L$. If a node of $\mathcal{L}_E$ is a leaf that corresponds to a suffix $T[i..]$, we mark it with the meta-character $\overleftarrow{T}[i,d] = t_{i+1}t_{i+2}\dots t_{i+d}$ which is interpreted as the color of the leaf for the suffix $T[i..]$. Colors are ordered by lexicographic order of underlying strings. If $S = s_1 \dots s_j$ is a string with $j < d$, then S defines an interval of colors, denoted $[minc(S), maxc(S)]$, corresponding to all character strings of length d with prefix S. Recall that there are $\log^{1/4} n$ different colors. On list $\mathcal{L}_E$, we maintain the data structure of Lemma 1 for colored range reporting queries.

The update of $\mathcal{T}_L$ and $\mathcal{L}_E$ is done as follows. After reading character t_i where $i = qd$ for $q \geq 1$, we add suffix $T[i..]$, viewed as a string over the meta-alphabet of cardinality $\log^{1/4} n$, to $\mathcal{T}_L$ following the algorithm described in Section 2.2. In addition, we have to update $\mathcal{L}_E$, i.e. to insert to $\mathcal{L}_E$ the new leaf holding the suffix $T[i..]$ colored with $t_{i+1}t_{i+2}\dots t_{i+d}$. (Note that at this point the algorithm "looks ahead" for the forthcoming d letters of T.) If a new internal node has been inserted into $\mathcal{T}_L$, we also update the list $\mathcal{L}_E$ accordingly.

Since the meta-alphabet size is only $\log^{1/4} n$, navigation in $\mathcal{T}_L$ from a node to a child can be supported in $O(1)$ time. Observe that the children of any internal node $v \in \mathcal{T}_L$ are naturally ordered by the lexicographic order of edge labels. We store the children of v in a data structure $\mathcal{P}_v$ which allows us to find in time $O(1)$ the child whose edge label starts with a string (meta-character) $S = s_1 \dots s_d$. Moreover, we can also compute in time $O(1)$ the "smallest" and the "largest" child of v whose edge label starts with a string $S = s_1 \dots s_j$ with $j \leq d$. $\mathcal{P}_v$ will also support adding a new edge to $\mathcal{P}_v$ in $O(1)$ time. Data structure $\mathcal{P}_v$ can be implemented using e.g. atomic heaps [9]; since all elements in $\mathcal{P}_v$ are bounded by $\log^{1/4} n$, we can also implement $\mathcal{P}_v$ as described in [17].

We now consider a long query pattern $P = p_1 \dots p_m$ and show how the occurrences of P are computed. An occurrence of P is said to be a δ-occurrence if it starts in T at a position $j = qd + \delta$, for some q. For each δ, $0 \leq \delta \leq d-1$, we find all δ-occurrences as follows. First we "spell out" $P_\delta = p_{\delta+1} \dots p_m$ in $\mathcal{T}_L$ over the meta-alphabet, i.e. we traverse $\mathcal{T}_L$ proceeding by blocks of up to d letters of Σ. If this process fails at some step, then P has no δ-occurrences. Otherwise, we spell out P_δ completely, and retrieve the closest explicit descendant node v_δ, or a range of descendant nodes $v_\delta^l, v_\delta^{l+1}, \dots, v_\delta^r$ in the case when P_δ spells to an explicit node except for a suffix of length less than d. The whole spelling step takes time $O(|P|/d + 1)$.

Now we jump to the list $\mathcal{L}_E$ and retrieve the first occurrence of v_δ (or v_δ^l) and the second occurrence of v_δ (or v_δ^r) in $\mathcal{L}_E$. A leaf u of $\mathcal{T}$ corresponds to a δ-occurrence of P if and only if u occurs in the subtree of v_δ (or the subtrees of $v_\delta^l, \dots, v_\delta^r$) and the color of u belongs to $[minc(p_\delta \dots p_1), maxc(p_\delta \dots p_1)]$. In the list $\mathcal{L}_E$, these leaves occur precisely within the interval we computed. Therefore, all δ-occurrences of P can be retrieved in time $O(\log\log n + k_\delta)$ by a colored range reporting query (Lemma 1), where k_δ is the number of δ-occurrences. Summing up over all δ, all occurrences of a long pattern P can be reported in time $O(d(|P|/d + \log\log n) + k) = O(|P| + d\log\log n + k) = O(|P| + k)$, as $d = \log\log n/(4\log\sigma)$, $\sigma = O(1)$ and $|P| \geq (\log\log n)^2$.

