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# A Novel Granular Approach for Detecting Dynamic Online Communities in Social Network

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**Abstract.** The great surge in the research of community discovery in complex network is going on due to its challenging aspects. Dynamicity and overlapping nature are among the common characteristics of these networks which are the main focus of this paper. In this research, we attempt to approximate the granular human-inspired viewpoints of the networks. This is especially helpful when making decisions with partial knowledge. *In line with the principle of granular computing, in which precision is avoided, we define the micro and macro granules in two levels of nodes and communities, respectively. The proposed algorithm takes micro granules as input and outputs meaningful communities in rough macro community form. For this purpose, the micro granules are drawn toward each other based on a new rough similarity measure defined in this paper. As a result, the structure of communities is revealed and adapted over time, according to the interactions observed in the network, and the number of communities is extracted automatically.* The proposed model can deal with both the low and the sharp changes in the network. The algorithm is evaluated in multiple dynamic datasets and the results confirm the superiority of the proposed algorithm in various measures and scenarios.

**Keywords:** social network analysis; dynamic community detection; granular clustering; evolutionary clustering.

## 1 Introduction and motivation

The widespread usage of various internet-based platforms has provided new forms of social interaction and collaboration, which contributes to the creation of virtual communities. With rapid growth of online dynamic social networks, where users' joining in and withdrawing from communities is common, the study of recognizing dense group of entities is a very valuable research topic and has triggered the interest of a variety of groups from social

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scientists and economists, to physicians and politicians. Recognizing niche markets for targeted advertisements and detection of influential individuals in politics are among a few applications of the “dynamic community detection” research domain, which is studied in this paper.

In spite of the increased number of studies in the past decade on dynamic communities [1], the field still requires the special attention of researchers due to its challenging aspects. Today, people join multiple groups introduced by the circle of their friends and may stay for a while or leave the groups soon thereafter. There should be smooth changes in the discovered communities, where many individuals stay for a long time, while accounting for unforeseen dramatic shifts in these communities due to external reasons. The desired approach to handle this task should be computationally feasible and rapid, in order to adapt to changes. Incremental dynamic community detection exploits the information of past time steps to estimate the current community structure and helps to produce smooth results. Evolutionary clustering is one dominant incremental paradigm widely used which accounts for the historical data of the network. The approach use the information of past time steps to improve the clustering quality and also minimize the community drift compared to previous time steps to produce smooth results. A recent survey [2] studied the application of this paradigm with different approaches. However, optimization of these two criteria together makes the evolutionary clustering suitable for the network with small changes and prevents the application of this method to abrupt changes. On the other hand, most of these methods are designed for non-overlapping networks [2]. The requirement to specify the number of communities is another problem observed in most studies [3,4]. Finally, all the methods requiring high amount of computation or space are precluded in real dynamic community detection scenarios [5].

The goal of our research is to present a novel incremental approach for dynamic networks which can produce high quality communities while being able to handle the abrupt changes in the network. On the other hand, the approach should estimate the number of communities automatically and functions locally. Finally, dealing with uncertainty in categorization is another important aspect considered, which is also considered in this study. For this, we take inspiration from the following human behaviors:

- 1) Human beings have a granular view of the world, thus prohibiting precise boundaries;
- 2) Human decision-making is mostly intertwined with uncertainty and biased toward the circle of one’s friends; and

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3) They are able to categorize both the elements similar to the previous patterns stored in their minds and also allowing to integrate the new patterns by their representative-based categorization model [6].

Using these principles, we propose our novel community detection algorithm called *Granular ARTISON* to achieve the above-mentioned goals. Shortly, the contribution of the proposed algorithm can be listed as follows:

- It introduces a novel social network modeling based on granular concepts.
- It presents a new procedure for dynamic incremental clustering algorithms capable of detecting both low and abrupt changes in the network.
- It provides an explicit novel formula for embracing uncertainty in similarity matching process.
- It functions locally and is applicable to both weighted and binary networks.
- It outperforms the existing dynamic state-of-the-art evolutionary clustering algorithms in several experiments and various internal/external measures.

To elaborate the functionality and the performance of the proposed algorithm, the paper is outlined as follows. In Section 2, we review several related methods and background knowledge in two subsections of crisp dynamic and soft community detection algorithms. Section 3 explains our proposed approach and Section 4 represents the experiments to evaluate the proposed algorithm. Finally, we present our conclusions and future directions in Section 5.

## 2 Related Work

### 2.1 Crisp Community Detection in Temporal Networks

The applicability of community detection in different domains has stimulated this research domain during the past decades. According to research papers on the static community detection domain [7], the main categories are enumerated as follows: 1) partitioning methods, 2) hierarchical methods, 3) modularity optimization methods, 4) inference-based algorithms, 5) spectral methods, and 6) label propagation methods.

Mainly, community detection algorithms in temporal networks follow two main lines of research [1]. In one prominent category, called *independent* community detection approach, the community mining process is performed in each snapshot of the network separately. Then, some correspondence with communities is obtained using similarity/distance measures to determine the relationship among them [8]. The best advantage offered in this approach is the

possibility of reusing traditional clustering methods in each snapshot. However, since most algorithms are seed-based, the emergence of unstable communities that differ drastically in different snapshots is the main weakness of this approach.

