



# Selected Papers of the 31st International Workshop on Combinatorial Algorithms, IWOCA 2020

Leszek Gąsieniec<sup>1</sup> · Ralf Klasing<sup>2</sup> · Tomasz Radzik<sup>3</sup>

Published online: 5 September 2022

© The Author(s), under exclusive licence to Springer Science+Business Media, LLC, part of Springer Nature 2022

Since its inception in 1989 as AWOCA (Australasian Workshop on Combinatorial Algorithms), IWOCA has provided an annual forum for researchers who design algorithms for the myriad combinatorial problems that underlie computer applications in science, engineering and business.

This special issue is devoted to selected papers presented at the *31st International Workshop on Combinatorial Algorithms (IWOCA 2020)*, originally scheduled to take place during during 8-10 June 2020 in Bordeaux, France. Due to the COVID-19 pandemic, the symposium was run online on the originally set dates.

Based on the program committee discussions and the presentations during the conference, the editors invited several papers to be submitted for this special issue. All submitted articles underwent a full reviewing process according to the normal standards of the journal *Algorithmica*, and as a result, ten papers were selected for publication. These articles are expanded versions of the original conference submissions and illustrate the breadth of algorithmic topics and techniques presented at *IWOCA 2020*. The selected articles can be found in the online collection for this [Special Issue on Combinatorial Algorithms \(IWOCA 2020\)](#).

In the paper *On the complexity of Broadcast Domination and Multipacking in digraphs*, Florent Foucaud, Benjamin Gras, Anthony Perez and Florian Sikora study the complexity of two covering and packing distance-based problems BROADCAST DOMINATION and MULTIPACKING in digraphs, which are dual to each other. In contrast to undirected graphs, they prove that for digraphs both problems are NP-complete,

---

✉ Ralf Klasing  
ralf.klasing@labri.fr

Leszek Gąsieniec  
L.A.Gasieniec@liverpool.ac.uk

Tomasz Radzik  
tomasz.radzik@kcl.ac.uk

<sup>1</sup> University of Liverpool, Liverpool, United Kingdom

<sup>2</sup> CNRS and University of Bordeaux, Talence, France

<sup>3</sup> King's College London, London, United Kingdom

even when inputs are restricted to planar layered acyclic digraphs of small maximum degree. Moreover, when parameterized by the cost or size of the solution, they show that the problems are respectively  $W[2]$ -hard and  $W[1]$ -hard, and that the first problem, BROADCAST DOMINATION, is FPT on acyclic digraphs but does not admit a polynomial kernel, unless the polynomial-time hierarchy collapses to its third level. They also show that both problems are FPT when parameterized by the cost or size of the solution together with the maximum out-degree. Finally, they give polynomial-time algorithms for some subclasses of acyclic digraphs.

The paper *On Proper Labellings of Graphs with Minimum Label Sum* by Julien Bensmail, Foivos Fioravantes and Nicolas Nisse studies proper labellings of the edges of a graph (adjacent vertices have different sums of the incident labels) with the optimization requirement of minimizing the sum of assigned labels. Equivalently, augment a given graph  $G$  with the smallest possible number of edges to obtain a locally irregular multigraph  $M^*$ . Without the optimization requirement, the famous 1-2-3 Conjecture suggests that we can get a proper labelling of  $G$  using only labels 1, 2 and 3, or equivalently that we can get a locally irregular multigraph by adding to each edge in  $G$  at most two further parallel edges. The authors prove however that if we require the overall minimum number of edges, then there is no absolute constant  $k$  such that an optimal multigraph  $M^*$  can always be obtained from  $G$  by replacing each edge with at most  $k$  parallel edges. They investigate several aspects of this problem, covering algorithmic and combinatorial aspects. For example, they prove that the problem of designing proper labellings with minimum label sum is  $\mathcal{NP}$ -hard in general, but solvable in polynomial time for graphs with bounded treewidth. The paper concludes with a conjecture that for almost every connected graph  $G$ , there should be a proper labelling with label sum at most  $2|E(G)|$ , which is verified for several classes of graphs.