3.2 Medium-Size Patterns

Now we show how to answer matching queries for patterns P where $(\log^{(3)} n)^2 \leq |P| < (\log\log n)^2$. In a nutshell, we apply the same method as in Section 3.1 with the main difference that the sparse suffix tree will store only *truncated suffixes* of length $(\log\log n)^2$, i.e. prefixes of suffixes bounded by $(\log\log n)^2$ characters. We store truncated suffixes starting at positions spaced by $\log^{(3)} n = \log\log\log n$ characters. The total number of different truncated suffixes is at most $\sigma^{(\log\log n)^2}$. This small number of suffixes will allow us to search and update the data structures faster compared to Section 3.1. We now describe the details of the construction.

We store all truncated suffixes that start at positions qd', for $q \geq 1$ and $d' = \log^{(3)} n$, in a tree $\mathcal{T}_M$. $\mathcal{T}_M$ is organized in the same way as the standard suffix tree; that is, $\mathcal{T}_M$ is a compacted trie for substrings $T[qd'..qd' - (\log\log n)^2 + 1]$, where these substrings are regarded as strings over the meta-alphabet $\Sigma^{d'}$.[2] Observe that the same truncated suffix can occur several times. Therefore, we augment each leaf v with a list of *colors* $Col(v)$ corresponding to left contexts of the corresponding truncated suffix S. More precisely, if $S = T[qd'..qd' - (\log\log n)^2 + 1]$ for some $q \geq 1$, then $\overleftarrow{T}[qd', d']$ is added to $Col(v)$. Note that the number of colors is bounded by $\sigma^{\log^{(3)} n}$. Furthermore, for each color *col* in $Col(v)$, we store all positions $i = qd'$ of T such that S occurs at i and $\overleftarrow{T}[i, d'] = col$. Similar to Section 3.1, we maintain a colored list $\mathcal{L}_M$ that stores the Euler tour traversal of $\mathcal{T}_M$. For each internal node, $\mathcal{L}_M$ contains two elements. For every leaf v and for each value *col* in its color list $Col(v)$, $\mathcal{L}_M$ contains a separate element colored with *col*. Observe that since the size of $\mathcal{L}_M$ is bounded by $O(\sigma^{(\log\log n)^2 + \log^{(3)} n})$, updates of $\mathcal{L}_M$ can be supported in $O(\log\log(\sigma^{(\log\log n)^2})) = O(\log^{(3)} n)$ time and colored reporting queries on $\mathcal{L}_M$ can be answered in $O(\log^{(3)} n + k)$ time (Lemma 1).

Truncated suffixes are added to $\mathcal{T}_M$ using a method similar to that of Section 3.1. After reading a symbol $t_{qd'}$ for some $q \geq 1$, we insert $S_{\text{new}} = T[qd'..qd' - (\log\log n)^2 + 1]$ colored with $\overleftarrow{T}[qd', d']$ into the tree $\mathcal{T}_M$. Insertion of S_{new} is done as described in Section 2.2, and the list $\mathcal{L}_M$ is updated accordingly. If $\mathcal{L}_M$ already contains a leaf with string value S_{new} and color $\overleftarrow{T}[qd', d']$, we add qd' to the list of its occurrences, otherwise we insert a new element into $\mathcal{L}_M$ and initialize its location list to qd'. Altogether, the addition of a new truncated suffix S_{new} requires $O(\log\log|\mathcal{T}_M|) = O(\log^{(3)} n)$ time.