In the other category, called *incremental community detection*, the algorithms incorporate the information obtained in other snapshots for extracting communities of the current time step. This approach improves the time and computational complexity compared to the independent community detection approach [9]. An important series of works proposed in this category are *evolutionary* community detection algorithms, in which two potentially contradicting criteria or cost functions in an additive equation should be optimized [10]. The first term relates to the correspondence of clustering results to current data as much as possible (*clustering quality*), and the second involves keeping the shifts of the results between current clustering and the previous time step as low as possible (*history quality*) to allow for *temporal smoothness* between clustering results in consecutive time steps. A weighting parameter  $\alpha$  ( $0 < \alpha < 1$ ) (also known as smoothing factor) is used to weigh the two potentially contradicting criteria (Eq. (1)). Hence, the approach is best matched with networks with low changes, because there is deteriorating clustering quality in networks with rapid changes due to additive cost function.

$$Cost_{total} = \alpha SC(G_t, C_t) + (1 - \alpha) TC(C_{t-1}, C_t) \quad (1)$$

where  $SC(G_t, C_t)$  ( $SC$  for *snapshot cost*) measures the quality of the community  $C_t$  and  $TC(C_{t-1}, C_t)$  ( $TC$  for *temporal cost*) measures the drift of the community discovered in two consecutive time steps. Several improved frameworks like *AFFECT*[5] are proposed, where the weighting factor is estimated using a statistical method according to the observed changes in the network.

Amendment of traditional static clustering models using the incremental approach is introduced in the literature in different categories of partitioning [10], agglomerative hierarchical clustering such as modularity-based clustering [11], spectral-based clustering [5], label propagation-based algorithms [12], and inference-based algorithms [13].

## 2.2 Soft Community Detection

One can recognize two broad categories of node-based [14] and link-based algorithms that address the problem of multiple belongingness of nodes to different communities in social networks [15]. In a more detailed view [14], node-based methods are subcategorized into seed-based and local expansion algorithms [16], clique expansion algorithms like CPM [17] as the

pioneer of overlapping dynamic algorithms, label propagation methods like dynamic overlapping SLPA[18] and other inherently dynamic and overlapping algorithms such as *AFOCS* [19]. In the second main category, i.e., link communities, clustering is performed on links instead of nodes [20]. Here, link communities are mapped to node communities by finding nodes incidents to links within each community.

Further, there is an important class of soft clustering methods that utilize concepts specifically designed for vague and uncertain situations related to our proposed model. Decision-making under uncertain situation is identified as a basic concept underlying the human conception [21] and also as a remarkable human ability to make rational decisions with partial knowledge. Yager and Filev [22] believe that the “human has developed a granular view of the world” and “objects with which mankind perceives, measure and conceptualize and reasons are granular.” This is totally consistent with the simplified representation of the real world by humans in which the precision in different levels of a social network is not a meaningful concept. Thus, the granulation concept is heavily used to present the situations involving uncertain and vague information. Here, instead of defining exact entities, the granules are defined. Granules are the clump of objects drawn together by some indiscernibility, functionality or similarity function. The inter-relation and intra-relation among granules are responsible for grouping smaller granules into larger ones, or decomposing a large granule into smaller units. Any method in this line such as rough set and fuzzy sets is regarded as a subcategory of granular computing [23] with different aggregation functions for the construction of the granules. [Application of fuzzy methods in decision making is already discussed in several papers \[24-27\]](#). Fuzzy set theory addresses graded knowledge by fuzzy membership, and rough set theory defines knowledge granulation by the interdisciplinary relation and two sets of upper and lower bounds. A short description of these techniques is presented in the following definitions.

**Definition 1.** (*Fuzzy Set*). Let  $X$  be a set of objects. A fuzzy set  $A$  in  $X$  is a set of pairs  $A = \{(x, \mu_A(x)) \mid x \in X\}$  where  $\mu_A : X \rightarrow M$  is called the membership function of  $x$  in  $A$  mapping  $X$  into the membership space  $M$  ( $M = [0,1]$ ). Membership indicates the degree of similarity of an object  $x$  to an imprecise concept characterized by the fuzzy set  $A$ . The set of all elements having a positive membership in fuzzy set  $A$  constitutes its support set, i.e.,

$$\text{support}(A) = \{x \mid \mu_A(x) > 0\} \quad (2)$$

**Definition 2.** (*Rough Set*). A granule in a rough set corresponds to a clump of objects drawn together by some indiscernibility, functionality or similarity function. Let  $E$  be such an equivalence relation on  $X$ . Any subset  $R \subseteq X$  in the approximation space  $(X, E)$  is represented by its lower and upper approximations. The lower approximation  $\underline{R}$  is the union of all the elementary sets that are subsets of  $X$  and the upper approximation  $\overline{R}$  is the union of all the elementary sets that have a non-empty intersection with  $X$ . Finally,  $bnd(R) = \overline{R} - \underline{R}$  is the boundary region of  $A$  where hesitant items lie; i.e.:

$$\underline{R}(X) = \{x \mid x \in V, [x]_R \subseteq X\} \quad (3)$$

$$\overline{R}(X) = \{x \mid x \in V, [x]_R \cap X \neq \emptyset\} \quad (4)$$

Information granules in fuzzy clustering arise by minimizing an objective function which expresses the spread of data  $X = \{x_1, x_2, \dots, x_n\}$  around prototypes  $Q = \sum_{i=1}^c \sum_{k=1}^N u_{ik}^m \|x_k - c_i\|^2$ , where  $\ell$  stands for the number of clusters. The clusters are described in terms of a family of prototypes  $C = \{c_1, c_2, \dots, c_c\}$ . Numerous research fall in this category; however, most fuzzy-based methods require a priori knowledge about networks, e.g., the number of communities [28] or other fine-tuned parameters such as probability threshold [29].