The paper *Strongly Stable and Maximum Weakly Stable Noncrossing Matchings* by Koki Hamada, Shuichi Miyazaki and Kazuya Okamoto refers to a paper by Ruangwises and Itoh from IWOCOA 2019, which introduced *stable noncrossing matchings*. Participants of each side are aligned on two parallel lines and matching edges are not allowed to cross each other. This definition led to two stability notions, *strongly stable noncrossing matching (SSNM)* and *weakly stable noncrossing matching (WSNM)*, and Ruangwises and Itoh proved that a WSNM always exists and can be found in  $O(n^2)$  time, leaving open the questions of the complexities of determining existence of an SSNM and finding a largest WSNM. The paper by Hamada et al. in this special issue shows that both problems are solvable in polynomial time. The proposed algorithms are applicable to extensions where preference lists may include ties, except for one case which is shown to be NP-complete.

Mader conjectured in 2010 that for any tree  $T$  of order  $m$ , every  $k$ -connected graph  $G$  with minimum degree at least  $\lfloor \frac{3k}{2} \rfloor + m - 1$  contains a subtree  $T' \cong T$  such that  $G - V(T')$  is  $k$ -connected. This conjecture has been proved for  $k = 1$ , remains open for general  $k \geq 2$ , and partially affirmative answers have been shown for  $k = 2$ . In the paper *Connectivity Keeping Trees in 2-Connected Graphs with Girth Conditions*, Toru Hasunuma first extends the previously known subclass of trees for which Mader's conjecture for  $k = 2$  holds, and then considers 2-connected graphs with girth conditions. The presented results include showing that Mader's conjecture

is true for every 2-connected graph  $G$  with  $g(G) \geq \delta(G) - 6$ , where  $g(G)$  and  $\delta(G)$  denote the girth of  $G$  and the minimum degree of a vertex in  $G$ , respectively. Mader's conjecture is interesting not only from a theoretical point of view but also from a practical point of view, since it may be applied to fault-tolerant problems in communication networks.

In the paper *Connected Reconfiguration of Lattice-Based Cellular Structures by Finite-Memory Robots*, Sándor Fekete, Eike Niehs, Christian Scheffer and Arne Schmidt provide algorithmic methods for connected reconfiguration of lattice-based cellular structures by finite-state robots. This model is motivated by large-scale constructions in space. They present algorithms that are able to detect and reconfigure arbitrary polyominoes, while preserving connectivity of a structure during reconfiguration. Mathematical proofs and performance guarantees are provided, and the developed techniques include determining a bounding box, scaling a given arrangement, and adapting more general algorithms for transforming polyominoes.

Given a graph  $G = (V, E)$ , a subset  $A \subseteq V$ , and integers  $k$  and  $\ell$ , the  $(A, \ell)$ -PATH PACKING problem asks to find  $k$  vertex-disjoint paths of length  $\ell$  that have endpoints in  $A$  and internal points in  $V \setminus A$ . The paper *Parameterized Complexity of  $(A, \ell)$ -Path Packing* by Rémy Belmonte, Tesshu Hanaka, Masaaki Kanzaki, Masashi Kiyomi, Yasuaki Kobayashi, Yusuke Kobayashi, Michael Lampis, Hirota Ono and Yota Otachi studies the parameterized complexity of this problem with parameters  $|A|$ ,  $\ell$ ,  $k$ , treewidth, pathwidth, and their combinations, presenting sharp complexity contrasts with respect to these parameters. Among other results, it is shown that the problem is polynomial-time solvable when  $\ell \leq 3$ , but NP-complete for constant  $\ell \geq 4$ . It is also shown that the problem is W[1]-hard when parameterized by pathwidth +  $|A|$ , but fixed-parameter tractable when parameterized by treewidth +  $\ell$ .

