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Timetable optimization for single bus line involving fuzzy travel time

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Abstract Timetable optimization is an important step for bus operations management, which essentially aims to effectively link up bus carriers and passengers. Generally speaking, bus carriers attempt to minimize the total travel time to reduce its operation cost, while the passengers attempt to minimize their waiting time at stops. In this study, we focus on the timetable optimization problem for a single bus line from both bus carriers' perspectives and passengers' perspectives. A bi-objective optimization model is established to minimize the total travel time for all trips along the line and the total waiting time for all passengers at all stops, in which the bus travel times are considered as fuzzy variables due to a variety of disturbances such as weather conditions and traffic conditions. A genetic algorithm with variable-length chromosomes is devised to solve the proposed model. In addition, we present a case study that utilizes real-life bus transit data to illustrate the efficacy of the proposed model and solution algorithm. Compared with the timetable currently being used, the optimal bus timetable produced from this study is able to reduce

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the total travel time by 26.75% and the total waiting time by 9.96%. The results demonstrate that the established model is effective and useful to seek a practical balance between the bus carriers' interest and passengers' interest.

Keywords Timetable optimization · Fuzzy variable · Travel time · Waiting time · Genetic algorithm

1 Introduction

Public transit, one of the most vital modes for people's daily traveling, and indeed for people's daily life, is highly praised by people and government due to its characteristics of cheapness, convenience and greenness. The public transit operations management is a sophisticated process which involves routes designing, timetable optimization, vehicle scheduling and crew scheduling (Guihaire and Hao, 2008). Among them, timetable optimization is a critical step to determine the departure times at first stop, and the arrival and departure times at the subsequent stops for all trips, influencing both bus carriers' cost and passengers' satisfaction. To obtain an attractive timetable, it is of great importance to consider the bus carriers' interest and passengers' interest.

Existing literature on bus timetable optimization mainly focuses on considering bus carriers' perspectives, such as the carrier's profits, bus transit reliability, service quality and travel time. For example, Yan and Chen (2002) introduced a mechanism to maximize the profits of a bus carrier by constructing a model based on a multiple time-space network. Yan et al. (2006) proposed a method to minimize the total cost of fleet flows and the expected cost by establishing a stochastic-demand scheduling model. Arhin et al. (2016) developed a technique to further bus transit reliability by studying a multi-objective re-synchronizing of bus timetable. Vissat et al. (2015) attempted to achieve a better timetable for bus service running with lower financial risk of penalties but higher punctuality and reliability by the use of a stochastic model. Based on analytical development and micro simulations, Salicrú et al. (2011) presented an approach to generate run-time values in an effort to optimize run-time and improve the operating process. In order to improve the bus operator's transit service, Yan et al. (2012) designed a robust optimization model aiming to minimize the sum of random schedule deviation and its variability. Wu et al. (2016) studied the re-planning issue of a bus network timetable, considering headway-sensitive passenger demand, uneven headway, service regularity, and flexible synchronization. Zhao and Zeng (2008) studied the route network design, vehicle headway and timetable assignment thoroughly. Chen et al. (2015) considered minimizing bus travel time and examined the optimal stopping criteria for limited-stop bus services. We attempt to minimize the total travel time for all trips along a given bus line.

There are also a number of studies on timetable optimization from the perspectives of passengers to minimize their waiting time. For instance, Amin-Naseri and Baradaran (2014) developed a simulation model to evaluate the performance of proposed formulations estimating the average waiting time at stops. Parbo et al. (2014) proposed a timetable optimization model minimizing the waiting time when transferring. Ceder et al. (2001) devised a method for minimizing passengers' waiting time at the transfer nodes, by establishing a model to maximize the synchronization of a given network of buses. Wu et al.(2015) considered a bus optimization problem to minimize the total waiting time cost for transferring passengers, boarding passengers and carrying through passengers. Based on the above literature review, it is clear that timetable optimization from either bus carriers' perspectives or passengers' perspectives has drawn great attention and become a much sought after topic in recent years. However, the study of optimizing timetable to conjunctively satisfy the requirements of both bus carriers and passengers is rather limited. Thus, we pursue to establish a novel approach for optimizing the timetable from both bus carriers' perspectives and passengers' perspectives by seeking a balance between these requirements.

In practice, random events like traffic jams and weather conditions may lead to certain buses not to finish their trips on time. By considering the nature of such randomness, the deviations between the theoretical optimization results and practical implementation outcomes can be minimized, if not totally avoided. Researchers have conducted a few studies for bus timetable optimization problems while presuming the bus travel times as random variables. Assuming the waiting time and travel time to be random variables, Tong and Wong (1999) formulated a dynamic transit assignment model over a transit network. Liu et al. (2013) studied a bus stop-skipping scheme with assumed random travel times. Chen et al. (2015) considered the vehicle capacity and random travel times and addressed the optimal stopping design of limited-stop bus services, allowing each bus vehicle to skip certain stops if desired. Wei and Sun (2017) investigated an uncertain bi-level programming model for multi-modal regional bus timetables and vehicle dispatch, again by assuming travel times as random variables.

There also exists work in the literature that represents bus travel times as fuzzy variables in bus scheduling. For example, considering fuzzy travel time, Djadane et al. (2007) and Brito et al. (2010) devised a mechanism to resolve the vehicle routing problems. Ban et al. (2014) examined the issue of traffic assignment with fuzzy travel time. Considering various uncertain and imprecise factors such as weather conditions and traffic accidents in practical applications, we characterize bus travel times as fuzzy variables in this work. In particular, we study the problem of bus timetable optimization within the framework of credibility theory, a branch of mathematics for studying the behavior of fuzzy phenomena (Liu and Liu, 2002). This design decision is made owing to the popularity, availability and suitability of this framework.

Generating timetables has been an active research area for a long time, but real-world applications of the theoretical results have been rather limited (Sels et al., 2016). Due to the lack of reliable and detailed bus transit data, most of the studies on timetable optimization have been conducted through numerical tests and theoretical analysis (Ceder et al., 2001; Yan et al., 2006; Zhao and Zeng, 2008; Yan et al., 2012). However, with the development of data collection and analysis technologies, investigations into the development of timetable optimization models supported with case studies that are particlly based on real-life data have recently been reported (Ceder and Philibert, 2014; Parbo et al., 2014; Hassold and Ceder, 2014; Sun et al., 2015; Arhin et al., 2016). In this paper, following this much desired trend, we take advantage of IC card data and GPS data to compute spatial-temporal travel times among adjacent stops, formulating a fuzzy bi-objective bus timetable optimization model and devising its associated solution algorithm.