Liu et al. [30] proposed a granular-computing based clustering algorithm where a granule is a subset of data with similar features according to their distances. For a complete review of methods in this category, please refer to [31]. Kundu [32] supposes granule construction around a node with fuzzy boundaries. In this approach, granules are constructed around each node with a fuzzy boundary, which takes its membership degree according to the following equation:

$$\mu_c(v) = \begin{cases} 0 & \text{for } d(c, v) > r \\ \frac{1}{1 + d(c, v)} & \text{otherwise} \end{cases}$$

where the distance measure  $d(c, v)$  can be any metric, e.g., the weighted hop distance from node  $v$  to the center node  $c$  as mentioned in the paper. For the application of clustering, the authors find the granular embeddedness of all granules in the network, where this value for a pair of nodes  $a$  and  $b$  is defined as:

$$\varepsilon(a, b) = |A_a \cap A_b| = \sum_{v \in V} \min(\mu_a(v), \mu_b(v)) \quad (5)$$

where  $A_a$  and  $A_b$  are the fuzzy sets representing the granules having the center nodes  $a$  and  $b$ . Then it takes a hierarchical agglomerative approach and merges similar granules together. Dillen and Chakraborty [33] used the FGSN framework to present their modularity-based community detection algorithm. However, the algorithm is designed for non-overlapping cases.

Granulation based on rough sets has found its application to clustering. In this approach, granules are chunks of objects drawn together by a similarity function. First, Lingras and West [34] used the rough set concept in k-means clustering. Mean computation, assignment of objects to the approximations, and checking the stopping criterion are the three key features of rough k-means. The calculation of the modified centroid in rough k-means is given by the following equation:

$$c_i = \left\{ \omega_{lower} \times \frac{\sum_{x \in \underline{R}(c_i)} x}{|\underline{R}(c_i)|} + \omega_{Upper} \times \frac{\sum_{x \in \overline{R}(c_i)} x}{|\overline{R}(c_i)|} \right. \quad (6)$$

where  $\omega_L + \omega_U = 1$  and  $\omega_L$  and  $\omega_U$  correspond to the relative importance of the lower and upper bound. For assigning the objects to the upper and lower bound of clusters, the ratio of  $d(x, c_i) / d(x, c_j)$ ,  $1 \leq i, j \leq k$  is used, where  $d(x, c_i)$  is the distance between any object  $x$  and the centroid of cluster  $c_i$ . Suppose that  $d(x, c_i) = \min_{1 \leq j \leq k} d(x, c_j)$  and  $T = \{j : d(x, c_i) / d(x, c_j) \geq threshold \text{ and } i \neq j\}$ . If  $T \neq \emptyset$ ,  $x \in \overline{R}(c_i)$ ,  $x$  is not part of any lower bound. Otherwise, if  $T = \emptyset$ ,  $x \in \underline{R}(c_i)$ . This algorithm and its derivation depend on the three parameters of  $\omega_L$ ,  $\omega_U$ , and *threshold*. All the derivation of k-means using soft methods inherits the problems of k-means, including the requirement of passing the number of communities to the algorithm, and their sensitivity to initial seeds. Peters and Weber [35] proposed a preliminary idea of dynamic rough clustering based on rough k-means [34]. The algorithm inherits the same challenges of rough k-means, i.e., determining the optimal number of clusters and approximation weights in each snapshot. Gupta et al. [36] used rough set concepts in their clustering approach, where a granule is formed by using a neighborhood connectedness around each node. Then, a measure called relative connectedness of the neighborhood subset is calculated for each node and all the nodes having the same measure are merged together. [A comparison of classical k-means and its fuzzy/rough version is discussed in \[37\] and granular neural network is also investigated in \[38\].](#)

As the related work in this section explains, granular clustering realized using the fuzzy set and rough set suffer from the complexity and tuning of various parameter used in the model. In the following we explain our work which is able to detect the number of communities automatically and has fewer parameters compared to rough clustering method.

### 3 Proposed Model: Rough Granular Social Network Community Mining

Taking a human perspective of the social network in which granulation is used to perceive, measure and conceptualize objects in the world, we leverage the concept of granules in different levels of nodes and communities to design our adaptive community mining algorithm called “*Rough Granular Social Network Community Detection Approach.*” The structure of the macro granules incrementally emerges based on the interaction of close-enough micro granules which are joined together over time to construct the higher level modules of the network. This process involving vagueness and rough concepts is used to model this uncertainty. The following section describes some notation and definitions which will be needed later to describe the mechanism of the construction of the final macro granules in temporal networks.