A query for a pattern $P = p_1 \dots p_m$, such that $(\log^{(3)} n)^2 \leq m < (\log\log n)^2$, is answered in the same way as in Section 3.1. For each $\rho = 0, \dots, \log^{(3)} n - 1$, we find locus nodes $v^l_\rho, \dots, v^r_\rho$ (possibly with $v^l_\rho = v^r_\rho$) of $P_\rho = p_{\rho+1} \dots p_m$. Then, we find all elements in $\mathcal{L}_M$ occurring between the first occurrence of v^l_ρ and the second occurrence of v^r_ρ and colored with a color *col* that belongs to $[minc(p_\rho \dots p_1), maxc(p_\rho \dots p_1)]$. For every such element, we traverse the associated list of occurrences: if a position i is in the list, then P occurs at position $(i+\rho)$. The total time needed to find all occurrences of a medium-size pattern P is $O(d'(|P|/d' + \log^{(3)} n) + k) = O(|P| + (\log^{(3)} n)^2 + k) = O(|P| + k)$ since $|P| \geq (\log^{(3)} n)^2$.

3.3 Short Patterns

Finally, we describe our indexing data structure for patterns P with $|P| < (\log^{(3)} n)^2$. We maintain the tree $\mathcal{T}_S$ of truncated suffixes of length $\Delta = (\log^{(3)} n)^2$ seen so far in the text. For every position i of T, $\mathcal{T}_S$ contains the substring $T[i..i - \Delta + 1]$. $\mathcal{T}_S$ is organized as a compacted trie. We support queries and updates on $\mathcal{T}_S$ using tabulation. There are $O(2^{\sigma^\Delta})$ different trees, and $O(\sigma^\Delta)$ different queries can be made on each tree. Therefore, we can afford explicitly storing all possible trees $\mathcal{T}_S$ and tabulating possible tree updates. Each internal node of a tree stores pointers to its lefmost

[2] For simplicity we assume that $\log^{(3)} n$ and $\log\log n$ are integers and $\log^{(3)} n$ divides $\log\log n$. If this is not the case, we can find d' and d that satisfy these requirements such that $\log\log n \leq d \leq 2\log\log n$ and $\log^{(3)} n \leq d' \leq 2\log^{(3)} n$.

and rightmost leaves, the leaves of a tree are organized in a list, and each leaf stores the encoding of the corresponding string Q.

The *update table* $\mathbf{T}_u$ stores, for each tree $\mathcal{T}_S$ and for any string Q, $|Q| = \Delta$, a pointer to the tree $\mathcal{T}'_S$ (possibly the same) obtained after adding Q to $\mathcal{T}_S$. Table $\mathbf{T}_u$ uses $O(2^{\sigma^\Delta}\sigma^\Delta) = o(n)$ space. The *output table* $\mathbf{T}_o$ stores, for every string Q of length Δ, the list of positions in the current text T where Q occurs. $\mathbf{T}_o$ has $\sigma^\Delta = o(n)$ entries and all lists of occurrences take $O(n)$ space altogether.

When scanning the text, we maintain the encoding of the string Q of Δ most recently read symbols of T. The encoding is updated after each symbol using bit operations. After reading a new symbol, the current tree $\mathcal{T}_S$ is updated using table $\mathbf{T}_u$ and the current position is added to the entry $\mathbf{T}_o[Q]$. Updates take $O(1)$ time.

To answer a query P, $|P| < \Delta$, we find the locus u of P in the current tree $\mathcal{T}_S$, retrieve the leftmost and rightmost leaves and traverse the leaves in the subtree of u. For each traversed leaf v_l with label Q, we report the occurrences stored in $\mathbf{T}_o[Q]$. The query takes time $O(|P| + k)$.

4 Real-Time Indexing

The indexes for long and medium-size patterns, described in Sections 3.1 and 3.2 respectively, do not provide real-time indexing solutions for several reasons. The index for long patterns, for example, requires to look ahead for the forthcoming d symbols when processing symbols t_i for $i = qd$, $q \geq 1$. Furthermore, for such i, we are unable to find occurrences of query patterns P starting at positions $t_{i-1} \dots t_{i-d+1}$ before processing t_i. A similar situation holds for medium-size patterns. Another issue is that in our previous development we assumed the length n of T to be known, whereas this may of course not be the case in the real-time setting. In this Section, we show how to fix these issues in order to turn the indexes real-time. Firstly we show how the data structures of Sections 3.1 and 3.2 can be updated in a real-time mode. Then, we describe how to search for patterns that start among most recently processed symbols. We describe our solutions to these issues for the case of long patterns, as a simple change of parameters provides a solution for medium-size patterns too. Finally, we will show how we can circumvent the fact that the length of T is not known in advance.