The main contributions of this paper are threefold. Firstly, we consider the bus timetable optimization problem for a single bus line from both bus carriers' perspectives and passengers' perspectives, to minimize the time cost of bus carriers and passengers, measured by the total travel time and total waiting time respectively. Secondly, the bus travel times between stops are considered as fuzzy variables, thereby handling the daily spatial-temporal travel time as a fuzzy sample in a credibility space. Thirdly, the bus timetable optimization model is empirically evaluated using real-life data.

The remainder of this paper is organized as follows. Section 2 formulates two modeling concepts: fuzzy total travel time and fuzzy total waiting time, and then constructs a bus timetable optimization model to minimize these objectives. In Section 3, a genetic algorithm with variable-length chromosomes is designed to solve the proposed model. Section 4 presents a case study to illustrate the efficacy of the model and the solution algorithm. Section 5 concludes the work and discusses further research. To aid the understanding of the paper, Appendix A summarizes basic and relevant concepts and definitions in credibility theory.

2 Fuzzy bi-objective timetable optimization: Model

An efficient timetable should result in lower travel time and lower waiting time, respectively representing the interest of bus carriers and that of passengers. In this section, we formulate a bi-objective bus timetable optimization model which minimizes the total travel time for all trips along a given bus line and the total waiting time for all passengers at all stops.

2.1 Notations

Suppose that there are I stops along a bus line, and two depots near the terminal stops. Note that for simplifying the model and notation system, the travel times between a depot and a terminal stop is not considered. The following notations are defined and used in what follows (See Table 1).

Table 1: List of notations

Notations	Description
Indices	
i	Stop index, $1 \leq i \leq I$
k	Trip index, $1 \leq k \leq K$
l	Segment index, $1 \leq l \leq L$
θ	Day index, $1 \leq \theta \leq \Theta$
Parameters	
τ_{il}^{θ}	Travel time from stop i to stop $i+1$ if the departure time belongs to the l-th time segment at day θ
T_s	Departure time for the first trip
T_e	Departure time for the last trip
H_{min}	Minimum headway among two adjacent trips
H_{max}	Maximum headway among two adjacent trips
δ	Length of a time segment
Fuzzy Parameters	
τ_{il}	Travel time from stop i to stop $i+1$ if the departure time belongs to the l-th time segment, which takes district values in $\{\tau_{il}^1, \tau_{il}^2, \cdots, \tau_{il}^{\Theta}\}$
Decision Variables	
K	Number of trips, $K \in [K^l, K^u]$ where K^l and K^u are lower and upper bound, respectively
t_{k1}	Departure time from stop 1 for the k-th trip, $2 \leq k \leq K-1$
Intermediate Variables	
t_{ki}	Departure time from stop i of the k-th trip, $2 \leq i \leq I, 1 \leq k \leq K$

2.2 Travel times among adjacent stops

As a mature bus network has steady traffic conditions, the past traffic conditions can be used to hypothesize the forthcoming situations. To characterize the spatial-temporal characteristics on traffic conditions, we divide the time horizon $[T_s, T]$ into a sequence of segments with euqal length δ, denoted by $[T_s + (l-1)\delta, T_s + l\delta), 1 \leq l \leq L$, where T_s is the departure time from stop 1 for the first trip, T is the end of the daily operating time and L is the number of segments satisfying $T - T_s = L\delta$. In practice, the traffic conditions are complex and changing real-time, frequently leading to the temporal (and spatial) variations on the bus travel time. Even for the same departure time, the travel times from stop i to stop $i+1$ in different days may be different. We can estimate the forthcoming spatial-temporal travel times based on the past values $\{\tau_{il}^{\theta} | 1 \leq i \leq I, 1 \leq l \leq L, 1 \leq \theta \leq \Theta\}$, approximately calculated using the GPS data at the last Θ days.

Generally speaking, the traffic conditions over the last Θ days have different influences upon the traffic conditions of the forthcoming day. The traffic conditions of the days near the forthcoming day have higher similarity with the forthcoming day, although not in a precisely definable manner. Therefore, the forthcoming travel times can be expressed as a fuzzy variable τ_{il}, taking district values in $\{\tau_{il}^1, \tau_{il}^2, \cdots, \tau_{il}^{\Theta}\}, 1 \leq \theta \leq \Theta$ with credibilities $\{v_1, v_2, \cdots, v_{\Theta}\}$. For $2 \leq i \leq I$ and $1 \leq k \leq K$, the departure time from stop i of the k-th trip can be depicted as follows

$$t_{ki} = \begin{cases} t_{k,i-1} + \tau_{i1}, \text{ if } t_{k,i-1} \in [T_s, T_s + \delta) \\ t_{k,i-1} + \tau_{i2}, \text{ if } t_{k,i-1} \in [T_s + \delta, T_s + 2\delta) \\ \vdots \\ t_{k,i-1} + \tau_{iL}, \text{ if } t_{k,i-1} \in [T_s + (L-1)\delta, T_s + L\delta). \end{cases} \tag{1}$$

The credibilities $\{v_1, v_2, \cdots, v_\Theta\}$ is determined by the similarities between the past days and the forthcoming day. The days near the forthcoming day have higher credibilities with the forthcoming day. Additionally, the weekday's timetable and the weekend's timetable should be optimized separately.

2.3 Total travel time

Transit travel time influences service quality, operating cost and efficiency (Bertini and El-Geneidy, 2004). Bus carriers generally attempt to supply passengers with less total travel time, which will make their customers happy and reduce their own cost as well, due to the decrease in the total working time of bus, driver and conductor. To reflect this observation we consider guaranteeing the interests of bus carriers by minimizing the total travel time

$$T_r(\theta) = \sum_{k=1}^{K} (t_{kI} - t_{k1}), 1 \le \theta \le \Theta, \tag{2}$$

where K denotes the total number of trips. In an actual operation, bus carriers must provide transport service to passengers with trip times greater than or equal to a minimum value determined by the maximum headway. Therefore, the number of trips K is a variable which takes values in $[K^l, K^u]$, where K^l is the minimum number of trips for ensuring the transport quality, and K^u is the maximum possible number of trips based on the carrier's resources.

It is obvious that decreasing the number of trips will reduce the total travel time of all trips along the line. Additionally, traffic congestions of metropolitan always occur during rush hours. Thus, the trips departing in rush hours are usually caught in traffic, which will increase the total travel time. Hence, this objective impels the timetable to reduce the frequency of trips in rush hours in order to decrease the total travel time.