#### 3.1 RGSN: Model Description

Let us consider a temporal weighted/unweighted network by several static snapshots of the network  $\Delta = (G^1, G^2, \dots, G^s)$ . A snapshot of the network is represented by  $G^t = (X^t, E^t)$  where  $X^t$  is the set of nodes available in time step  $t$  and  $E^t \in (X^t \times X^t)$  is the set of edges available. The number of nodes may be changed in different time steps allowing for insertion and deletion of nodes. The interaction of nodes with each other can be represented in an adjacency matrix  $W^t = (w_{ij}^t)_{n \times n}$  where  $w_{ij}^t = 1$  denotes there is an edge between node  $i$  and  $j$  and other values denotes their weight of interaction. We denote the members of the network in time step  $t$  as  $X^t = \{x_1, \dots, x_n\}$  where each member includes its attribute including node id ( $v_i$ ), *timestamp* ( $t$ ) and *weight* of the interaction with their neighborhood  $\Gamma(x_i)$  where  $\Gamma(x_i) = \{x_j \in X \mid (x_i, x_j) \in E\}$ , i.e.  $(w_{ij}^t)$ . Further,  $\Omega = \{C_1, \dots, C_k\}$  is a family of clusters classifying objects into  $k$  communities.

A social network as an instance of a complex system can be characterized by the interaction that occurs between two levels of the network. From this viewpoint, the interaction

among the basic micro components of the network, i.e., the nodes, allows for the creation of the some macro structure of the network, i.e., communities, at the higher level of the system. This micro-macro mechanism is heavily discussed in emergence theories and third wave of system theories, which are well-linked to social sciences [39]. Inspired by this mechanism, we model the network at both levels of micro and macro structures as granules. Let a micro granule be associated with each node, i.e.,  $x_i \in X$  be  $\varphi(x_i)$ . In addition, the final community structure be represented by macro granule structure  $\Omega$ . Now, we model the network by a rough granular framework represented by a pair  $S = (\varphi, \Omega)$  where  $\varphi$  is a finite set of rough micro granules around each node, i.e.,  $\varphi = \{\bigcup \varphi(x_i) \mid x_i \in X\}$ , and  $\Omega$  is a finite set of rough macro granules constructed over time by the interaction of close-enough micro granules. In the following section, we present some definitions used in the proposed model.

### 3.1.1 Rough micro granule

Today, the importance of the environment within which each individual lives, has a tremendous effects on his/her decision-making process. This is long discussed in different modern social theories and is observed as de-facto. Taking this viewpoint, we model each individual as a micro granule represented together with his/her direct neighborhood. To realize the granular structure of each individual, the rough set concept is used. In the rough terminology, the members of the set are splits into two parts, deterministic and indeterministic, drawn together by similarity function. Thus, each center node is placed in the lower approximation of micro granules, and other neighbors are associated with the granule by a variation of similarity which is more highlighted if the network is weighted (neighbors with higher volumes of interaction are more similar to the center node). Hence, we give the following definition for *rough micro granule*:

**Definition 1.** (*Rough micro granule*). A micro granule around a center node  $x_i$  and annotated by  $\varphi(x_i)$  consists of the attributes for the center node  $x_i$  and its direct neighborhood class  $\Gamma(x_i)$ . Thus, the network can be represented by a set of micro granules constructed around each node ( $G^t = \{\varphi(x_i), \dots, \varphi(x_n)\}$  for all  $v_i \in V$ ). The attribute of each node in the granule is related to the network properties of these nodes. We have specified the following attributes for each member of the micro granules:

$$\varphi(x_i) = \begin{cases} \varphi_V(x_i) = \{v_i \cup v_j \mid (v_i, v_j) \in E\}, \\ \varphi_W(x_i) = \{\sum w_{ij} \cup w_{ij} \mid j \in \Gamma(x_i)\}, \\ \varphi_T(x_i) = t, \\ \varphi_\mu(x_i) = \{1 \cup \frac{w_{ij}}{\sum_{j \in \Gamma(x_i)} w_{ij}}\} \end{cases} \quad (7)$$

where

- $\varphi_V(x_i)$  stores the vertex vector ( $V$ ) comprised of the node ids for the center node  $x_i$  and its neighbors;
- $\varphi_W(x_i)$  stores the weight vector ( $W$ ) for each member of vector  $V$ . For the neighbor node  $j \in \Gamma(x_i)$ , this is equal to the weight of each neighbor  $j$  connected to the center node ( $w_{ij}$ ) and for the center node ( $x_i$ ), this will be the sum of weights  $\sum w_{ij}$ .
- $\varphi_T(x_i)$  records the time step that the nodes in the vector  $V$  are observed.
- Finally,  $\varphi_\mu(x_i)$  stores the *participation* vector ( $\mu$ ) for each member of vector  $V$  derived by the calculation of the weight value normalized by the sum of weights of the center node. The center node gets the highest participation point of 1, and other members acquire some value less than one proportional to their weight, i.e.

$\frac{w_{ij}}{\sum_{j \in \Gamma(x_i)} w_{ij}}$ . The attribute is similar to the membership degree in fuzzy set, and is an

implication of the belongingness of each member to the micro granule. The graphical representation of this structure is illustrated in Figure 1.