In the algorithm of Section 3.1, the text is partitioned into blocks of length d, and the insertion of a new suffix $T[i..]$ is triggered only when the leftmost symbol t_i of a block is reached. The insertion takes time $O(d)$ and assumes the knowledge of the forthcoming block $t_{i+d} \dots t_{i+1}$. To turn this algorithm real-time, we apply a standard deamortization technique. We distribute the cost of the insertion of suffix $T[i-d..]$ over d symbols of the block $t_{i+d} \dots t_{i+1}$. This is correct, as by the time we start reading the block $t_{i+d} \dots t_{i+1}$, we have read the block $t_i \dots t_{i-d+1}$ and therefore have all necessary information to insert suffix $T[i-d..]$. In this way, we spend $O(1)$ time per symbol to update all involved data structures.

Now assume we are reading a block $t_{i+d} \dots t_{i+1}$, i.e. we are processing some symbol $t_{i+\delta}$ for $1 \leq \delta < i$. At this point, we are unable to find occurrences of a query pattern P starting at $t_{i+\delta} \dots t_{i+1}$ as well as within the two previous blocks, as they have not been indexed yet. This concerns up to $(3d-1)$ most recent symbols. We then introduce a separate procedure to search for occurrences that start in $3d$ leftmost positions of the already processed text. This can be done by simply storing T in a compact form T_c where every $\log_\sigma n$ consecutive symbols are packed into one computer word[3]. Thus, T_c uses $O(|T|/\log_\sigma n)$ words of space. Using T_c, we can test whether

[3]In fact, it would suffice to store $3d-1$ most recently read symbols in compact form.

$T[j..j-|P|+1] = P$, for any pattern P and any position j, in $O(\lceil |P|/\log_\sigma n\rceil) = o(|P|/d) + O(1)$ time. Therefore, checking $3d$ positions takes time $o(|P|) + O(d) = O(|P|)$ for a long pattern P.

We now describe how we can apply our algorithm in the case when the text length is not known beforehand. In this case, we assume $|T|$ to take increasing values $n_0 < n_1 < \ldots$, as long as the text T keeps growing. Here, n_0 is some appropriate initial value and $n_i = 2n_{i-1}$ for $i \geq 1$.

Suppose now that n_i is the currently assumed value of $|T|$. After we reach character $t_{n_i/2}$, during the processing of the next $n_i/2$ symbols, we keep building the index for $|T| = n_i$ and, in parallel, rebuild all the data structures under assumption that $|T| = n_{i+1} = 2n_i$. In particular, if $\log\log(2n_i) \neq \log\log n_i$, we build a new index for long patterns, and if $\log^{(3)}(2n_i) \neq \log^{(3)} n_i$, we build a new index for meduim-size and short patterns. If $\log_\sigma(2n_i) \neq \log_\sigma n_i$, we also construct a new compact representation T_c introduced earlier in this section. Altogether, we distribute the construction cost of the data structures for $T[n_i..1]$ under assumption $|T| = 2n_i$ over the processing of $t_{n_i/2+1}\ldots t_{n_i}$. Since $O(n_i) = O(n_i/2)$, processing these $n_i/2$ symbols remains real-time. By the time t_{n_i} has been read, all data structures for $|T| = 2n_i$ have been built, and the algorithm proceeds with the new value $|T| = n_{i+1}$. Observe finally that the intervals $[n_i/2+1, n_i]$ are all disjoint, therefore the overhead per letter incurred by the procedure remains constant. In conclusion, the whole algorithm remains real-time. We finish with our main result.

Theorem 2 *There exists a data structure storing a text T over a constant-size alphabet that can be updated in $O(1)$ worst-case time after prepending a new symbol to T. This data structure supports reporting all occurrences of a pattern P in the current text T in $O(|P|+k)$ time, where k is the number of occurrences.*

5 Conclusions

In this paper we presented the first real-time indexing data structure that supports reporting all pattern occurrences in optimal time $O(|P|+k)$. As in the previous works on this topic [14, 3, 5], we assume that the input text is over an alphabet of constant size. It may be possible to extend our result to alphabets of poly-logarithmic size.

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