2.4 Total waiting time

From the perspectives of passengers, they prefer to minimize the waiting time at stops. For each stop, the departure time difference between two adjacent trips is used to characterize the average waiting time for all passengers based on the (practically reasonable) assumption that passengers arrive at stops randomly with a uniform distribution, that is

$$\Delta_{ki} = t_{ki} - t_{k-1,i},\ 1 \le i \le I, 2 \le k \le K.$$

In order to ensure the fairness among passengers, we try to decrease the maximal time difference among adjacent trips. Since the time difference between two adjacent trips for each stop is generally affected by the distance between the two stops that flank that stop, we minimize the average waiting time for all stops. We consider guaranteeing the passengers' satisfaction of a fixed bus line by minimizing the total waiting time for all passengers at all stops as follows

$$T_w(\theta) = \sum_{i=1}^{I} \left(\max_{2\le k\le K} \ \Delta_{ki} \right), 1 \le \theta \le \Theta. \tag{3}$$

This objective attempts to increase the total number of trips K in an effort to decrease the total waiting time. In addition, the traffic congestions may lead to the situation where buses can not arrive the depots in time and then depart according to timetable. Thus, the passengers usually wait for longer time at stops after the rush hours. This objective in effect impels the timetable to reduce the frequency of trips in rush hours.

2.5 Proposed model

From bus carriers' perspectives any timetable should be set up as an objective to reduce the number of required trips. However, this contradicts with the objective from passengers' perspectives which should pursue more bus trips in order to decrease the passengers' waiting time. Given such a clear conflicting requirement between the two objectives, a balance need to be achieved. In accordance with the requirements and conditions, a bi-objective bus timetable optimization is herein proposed such that

$$\begin{aligned}
\min \ & E\left[T_r(\theta)|1 \le \theta \le \Theta\right] && (4)\\
\min \ & E\left[T_w(\theta)|1 \le \theta \le \Theta\right] && (5)\\
\text{s.t.} \ & H_{min} \le t_{k1} - t_{k-1,1} \le H_{max}, 2 \le k \le K && (6)\\
& t_{ki} = t_{k,i-1} + \tau_{il}, 1 \le k \le K, 2 \le i \le I, 1 \le l \le L && (7)\\
& t_{11} = T_s && (8)\\
& t_{K1} = T_e. && (9)
\end{aligned}$$

The first objective (Eqn. 4) in the above minimizes the expected travel time of all trips along the given line. The second objective (Eqn. 5) minimizes the expected waiting time for all passengers at all stops. The third constraint (Eqn. 6) defines the minimum and maximum headways between two adjacent trips. The fourth (Eqn. 7) constrains the departure times from different stops.

The final two constraints (Eqn. 8 and 9) specify the departure times for the first trip and the last.

In order to address this bi-objective bus timetable optimization problem, we take a compromise approach. Firstly, we calculate f_{1max}, f_{1min}, f_{2max} and f_{2min} to denote the maximum expected travel time, the minimum expected travel time, the maximum expected waiting time and the minimum expected waiting time, respectively. Then, we use $\lambda \in [0, 1]$ to denote the relative importance degree on the two objectives. Furthermore, the compromise model is formulated to minimize the following linearly weighted objective function:

$$\min \quad \lambda \frac{f_1 - f_{1min}}{f_{1max} - f_{1min}} + (1 - \lambda) \frac{f_2 - f_{2min}}{f_{2max} - f_{2min}}, \tag{10}$$

In this compromise model, if the two objectives are equally important, we take $\lambda = 0.5$; if the first objective is more important than the second, we take $\lambda > 0.5$; otherwise, we take $\lambda < 0.5$.

As such, the proposed model is an integer nonlinear programming. Due to the fuzzy travel time in the model, we need to use fuzzy simulations to approximate the expected travel time and the expected waiting time. Thus, we rely on a heuristic optimization algorithm by integrating the fuzzy simulation and GA to solve the proposed model in the next section.

3 Fuzzy bi-objective timetable optimization: Solution method

A genetic algorithm (GA) is a computational method for simulating the biological evolution process of natural selection and genetic mechanism. Since genetic algorithm first proposed by Holland (1975), it has been widely studied, experimented and applied by many researchers (Pillai et al., 2017; Ting et al., 2017). Owing to the robustness and success of GA in providing good solutions to many complex optimization problems, the algorithm has been widely applied to solve various scheduling problems in the field of public transit (Liu et al., 2013; Zuo et al., 2015; Huang et al., 2016), including the application of GA to solve timetable optimization problems (Niu and Zhou, 2013; Wu et al., 2015). Following this trend, this paper designs a genetic algorithm with variable-length chromosomes in the attempt to resolve the proposed fuzzy bi-objective model.

3.1 Representation structure

A chromosome $v = (t_{11}, t_{21}, \cdots, t_{K1})$ (See Fig. 1) consists of the departure time of every trip. K is the number of trips in a day which is permitted in an interval $[K^l, K^u]$, and $t_{11} = T_s$ and $t_{K1} = T_e$ are used to denote the departure time of the first and last bus trip in a day, respectively.

t_{11}	t_{21}	t_{31}	t_{41}	t_{51}	...	$t_{K-1,1}$	t_{K1}

Fig. 1: Chromosome structure for fuzzy bi-objective timetable optimization

3.2 Initialization

An integer *pop_size* is defined as the size of population and a real number K from $[K^l, K^u]$ is generated randomly. We use the MATLAB function randfixedsum to randomly generate $K-1$ numbers $H_{k-1,k}$ $(k = 2, 3, \cdots, K)$ from $[H_{min}, H_{max}]$, satisfying $\sum_{k=2}^{K} H_{k-1,k} = T_e - T_s$. Then a chromosome $v = (t_{11}, t_{21}, \cdots, t_{K1})$ with $t_{11} = T_e$, $t_{k1} = t_{k-1,1} + H_{k-1,k}$ and $t_{K1} = T_e$ is obtained. This is in order to obtain a feasible chromosome. Repeat the above procedures for *pop_size* times and denote the generated chromosomes as $\mathbf{v}_i$, $i = 1, 2, \cdots, pop\_size$. Then, return the initialized population $\mathbf{v}_i$, $i = 1, 2, \cdots, pop\_size$.

3.3 Evaluation function

The evaluation function assigns each chromosome a probability of reproduction so that its likelihood of being selected (to act as a potential parent) is proportional to its fitness relative to the other chromosomes in the population. That is, the chromosomes with higher fitness will have more chance to produce offspring. For a real number $\alpha \in (0, 1)$, we define the evaluation function as follows

$$Eval(\mathbf{v}_i) = \alpha(1-\alpha)^{i-1}, \ i = 1, 2, \cdots, pop\_size.$$

Then, we rearrange these *pop_size* chromosomes from higher quality to lower based on the order relationship, i.e., the evaluation function, with the resultant $\mathbf{v}_1$ being the best chromosome and $\mathbf{v}_{pop\_size}$ being the worst.