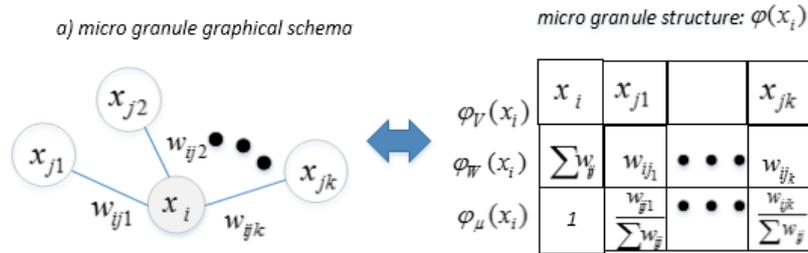


Figure 1. Micro granule structure of vertex  $x_i$

### 3.1.2 Rough macro granule

Naturally, each community may hold members shared by other communities producing an overlapping structure. The belongingness degree of individuals to different communities varies. Hence, we model each community in the framework of rough sets. In this model, the

members which are determined *surely* to belong to a community constitute the lower approximation of the rough macro granule intended to represent that community, and the members in the overlapping region with other communities constitute its upper approximation.

**Definition 3.** (*Rough macro granule*). Let  $\Omega^t(l)$  ( $l=1, \dots, k$ ) be the  $l$ 'th community discovered in time step  $t$  and the lower and upper approximation of this community be  $\underline{\Omega}_l^t$  and  $\overline{\Omega}_l^t$  respectively. The *lower approximation of the rough macro granule* set are the members which are determined to be definitely in the community  $\Omega_l$ . On the other hand, the upper approximation set of the macro granule  $l$  ( $\overline{\Omega}_l$ ) is constituted of the members which are *probably* inside the community. These members are in the overlapping regions with other communities and some of them may later be added to the lower approximation of the micro granule  $l$  ( $\underline{\Omega}_l$ ). This is mathematically expressed in the following equations:

$$\begin{aligned} \underline{\Omega}^t(l) &= \{x_j \mid x_j \in \text{Support}(\Omega(x_p)) \wedge x_j \in \text{Support}(\varphi(x_q)); \\ &\forall \Omega(x_p) \in \Omega_i \text{ and } \forall \varphi(x_q) \in \Omega_j; i \neq j\} \end{aligned} \quad (8)$$

$$\overline{\Omega}^t(l) = \{x \mid x \in \text{Support}(\varphi(x_p)); \forall \varphi(x_p) \in \Omega_i\}. \quad (9)$$

Both the lower and the upper approximation structure of the macro granules have similar attributes to the micro granules, i.e.,  $\overline{\Omega}^t(l) : \{\overline{\Omega}_V^t(l), \overline{\Omega}_W^t(l), \overline{\Omega}_\mu^t(l)\}$ . The values of these attributes are updated from the micro granule structures joined into the macro granule structure. A detailed explanation of each attribute is described in the update process.

### 3.2 Algorithm Description

In this section, we outline how the final communities comprised of densely connected nodes are formed in temporal context. Exploiting the granular model of social network as stated in Section 3.1, we first present a general view of the algorithm in a short step-by-step schema. Next, we provide a detailed explanation of different steps. The high-level procedure of rough community detection method is represented in Pseudo code 1.

**Pseudo-code 1.** High level description of the proposed algorithm

**Input:** Graph at  $t^{\text{th}}$  snapshot ( $G^t$ ) or streaming data

**Step 1.** Granule Initialization

1 a. Initialization of macro granules (prototypes) using the previous macro  
2 granules (for  $t > 1$  )

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4 b. Initialization of the next micro granule by receiving new data entry and its  
5 available neighborhood information (Eq. 8-10)  
6

7 **Step 2.** *Similarity calculation*

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9 a. Calculate the rough granule embeddedness of the micro granule into available  
10 macro granules (Eq. 11)

11  
12 b. Find the best candidate macro granule by the calculation of the maximum  
13 weighted rough granule embeddedness (Eq. 14)  
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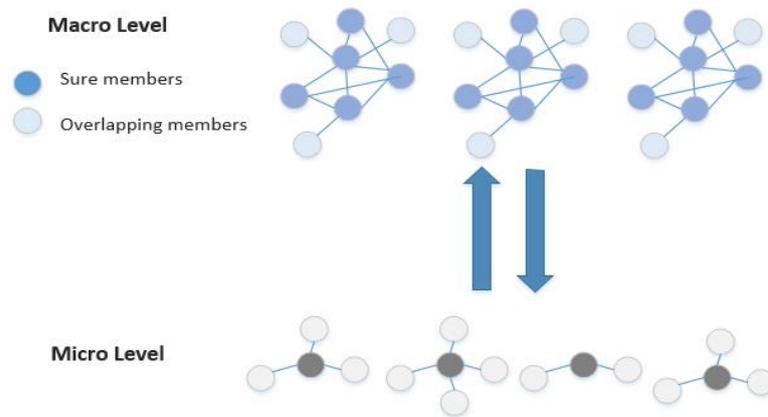
16 **Step 3.** *Update community structure*

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18 **Step 4.** *Termination or Go back to Step 1.*