In online edge- and node-deletion problems, the input arrives node by node and an algorithm has to delete nodes or edges in order to keep the input graph in a given graph class  $\Pi$  at all times. In the paper *Online Node- and Edge-Deletion Problems with Advice*, Li-Hsuan Chen, Ling-Ju Hung, Henri Lotze and Peter Rossmanith consider hereditary properties  $\Pi$ , for which optimal online algorithms exist and which can be characterized by a set of forbidden subgraphs  $\mathcal{F}$  and analyze the advice complexity of getting an optimal solution. They give almost tight bounds for two cases: the DELAYED CONNECTED  $\mathcal{F}$ -NODE DELETION PROBLEM where all graphs of the family  $\mathcal{F}$  have to be connected, and the DELAYED  $H$ -NODE DELETION PROBLEM where there is only one forbidden induced subgraph  $H$  that may be connected or not. The paper also gives tight bounds on the DELAYED CONNECTED  $\mathcal{F}$ -EDGE DELETION PROBLEM where one has an arbitrary number of forbidden connected graphs, and presents a separate analysis for the DELAYED CONNECTED  $H$ -EDGE DELETION PROBLEM.

Given a graph  $G$ , and terminal vertices  $s$  and  $t$ , the TRACKING PATHS problem asks to compute a minimum number of vertices to be marked as trackers, such that the sequence of trackers encountered in each  $s$ - $t$  path is unique. This problem is NP-hard in both directed and undirected graphs in general. The paper *Polynomial Time Algorithms for Tracking Path Problems* by Pratibha Choudhary gives a collection of polynomial time algorithms for some restricted versions of TRACKING PATHS. It proves that TRACKING PATHS is polynomial time solvable for chordal graphs and tournament graphs. It proves that TRACKING PATHS is NP-hard in graphs with bounded

maximum degree  $\delta \geq 6$ , and gives a  $2(\delta + 1)$ -approximate algorithm for such graphs. It also analyzes the version of TRACKING PATHS where paths are tracked using edges instead of vertices, giving a polynomial time algorithm. Finally it gives a polynomial algorithm which, given an undirected graph  $G$ , a tracking set  $T \subseteq V(G)$ , and a sequence of trackers  $\pi$ , returns the unique  $s$ - $t$  path in  $G$  that corresponds to  $\pi$ , if one exists.

In the  $k$ -mismatch problem we are given a pattern of length  $n$  and a text and one must find all locations where the Hamming distance between the pattern and the text is at most  $k$ . A series of recent breakthroughs have resulted in an ultra-efficient streaming algorithm for this problem. In the paper *Streaming Dictionary Matching with Mismatches*, Paweł Gawrychowski and Tatiana Starikovskaya consider a strictly harder problem called dictionary matching with  $k$  mismatches. In this problem, one is given a dictionary of  $d$  patterns, where the length of each pattern is at most  $n$ , and one must find all substrings of the text that are within Hamming distance  $k$  from one of the patterns. The authors show a streaming algorithm for this problem with  $O(kd \log^k d \text{polylog } n)$  space and  $O(k \log^k d \text{polylog } n + |\text{output}|)$  time per position of the text. The algorithm is randomised, outputting correct answers with high probability. On the lower bound side, they show that any streaming algorithm for dictionary matching with  $k$  mismatches requires  $\Omega(kd)$  bits of space.

In the paper *Dynamic Averaging Load Balancing on Cycles*, Dan Alistarh, Giorgi Nadiradze and Amirmojtaba Sabour consider the following dynamic load-balancing process: given an underlying graph  $G$  with  $n$  nodes, in each step  $t \geq 0$ , a random edge is chosen, one unit of load is created, and placed at one of the endpoints. In the same step, assuming that loads are arbitrarily divisible, the two nodes *balance* their loads by averaging them. The quantity of interest is the expected gap between the minimum and maximum loads at nodes as the process progresses, and its dependence on  $n$  and the graph structure. The authors consider cycle graphs and introduce a new potential analysis technique, which enables to bound the difference in load between  $k$ -hop neighbors on the cycle, for any  $k \leq n/2$ . Using this technique, they prove their main result that the expected gap is  $O(\sqrt{n} \log n)$ , improving the previous bound of  $O(n \log n)$ . They also show how their analysis can be extended to some special class of regular graphs. Additionally, the paper provides experimental evidence that the proven upper bound on the gap in cycle graphs is tight up to a logarithmic factor.

The editors would like to thank the authors and reviewers for their excellent and timely work, and Ming-Yang Kao, the former editor-in-chief of *Algorithmica*, for his support and the opportunity to edit this special issue.

**Publisher's Note** Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.