3.4 Selection process

The method of spinning the roulette wheel is used to select chromosomes which breed a new generation. The chromosomes with larger fitness are more likely to be selected and the selection process is summarized in Algorithm 1.

Algorithm 1:	Selection Process
Step 1	Calculate the reproduction probability q_i for each chromosome $\mathbf{v}_i$ $$q_0 = 0, \quad q_i = \sum_{j=1}^{i} Eval(\mathbf{v}_j), \quad i = 1, 2, \cdots, pop\_size.$$
Step 2	Generate a random number r_i in $(0, q_{pop\_size}]$.
Step 3	Select the chromosome $\mathbf{v}_i$ such that $q_{i-1} < r_i \leq q_i$.
Step 4	Repeat the second and third steps *pop_size* times and obtain *pop_size* chromosomes.

3.5 Crossover operation based on multi-periods

We define a parameter P_c to denote the probability of crossover. Generate a random number r_i from $[0, 1]$ with $i = 1, 2, \cdots, pop\_size$, then select the chromosome $\mathbf{v}_i$ if $r_i \leq P_c$. Without loss of generality, the crossover operation is introduced on a pair of chromosomes $\mathbf{v}_1 = (t^1_{11}, t^1_{21}, \cdots, t^1_{K_1,1})$ and $\mathbf{v}_2 = (t^2_{11}, t^2_{21}, \cdots, t^2_{K_2,1})$ where $K_1, K_2 \in [K^l, K^u]$. Firstly, divide the operating time into N segments $[T_j, T_{j+1})$ $(j = 1, 2, \cdots, N)$, in which $T_1 = T_s$ and $T_{N+1} = T_e$. Secondly, generate N real numbers m_j from $[T_j, T_{j+1} - t)$ randomly (where t represents a unit time, typically an hour). Then m_j are used as the time start points of individual sections (a unit time per section) and hence, the sections P_j are determined. Thirdly, for the two chromosomes, determine the sections in the periods P_j, swapping the corresponding sections by exchanging the difference of two adjacent departure times, i.e., $t_{k1} - t_{k-1,1}$ in P_j, then two new chromosomes are obtained. Finally judge the length of chromosomes: if the length of the new chromosome $K_3(K_4) \in [K^l, K^u]$, then a pair of child is obtained (See Fig. 2). Otherwise, repeat the above process until two qualified chromosomes are obtained.

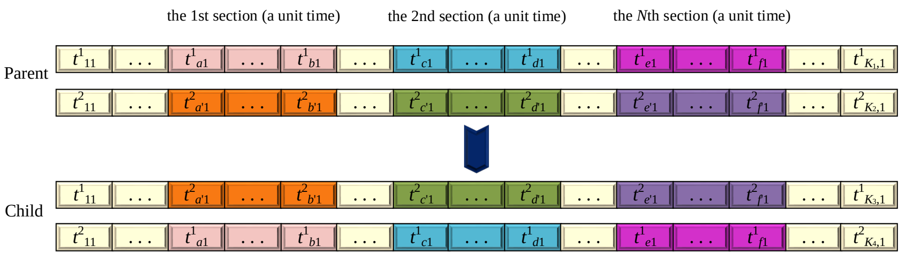

Fig. 2: Crossover process

3.6 Mutation operation based on multi-periods

We define a parameter P_m to denote the probability of mutation and repeat the following processes for pop_size times. Randomly generate a real number r_i from $[0, 1]$ with $i = 1, 2, \cdots, pop\_size$, then the ith chromosome is chosen as the parent for mutation if $r_i < P_m$. For each parent, firstly generate a real number n from $[T_s, T_e - t)$ (where t represents a time unit, again, typically an hour) randomly. Secondly, a section T (a unit time) is determined using n as the time start point of the section, containing the part $(t_{a1}, \cdots, t_{b1})$. Thirdly, operate the mutation process on the selected part $(t_{a1}, \cdots, t_{b1})$: randomly generate a real number q in $[H_{min}, H_{max}]$, and determine the first point $t_{a'1} = t_{a-1,1} + q$ in the section to be mutated. Repeat the third step to make the next point's value equal to the sum of the last point and generate a new random number q: $t_{(a+1)',1} = t_{a'1} + q$ until attaining a value $t_{b'1}$ satisfying that $(t_{b+1,1} - t_{b'1}) \in [H_{min}, H_{max}]$. Fourthly, repeat the above processes for N times and judge the length of the new chromosome K_2: if $K_2 \in [K^l, K^u]$, a child is obtained (See Fig. 3). Otherwise, repeat the above process until a qualified chromosome is obtained.

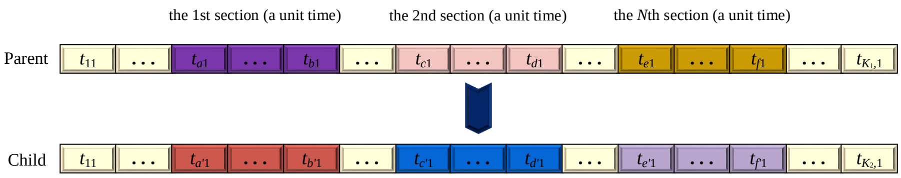

Fig. 3: Mutation operation

3.7 Procedure summary

Following the selection, crossover and mutation operations, a new population is generated. The GA will terminate after a given number of iterations running the above steps. The general procedure for the GA-based solution method is summarized in Algorithm 2.

Algorithm 2:	Genetic Algorithm
Step 1	Randomly initialize *pop_size* chromosomes.
Step 2	Calculate objective values for all chromosomes.
Step 3	Evaluate fitness of each chromosome using objective values.
Step 4	Select chromosomes by spinning roulette wheel.
Step 5	Update chromosomes using crossover and mutation.
Step 6	Repeat Step 2 to Step 5 for a given number of iterations.
Step 7	Report best found chromosome as optimal solution.

4 Case study

To illustrate how well our model and the associated solution method can be applied in reality, we present a case study utilizing real-life data obtained from a bus carrier in Beijing, China. The bus line, Yuntong 128 is of 21.44 km long with 31 stops including two depots, running from Beijing Business School Station in Changping District to Laiguangying North Station in Chaoyang District, as shown in Fig. 4.