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20 a. At the end of each time step, output the final disjoint/overlapping macro  
21 granules (communities)  
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24 Granular-ARTISON models the networks into granule structure in both the node (micro)  
25 and the community (macro) level, similar to human granular perception way of perceiving  
26 nodes and communities in the network. Next, the micro granules join the macro granule  
27 structure which they find the most *similar*. Group formation based on some similarity measure  
28 is verified in different studies. This is the way that is already followed by the representative or  
29 partitioning-based algorithms. If there is not any similarly *enough* macro granule to join, a new  
30 one will be created. The process of the similarity determination of each micro granule with the  
31 available macro granules is done through a two-step process, which comprises of first selecting  
32 some candidate communities (Eq. 11) and then refining the selection by finding the best match  
33 (Eq. 14). The two step process of determination is similar to the way humans are involved in  
34 the decision-making process [6]. The uncertainty is observed in different stages of network  
35 modelling and decision making. For this reason, we incorporate the uncertainty in different  
36 parts of the algorithm, specifically in the modeling of the nodes and communities and the  
37 similarity measure designed. The structure and nodes in the macro granules are modified and  
38 tuned based on the interactions of nodes observed. In the end, the most probable members of  
39 each community that have dense interactions with each other compared to other members will  
40 be stored in the lower approximation of each macro granule, and the other fringe items with  
41 fewer interaction are stored in the upper approximation of the macro granule. Hence, there may  
42 be members with different degree of memberships to a community. For the purpose of disjoint  
43 communities, only the members with high probability of belongingness to each community are  
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considered (lower approximation members of each macro granule) and for the overlapping case, the members present in both the lower and upper approximation are included. The members with high participation ratio are considered as the core nodes and the other members in upper approximation construct the overlapping region of each community. Throughout these processes, the number of communities is gradually found based on the interaction information. Having described a total schema of the algorithm, we go through a detailed explanation in the following sections. An illustration of the algorithm's components including micro-granule and macro granule in two levels of node and community is depicted in Figure 2 and will be discussed in the following parts.



**Figure 2.** A schema of the interactions observed in Granular-ARTISON algorithm in two micro and macro level (micro granule observed in the node level leads to the formation of macro granules which are adapted in different time steps)

### 3.2.1 Initialization

First, the basic elements of the framework, i.e., the micro granules, are constructed iteratively according to the information available (based on Eq. 6). After each micro granule is created, it searches for the similar macro granules (communities) in which to be included, as discussed in the next step. Further, at the beginning of each time step, the community prototypes are initialized with the rough macro granules obtained in the previous time step to preserve the temporal smoothness. Figure 3 shows the network structure composed of micro granules.

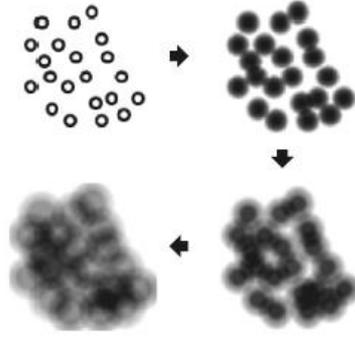


Figure 3. Node granulation concept using circle of connections

### 3.2.2 Rough granular similarity calculation

Humans are always intertwined with decision-making in uncertain situations. Here, we define a novel similarity measure that directly integrates the uncertainty of the decision-making process in its formulation. In this way, we favor the similarity obtained through sure level of rough set compared to similarity obtained by unsure level. The similarity calculation process is illustrated in the following.

First, we will determine for each node  $x_i$  under examination, along with its neighbors  $\Gamma(x_i)$  constructing the micro granule  $\varphi(x_i)$ , the most similar macro granules  $\Omega^l (l=1, \dots, k)$  for incorporating the micro granule. Obviously, at the beginning of the first timestamp, there is no macro granule available and the first macro granule ( $\Omega(1)$ ) is constructed based on the micro granule  $\varphi(x_i)$ . In this case, the center node  $x_i$  is placed in the lower approximation set of the macro granule, since we are in search of the most similar macro granule for this center member and each member is *surely* a subset of itself. The other neighbors of the center node ( $\Gamma(x_i)$ ) are *probably* members of this community and are placed in the upper approximation of this macro granule. They resemble some similarity with the sure member of the community (the center node) through the neighborhood relation. An example of micro granule is schematically represented in Figure 4.

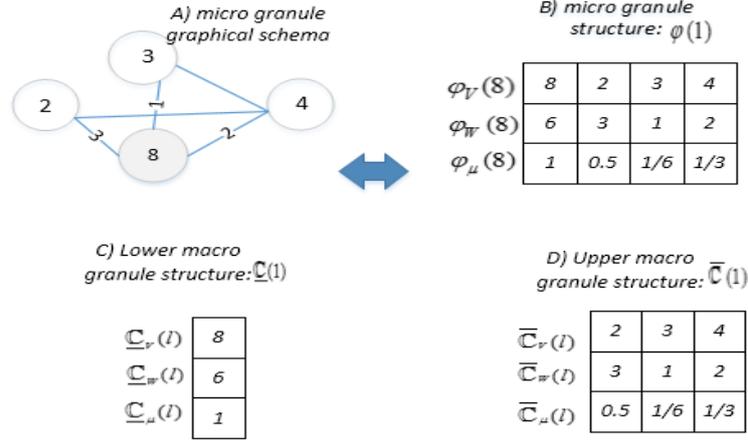


Figure 4. An example of the construction of the first macro granule based on the first micro granule

The appropriate similarity measure should be used to assess the similarity of each micro granule against multiple macro granules. Notice that the macro granules are presented in rough concepts, i.e., some members *surely* belong to the macro granule (placed in the lower approximation), and some others *probably* belong to this structure (upper approximation). Hence, it is rational to consider a *rough granular weighted similarity* measure to differentiate between the similarity values obtained from these two approximations and assign a higher weight to the term that results from the comparison with certain elements of the macro granules. The *rough granular weighted similarity* measure is formally defined as follows:

**Definition 4.** For two given micro granule  $\varphi(x_i)$  and macro granule  $\Omega_q$ , the *rough granular participation ratio* denoted as  $sim^\mu(\varphi(x_i), \Omega_q)$  is a weighted additive measure composed of the *rough granular lower participation ratio* and *rough granular upper participation ratio* measure linked by two different weights of  $(\alpha, \beta)$ , where  $\alpha > \beta$ ; i.e.,

$$sim^\mu(\varphi(x_i), \Omega(q)) = \alpha * \underline{sim}^\mu(\varphi(x_i), \underline{\Omega}(q)) + \beta * \overline{sim}^\mu(\varphi(x_i), \overline{\Omega}(q)). \quad (10)$$

These two terms are derived from the similarity value of the micro granule  $\varphi(x_i)$  in both the lower and the upper approximation of the macro granule  $\Omega(q)$ . How these two similarity measures are calculated is explained as follows.

The *rough granular lower (upper) participation ratio* measure for a given micro granule  $\varphi(x_i)$  and macro granule  $\Omega(q)$  is defined as the ratio of the participation degree of the micro granule  $\varphi(x_i)$  into the lower (upper) approximation of the macro granule  $\Omega(q)$ . This is calculated by the intersection of the participation ratio of the micro granule and the lower (upper) macro granule normalized by the value of this attribute in the micro granule, i.e.,

$$\underline{sim}^{\mu}(\varphi(x_i), \underline{\Omega}(q)) = \frac{\sum_{j \in \varphi_v(x_i) \cap \underline{\Omega}_v(q)} \varphi_{\mu_j}(x_i) \cap \underline{\Omega}_{\mu_j}(q)}{\sum \varphi_{\mu}(x_i)} \quad (11)$$

$$\overline{sim}^{\mu}(\varphi(x_i), \overline{\Omega}(q)) = \frac{\sum_{j \in \varphi_{\mu}(x_i) \cap \overline{\Omega}_q(v)} \varphi_{\mu_j}(x_i) \cap \overline{\Omega}_{\mu_j}(q)}{\sum \varphi_{\mu}(x_i)} \quad (12)$$

The micro granule is checked against all available macro granule structures to find the prototypes which have *rough granule participation ratio* higher than a threshold value. The threshold value is a minimal similarity level which is required for the inclusion of a micro granule into higher macro granule structure. Obviously, if there is not any macro granule satisfying the threshold criterion, a new community should be created to incorporate the micro granule. Otherwise, if there are several communities higher than the threshold value, the community which has the highest value of similarity is chosen to incorporate the micro granule. The decision making process is translated into the following relation:

$$\Omega^* = \max(sim^{\mu}(\varphi(x_p), \Omega(q)) > \delta) \quad (13)$$

In this way, the community that is most similar in terms of both activeness and embeddedness is selected to incorporate the micro granule members. We have chosen the threshold value of 0.3 which gives good results in various datasets.

### 3.2.3 Updating scheme

After the selection of the macro granule, which should integrate the micro granule members, an update scheme should take place to account for this assignment. The core node of the micro granule is added to the lower approximation of the selected macro granule. Further, the neighbors of the micro granule are added to the upper approximation of this macro granule. As described in Section 3.1.2, the macro granule structure has four fields of *ids*, *weights*, *timestamp*, and *participation ratio* in both the lower and the upper approximations  $\Omega(l) : \{\underline{\Omega}(l), \overline{\Omega}(l)\}$  which. Here, we give an explanation of the update process in these fields related to the lower approximation  $\underline{\Omega}^t(l) : \{\underline{\Omega}_V^t(l), \underline{\Omega}_W^t(l), \underline{\Omega}_{\mu}^t(l)\}$ :

- *Id* attribute: ( $\underline{\Omega}_V^t(l)$ ): this is the identity of each member present in the lower approximation of the macro granule  $l$ . The other attributes are defined for each member of this set ( $V$ ). In each update process, the center node is added to the

1 lower approximation of the macro granule, and the neighbors are added to the upper  
2 approximation (if not already present).

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4 • *Weight* ( $\underline{\Omega}_w^t(l)$ ): it indicates the sum of the weighted interactions observed during  
5 different iterations of the algorithm. The higher the weighted sum of a member is,  
6 the more weighted connection it has till the present time and is a better  
7 representative for that community. For the nodes added to the lower or upper  
8 approximation of the set, this attribute will be updated by adding previous values  
9 and the values of the newly added micro granule members. The final maximum  
10 value for each member will be the sum of weights of the member in the network.
- 11  
12 • *Normalized participation ratio* ( $\Omega_\mu^t(l)$ ): the attribute indicates the participation  
13 grade of the micro granule members in the macro granule structure. The values of  
14 this attribute get updated using the rough granular similarity measure obtained for  
15 The higher the value of this attribute for a member, the higher is the interaction  
16 level of this macro granule in the ; i.e. the node has the higher number of  
17 connections with the other members of the community.