4.1 Data preparation

The data utilized is recorded during weekdays from February 27, 2017 to March 24, 2017 from the bus carrier. Such GPS data of 20 weekdays has been analyzed for its statistical properties. Based on which, the number of bus trips in a day should be selected from the an interval [65,75] meeting the demand

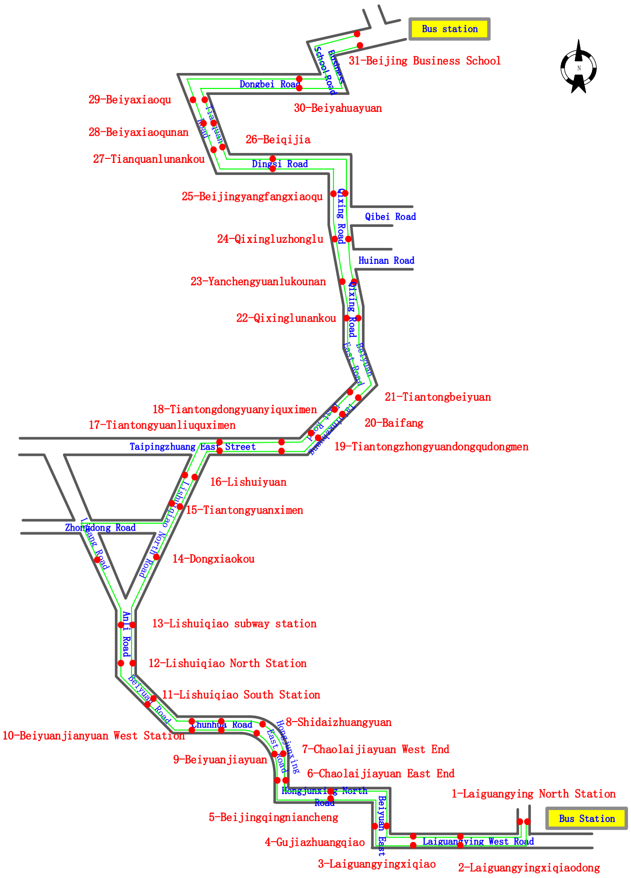

Fig. 4: Yuntong 128 bus route

of the bus carrier's service. In order to process the data of bus travel time between two adjacent stops, a time segment of 5 minutes is used to divide the operating time into 216 segments. We gather data statistics and compute the travel time from a stop to the next per segment per day. According to the bus operator's experience, the forthcoming travel time from stop i to stop $i+1$ in time segment l can be expressed as a simple fuzzy variable τ_{il}^{θ}, taking distinctive values from $\{\tau_{il}^{1}, \tau_{il}^{2}, \cdots, \tau_{il}^{20}\}$, whose credibilities are specified as follows

$$v(r) = \begin{cases} 0.2, \text{ if } r = \tau_{il}^{1}, \tau_{il}^{2}, \tau_{il}^{3}, \tau_{il}^{4}, \tau_{il}^{5} \\ 0.4, \text{ if } r = \tau_{il}^{6}, \tau_{il}^{7}, \tau_{il}^{8}, \tau_{il}^{9}, \tau_{il}^{10} \\ 0.6, \text{ if } r = \tau_{il}^{11}, \tau_{il}^{12}, \tau_{il}^{13}, \tau_{il}^{14}, \tau_{il}^{15} \\ 0.8, \text{ if } r = \tau_{il}^{16}, \tau_{il}^{17}, \tau_{il}^{18}, \tau_{il}^{19}, \tau_{il}^{20}. \end{cases}$$

This means that the last week near the forthcoming day has higher similarity on the traffic conditions with the forthcoming day, while the earlier weekday has lower similarity, which of course, reflects the commonsense.

4.2 Results

4.2.1 Experiments with fixed model parameters

Based on the reality and the bus operator's experience, the following is set such that $\lambda = 0.5$, $H_{min} = 8$ min and $H_{max} = 20$ min. Aiming to select the optimal values of P_c and P_m, we resolve the model by setting different solution parameters in GA for an initial comparison. The relative errors of the objective values are defined by

$$\frac{\text{Actual objective} - \text{Minimal objective}}{\text{Minimal objective}} \times 100\%.$$

The minimal objective values are empirically taken as the corresponding minimal of all the computational results returned. The detailed results while setting different P_c and P_m are listed in Table 2. The compromise objective value in No.13 is the best, thus the parameters in No.13, where $P_c = 0.5$ and $P_m = 0.3$ are chosen to be used in the GA for solving the model.

Table 2: Optimal values of P_c and P_m

No.	P_c	P_m	K	f_1	f_2	Compromise objective value	Relative error		
							f_1	f_2	Compromise objective value
1	0.8	0.2	65	3898.89	857.12	0.2188	0.30%	8.31%	5.14%
2	0.8	0.4	67	3999.63	800.96	0.2252	2.89%	1.21%	8.25%
3	0.8	0.6	65	3889.26	890.50	0.2275	0.05%	12.53%	9.33%
4	0.8	0.8	65	3940.05	1148.70	0.2403	1.36%	45.15%	15.48%
5	0.7	0.3	65	3907.60	909.79	0.2385	0.52%	14.96%	14.62%
6	0.7	0.5	65	3938.04	1408.50	0.2300	1.31%	77.98%	10.55%
7	0.7	0.7	66	3942.36	838.45	0.2234	1.42%	5.95%	7.37%
8	0.7	0.9	67	3934.89	1031.53	0.2293	1.23%	30.35%	10.19%
9	0.6	0.2	66	3945.53	820.59	0.2182	1.50%	3.69%	4.89%
10	0.6	0.4	65	3925.53	873.13	0.2308	0.98%	10.33%	10.91%
11	0.6	0.6	65	3887.26	895.64	0.2287	0.00%	13.18%	9.91%
12	0.6	0.8	66	3964.53	839.35	0.2292	1.99%	6.06%	10.18%
13	0.5	0.3	66	3943.79	791.36	0.2081	1.45%	0.00%	0.00%
14	0.5	0.5	65	3928.67	1061.61	0.2194	1.07%	34.15%	5.46%
15	0.5	0.7	65	3905.87	839.74	0.2147	0.48%	6.11%	3.19%
16	0.5	0.9	65	3964.99	1097.79	0.2394	2.00%	38.72%	15.08%

Apart from the above set values for the parameters P_c and P_m, the number of iterations is set to 500 for the experimental investigation. The proposed GA with these parameters is run, leading to the optimal timetable as detailed in Table 3, in which 67 bus trips ought to be arranged. The total travel time is 3991.80 minutes and the total waiting time at all stops is 828.30 minutes. The convergence of objective value is shown in Fig. 5 which indicates the proposed GA is effective to solve the proposed model.

4.2.2 Experiments by varying model parameters

By setting different H_{min} and H_{max}, we run the GA and obtain comparative results as listed in Table 4. Generally, with the increase of H_{min} and H_{max},

Table 3: Computed optimal timetable in fuzzy environment

t_{k1}	value	time	t_{k1}	value	time	t_{k1}	value	time	t_{k1}	value	time	t_{k1}	value	time
$t_{1,1}$	0	05:30	$t_{15,1}$	208	08:58	$t_{29,1}$	438	12:48	$t_{43,1}$	641	16:11	$t_{57,1}$	841	19:31
$t_{2,1}$	10	05:40	$t_{16,1}$	231	09:21	$t_{30,1}$	456	13:06	$t_{44,1}$	654	16:24	$t_{58,1}$	861	19:51
$t_{3,1}$	23	05:53	$t_{17,1}$	249	09:39	$t_{31,1}$	467	13:17	$t_{45,1}$	665	16:35	$t_{59,1}$	875	20:05
$t_{4,1}$	44	06:14	$t_{18,1}$	273	10:03	$t_{32,1}$	478	13:28	$t_{46,1}$	685	16:55	$t_{60,1}$	887	20:17
$t_{5,1}$	57	06:27	$t_{19,1}$	293	10:23	$t_{33,1}$	493	13:43	$t_{47,1}$	698	17:08	$t_{61,1}$	897	20:27
$t_{6,1}$	69	06:39	$t_{20,1}$	312	10:42	$t_{34,1}$	501	13:51	$t_{48,1}$	717	17:27	$t_{62,1}$	916	20:46
$t_{7,1}$	81	06:51	$t_{21,1}$	326	10:56	$t_{35,1}$	510	14:00	$t_{49,1}$	729	17:39	$t_{63,1}$	936	21:06
$t_{8,1}$	97	07:07	$t_{22,1}$	340	11:10	$t_{36,1}$	529	14:19	$t_{50,1}$	741	17:51	$t_{64,1}$	956	21:26
$t_{9,1}$	105	07:15	$t_{23,1}$	350	11:20	$t_{37,1}$	548	14:38	$t_{51,1}$	750	18:00	$t_{65,1}$	966	21:36
$t_{10,1}$	122	07:32	$t_{24,1}$	364	11:34	$t_{38,1}$	566	14:56	$t_{52,1}$	770	18:20	$t_{66,1}$	986	21:56
$t_{11,1}$	143	07:53	$t_{25,1}$	380	11:50	$t_{39,1}$	586	15:16	$t_{53,1}$	786	18:36	$t_{67,1}$	990	22:00
$t_{12,1}$	160	08:10	$t_{26,1}$	391	12:01	$t_{40,1}$	603	15:33	$t_{54,1}$	796	18:46			
$t_{13,1}$	171	08:21	$t_{27,1}$	410	12:20	$t_{41,1}$	618	15:48	$t_{55,1}$	813	19:03			
$t_{14,1}$	188	08:38	$t_{28,1}$	430	12:40	$t_{42,1}$	627	15:57	$t_{56,1}$	828	19:18			

the number of bus trips K and the total travel time T_r tend to decrease. When H_{min} and H_{max} are greater than or equal to 8 and 20 respectively, the results appear to be in stability.

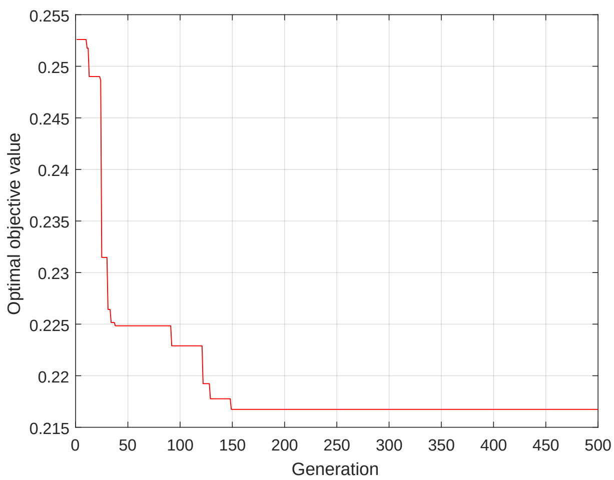

Fig. 5: Convergence curve of objective value

The comparison over the use of different λ values is also carried out, with the results listed in Table 5. As λ increases, the relative importance of f_1 increases also but that of f_2 decreases, resulting in the decrease of the number of bus trips K. The bus carrier normally prefers less bus trips in a day, while the passengers expect more bus trips to ensure less waiting time they cost at stops. The balance between the total travel time and the total waiting time is sought by the proposed model and solution algorithm to assure the interests of both bus carriers and passengers.

Table 4: Comparative results with different H_{min} and H_{max}

No.	H_{min}	H_{max}	K	f_1	f_2	Relative error of f_1	Relative error of f_2
1	6	18	72	4231.60	719.39	9.17%	0.00%
2	6	20	65	3901.35	860.24	0.65%	19.58%
3	6	22	65	3970.67	872.96	2.44%	21.35%
4	6	24	65	3889.48	900.82	0.34%	25.22%
5	7	18	69	4118.21	825.92	6.24%	14.81%
6	7	20	67	3968.18	1196.69	2.37%	66.35%
7	7	22	65	3906.34	864.31	0.78%	20.14%
8	7	24	65	3900.11	1098.54	0.62%	52.70%
9	8	18	66	3952.78	901.44	1.97%	25.31%
10	8	20	65	3908.67	874.01	0.84%	21.49%
11	8	22	65	3914.28	1289.75	0.98%	79.28%
12	8	24	65	3899.45	897.89	0.60%	24.81%
13	9	18	65	3904.78	1235.16	0.74%	71.70%
14	9	20	66	3944.96	867.55	1.77%	20.59%
15	9	22	65	3919.34	855.99	1.11%	18.99%
16	9	24	65	3876.24	849.96	0.00%	18.15%
17	10	18	65	3883.55	867.48	0.19%	20.58%
18	10	20	65	3919.13	825.99	1.11%	14.82%
19	10	22	66	3957.59	811.02	2.10%	12.74%
20	10	24	65	3890.44	866.27	0.37%	20.42%

Table 5: Comparative results with different λ

No.	λ	K	f_1	f_1	Relative error of f_1	Relative error of f_1
1	0	75	4312.95	1119.82	15.44%	49.10%
2	0.1	71	4191.95	751.07	12.20%	0.00%
3	0.2	69	4082.23	809.11	9.27%	7.73%
4	0.3	65	3977.28	1048.53	6.46%	39.60%
5	0.4	66	3944.00	848.16	5.57%	12.93%
6	0.5	67	3936.66	1104.06	5.37%	46.50%
7	0.6	65	3900.63	897.20	4.41%	19.46%
8	0.7	65	3874.89	952.72	3.72%	26.85%
9	0.8	65	3855.27	1003.63	3.19%	33.63%
10	0.9	65	3835.74	1362.85	2.67%	81.45%
11	1	65	3736.02	4686.95	0.00%	524.03%

4.3 Comparison

In order to verify the potential of the proposed model and the associated solution method, the resulting optimal timetable in the fuzzy environment is compared with that in the deterministic environment and also, with the currently used real-life timetable. We process the bus travel time between any two stops over a period of 20 days as the expected values and employ the GA in optimizing timetable in the deterministic environment. The optimized departure time of every bus trip in the deterministic environment is shown in Table 6, and the currently used timetable obtained from the bus carrier is presented in Table 7. Overall, there are 65 bus trips that should be arranged in the optimal timetable given the deterministic environment, and 67 bus trips in the currently used timetable.

In the currently used timetable, the total travel time for all trips alone the line is 5449.76 minutes and the total waiting time for all passengers at all stops is 919.97 minutes. In order to compare the results in the fuzzy and determin-

Table 6: Optimal timetable in deterministic environment

t_{k1}	value	time	t_{k1}	value	time	t_{k1}	value	time	t_{k1}	value	time	t_{k1}	value	time
$t_{1,1}$	0	05:30	$t_{14,1}$	166	08:16	$t_{27,1}$	365	11:35	$t_{40,1}$	610	15:40	$t_{53,1}$	770	18:20
$t_{2,1}$	20	05:50	$t_{15,1}$	176	08:26	$t_{28,1}$	385	11:55	$t_{41,1}$	625	15:55	$t_{54,1}$	785	18:35
$t_{3,1}$	40	06:10	$t_{16,1}$	186	08:36	$t_{29,1}$	405	12:15	$t_{42,1}$	640	16:10	$t_{55,1}$	800	18:50
$t_{4,1}$	57	06:27	$t_{17,1}$	198	08:48	$t_{30,1}$	425	12:35	$t_{43,1}$	652	16:22	$t_{56,1}$	815	19:05
$t_{5,1}$	72	06:42	$t_{18,1}$	210	09:00	$t_{31,1}$	445	12:55	$t_{44,1}$	664	16:34	$t_{57,1}$	830	19:20
$t_{6,1}$	87	06:57	$t_{19,1}$	225	09:15	$t_{32,1}$	465	13:15	$t_{45,1}$	676	16:46	$t_{58,1}$	850	19:40
$t_{7,1}$	102	07:12	$t_{20,1}$	240	09:30	$t_{33,1}$	485	13:35	$t_{46,1}$	688	16:58	$t_{59,1}$	870	20:00
$t_{8,1}$	111	07:21	$t_{21,1}$	255	09:45	$t_{34,1}$	505	13:55	$t_{47,1}$	700	17:10	$t_{60,1}$	890	20:20
$t_{9,1}$	120	07:30	$t_{22,1}$	270	10:00	$t_{35,1}$	525	14:15	$t_{48,1}$	712	17:22	$t_{61,1}$	910	20:40
$t_{10,1}$	129	07:39	$t_{23,1}$	285	10:15	$t_{36,1}$	545	14:35	$t_{49,1}$	724	17:34	$t_{62,1}$	930	21:00
$t_{11,1}$	138	07:48	$t_{24,1}$	305	10:35	$t_{37,1}$	565	14:55	$t_{50,1}$	736	17:46	$t_{63,1}$	950	21:20
$t_{12,1}$	147	07:57	$t_{25,1}$	325	10:55	$t_{38,1}$	580	15:10	$t_{51,1}$	746	17:56	$t_{64,1}$	970	21:40
$t_{13,1}$	156	08:06	$t_{26,1}$	345	11:15	$t_{39,1}$	595	15:25	$t_{52,1}$	758	18:08	$t_{65,1}$	990	22:00

Table 7: Currently used timetable

t_{k1}	value	time	t_{k1}	value	time	t_{k1}	value	time	t_{k1}	value	time	t_{k1}	value	time
$t_{1,1}$	0	05:30	$t_{15,1}$	166	08:16	$t_{29,1}$	385	11:55	$t_{43,1}$	640	16:10	$t_{57,1}$	800	18:50
$t_{2,1}$	20	05:50	$t_{16,1}$	176	08:26	$t_{30,1}$	405	12:15	$t_{44,1}$	652	16:22	$t_{58,1}$	815	19:05
$t_{3,1}$	40	06:10	$t_{17,1}$	186	08:36	$t_{31,1}$	425	12:35	$t_{45,1}$	664	16:34	$t_{59,1}$	830	19:20
$t_{4,1}$	57	06:27	$t_{18,1}$	198	08:48	$t_{32,1}$	445	12:55	$t_{46,1}$	674	16:44	$t_{60,1}$	850	19:40
$t_{5,1}$	72	06:42	$t_{19,1}$	210	09:00	$t_{33,1}$	465	13:15	$t_{47,1}$	684	16:54	$t_{61,1}$	870	20:00
$t_{6,1}$	84	06:54	$t_{20,1}$	225	09:15	$t_{34,1}$	485	13:35	$t_{48,1}$	694	17:04	$t_{62,1}$	890	20:20
$t_{7,1}$	93	07:03	$t_{21,1}$	240	09:30	$t_{35,1}$	505	13:55	$t_{49,1}$	704	17:14	$t_{63,1}$	910	20:40
$t_{8,1}$	102	07:12	$t_{22,1}$	255	09:45	$t_{36,1}$	525	14:15	$t_{50,1}$	714	17:24	$t_{64,1}$	930	21:00
$t_{9,1}$	111	07:21	$t_{23,1}$	270	10:00	$t_{37,1}$	545	14:35	$t_{51,1}$	724	17:34	$t_{65,1}$	950	21:20
$t_{10,1}$	120	07:30	$t_{24,1}$	285	10:15	$t_{38,1}$	565	14:55	$t_{52,1}$	734	17:44	$t_{66,1}$	970	21:40
$t_{11,1}$	129	07:39	$t_{25,1}$	305	10:35	$t_{39,1}$	580	15:10	$t_{53,1}$	746	17:56	$t_{67,1}$	990	22:00
$t_{12,1}$	138	07:48	$t_{26,1}$	325	10:55	$t_{40,1}$	595	15:25	$t_{54,1}$	758	18:08			
$t_{13,1}$	147	07:57	$t_{27,1}$	345	11:15	$t_{41,1}$	610	15:40	$t_{55,1}$	770	18:20			
$t_{14,1}$	156	08:06	$t_{28,1}$	365	11:35	$t_{42,1}$	625	15:55	$t_{56,1}$	785	18:35			

istic situations against the result in currently used timetable, the deviation ratios of the total travel time and the total waiting time are calculated by

$$\frac{\text{Optimal time} - \text{Current time}}{\text{Current time}} \times 100\%.$$

The comparative results are shown in Table 8 and Fig. 6. The total travel time and the total waiting time in the fuzzy situation, the deterministic situation and the currently used timetable are compared. The results are:

1. Comparing to the currently used timetable, the optimal timetable in the deterministic situation is able to reduce the total travel time by 16.03% but increases the total waiting time by 17.94%.
2. Comparing to the currently used timetable, the optimal timetable in the fuzzy situation is able to reduce the total travel time by 26.75% while also reducing the total waiting by 9.96%.
3. Compared to the optimal results in the deterministic situation, both of the total travel time and the total waiting time in fuzzy situation are significantly shorter.

Generally, both of the total travel time and the total waiting time resulted from the optimal timetable in the fuzzy situation are the shortest among the three compared. In the optimal timetable returned under the deterministic situation and also, in the currently used timetable, the arithmetic mean of the

Table 8: Comparative results in three situations

Situation	K	Total travel time (min)	Total waiting time (min)	Deviation ratio	
				Total travel time	Total waiting time
Current situation	67	5449.76	919.97	0.00%	0.00%
deterministic situation	65	4575.90	1085.00	-16.03%	17.94%
Fuzzy situation	67	3991.80	828.30	-26.75%	-9.96%

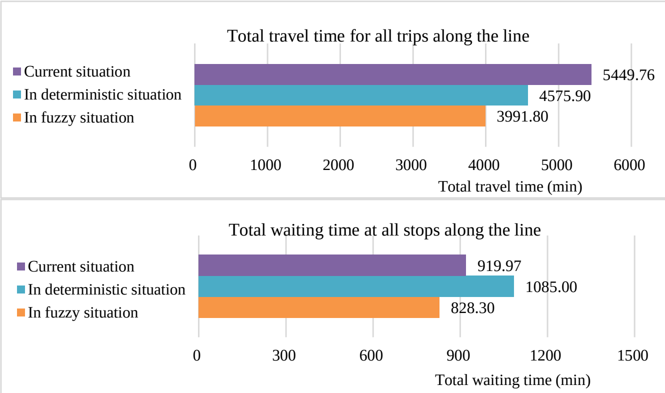

Fig. 6: Comparative results in three situations

travel times between any two stops over a period of several days is input into the model. The traffic congestions occur occasionally is a primary reason for increasing the average travel time between two stops, resulting in the increase of the total travel time and the total waiting time. Compared to the optimal timetables obtained in the fuzzy situation and that in the deterministic situation, the currently used timetable is featured with more bus trip times and a higher departing frequency in rush hours, which lead to both longer total travel time and longer total waiting time. From these results it is clear that the optimization under the fuzzy situation makes the optimal timetable more efficient and robust than that achievable by its counterparts, be they obtained in the deterministic situation or directly using the current real-life timetable.

5 Conclusion

In this paper, we have proposed a fuzzy bi-objective model for a bus timetable optimization problem which minimizes the total travel time and the total waiting time simutaneously. To incorporate the fuzziness in the modeling environment, we have processed multiple spatial-temporal travel time matrices based on real-life data acquired over several past days to statistically characterize the traffic conditions of the forthcoming situations. A genetic algorithm with variable-length chromosomes has been introduced here, for obtaining the global optimal solution to solve the proposed model. A case study utilizing real-life data has been presented to show that the model and algorithm are effective to seek a balance between the demands between bus carriers and passengers. Aiming at verifying the performance of the outcomes of this study, the optimal timetable attained in the fuzzy environment has been compared with that in the deterministic environment and the currently used real-life timetable. The results have demonstrated that the consideration of fuzziness makes the optimal timetable more robust than the other two.

This work has been focused on the optimization of the timetable for a single bus line in a fuzzy environment. However, in practice, every bus line is not isolated but exists in a shared transport network. Thus, in future work, we plan to address the more challenging problem of fuzzy bus timetable optimization within a transport network.

Compliance with ethical standards

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Conflict of interest

All authors declare that they have no conflict of interest in this research.

Human and animal rights

This article does not contain any studies with human or animal participants performed by the author.

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Appendix A

In this appendix, some basic concepts and definitions about credibility theory are introduced.

Let Θ be a nonempty set, and let $\mathcal{A}$ be its power set. Each element of $\mathcal{A}$ is called an event. Credibility measure is a set function from $\mathcal{A}$ to [0, 1]. For each event, its credibility indicates the chance that the event will occur. In order to ensure that the set function has certain mathematical properties, Li and Liu (2006) provided the following four axioms:

Axiom 1 *(Normality)* $\mathbf{Cr}\{\Theta\} = 1$.

Axiom 2 *(Monotonicity)* $\mathbf{Cr}\{A\} \leq \mathbf{Cr}\{B\}$ *for any events* $A \subseteq B$.

Axiom 3 *(Duality)* $\mathbf{Cr}\{A\}+\mathbf{Cr}\{A^c\} = 1$ *for any event* A.

Axiom 4 *(Maximality)* $\mathbf{Cr}\{\cup_i A_i\} = \sup_i \mathbf{Cr}\{A_i\}$ *for any collection of events* $\{A_i\}$ *with* $\sup_i \mathbf{Cr}\{A_i\} < 0.5$.

Let Θ be a nonempty set, $\mathcal{A}$ the power set, and $\mathbf{Cr}$ a credibility measure. Then the triplet $(\Theta, \mathcal{A}, \mathbf{Cr})$ is called a credibility space. Let ξ be a fuzzy variable on the credibility space $(\Theta, \mathcal{A}, \mathbf{Cr})$. Then, its expected value (Liu and Liu, 2002) is defined as

$$E[\xi] = \int_0^{+\infty} \mathbf{Cr}\{\xi \geq r\}dr - \int_{-\infty}^0 \mathbf{Cr}\{\xi \leq r\}dr,$$

provided that at least one of the two integrals is finite.

Example 1 Assume that ξ is a simple fuzzy variable taking district values in $\{x_1, x_2, \cdots, x_m\}$. If ξ has the following credibility function

$$v(x) = \begin{cases} v_1, & \text{if } x = x_1 \\ v_2, & \text{if } x = x_2 \\ \cdots & \cdots \\ v_m, & \text{if } x = x_m, \end{cases}$$

then it has expected value

$$E[\xi] = \sum_{i=1}^{m} w_i x_i, \tag{11}$$

where for each $1 \leq i \leq m$, the weight w_i is given by

$$w_i = \max_{x_j \leq x_i} v_j \wedge 0.5 - \max_{x_j < x_i} v_j \wedge 0.5 + \max_{x_j \geq x_i} v_j \wedge 0.5 - \max_{x_j > x_i} v_j \wedge 0.5.$$