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19 This community assignment scheme allows for an overlapping community structure. The  
20 certain members are in the lower approximation of the macro granule and overlapping members  
21 in the upper approximation.  
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## 36 **4 Experimental Results**

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38 In this section, we represent the results of the experiments carried out to assess the  
39 performance of the algorithm on several evolving synthetic [40], and real datasets[41-44]. First,  
40 we describe different aspects of evaluation including the introduction of datasets, algorithms  
41 and measures used. Then, the results of the experiments on synthetic and real datasets are  
42 investigated.  
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### 49 **4.1 Experiment setup**

#### 50 **4.1.1 Datasets**

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52 For synthetic dataset generation, we use the scenarios generated by the dynamic version  
53 of the frequently used LFR generator [40], which is one of the best generators producing similar  
54 properties to real networks. The availability of ground truth information makes these networks  
55 especially interesting for evaluation purposes. Each experiment is executed for 50 time and  
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1 average values are reported. Further, several famous benchmarks including Zachary Karate  
2 club [41], Doubtful Sound Dolphins [42], and the Risk Game network [45] are used to evaluate  
3 the accuracy of the algorithm. Finally, the performance of the proposed algorithm in the  
4 evolving network dataset is also examined by the NEC blog dataset [44]. Our programming  
5 environment is MATLAB.  
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#### 8 9 **4.1.2 Compared algorithms**

10 Granular-ARTISON is placed in the representative-based algorithm class, the best  
11 comparison is achieved by comparing it to other representative-based algorithms. For this  
12 reason, we choose two other evolutionary representative-based algorithms especially designed  
13 for dynamic settings. We use the state-of-the-art evolutionary framework called Adaptive  
14 Evolutionary Clustering (AFFECT [5]), extended to the k-means and spectral algorithm, in  
15 which the optimal smoothing factor is determined automatically using a statistical approach.  
16 Both algorithms require an estimation of the number of communities for each timestamp. For  
17 this purpose, we use the well-known silhouette width criterion for automatic determination of  
18 the number of communities. This measure determines how compact the distance of  
19 communities is for a given time step. The maximum width of this measure is used to assess the  
20 number of needed communities in k-means. Since both algorithms are random-based, they  
21 produce different results in experimental runs. Hence, the average value of different measures  
22 in these two algorithms are reported in the experiments. Finally, we use two other methods in  
23 real dataset evaluation section. These algorithms are in the category of modularity-based and  
24 label propagation-based algorithms called Louvain [46] and SLPA [18]. Both algorithms show  
25 high accuracy and are selected as representative algorithms in their category. We use the  
26 threshold value of  $\delta = 0.3$ , which yield good results using experiments.  
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43 For the evaluation of the proposed algorithm in overlapping case, two highly used  
44 algorithms in overlapping community detection domain with available code are selected. The  
45 first is Link-Clustering [20] algorithm as the pioneer link-based algorithm and the second is  
46 OCG [47] which has several similar functionalities to Granular-ARTISON. Specifically, OCG  
47 also uses a local similarity based algorithm where different segments of the network are merged  
48 into each other to construct communities.  
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#### 54 **4.1.3 Compared measures**

55 The standard approach for assessing the accuracy of community detection results is to  
56 exploit the external measures that compare the accuracy of detected communities with ground  
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1 truth communities. We use a range of measures including the Rand Index, NMI and the F-  
2 measure to determine the accuracy of the community detection algorithms in different  
3 scenarios, as will be explained. The NMI measure indicates how much knowing one of these  
4 communities helps predict the structure of the other and reduce prediction uncertainty. This  
5 measure has proven to be a robust and accurate similarity measure for many modalities. It is  
6 bounded in  $[0,1]$  to verify the complete sharing of the partitions found when the value is equal  
7 to 1 and the complete dependence of the partitions when it equals 0. The F-measure presents a  
8 harmonic means of precision and recall measures, where precision is the ratio of relevant  
9 objects (real community members detected) to the total number of objects detected, and recall  
10 is the ratio of relevant objects detected to the total number of objects based on ground truth.  
11 All these measures reach their best at 1 and their worst at zero value. Further, the internal  
12 measure of modularity is used to assess the quality of the communities in real networks. We  
13 use the well-known modularity measure for this purpose. **In addition, when the functioning of  
14 the algorithm in overlapping case is assessed, the overlapping-NMI [48] specifically measure  
15 is used which is designed to specifically assess the accuracy of clustering in overlapping case.  
16 Another measure in this context is the accuracy of the number of clusters each algorithm is able  
17 to detect which is used in the evaluation of algorithms in this category.**

## 32 **4.2 Artificial networks**

33 To evaluate the performance of the algorithm in dynamic settings, it is important to assess  
34 the algorithm on dynamic networks where ground truth information is available. Dynamic  
35 network environments incorporate different events of birth, merge and expansion/contraction  
36 sections. For this purpose, we use the dynamic synthetic dataset generator introduced by  
37 Greene et al. [40], which is derived from the well-known LFR benchmark network. The  
38 benchmark can create different events in dynamic scenarios and inherits the basic statistical  
39 properties of the real networks in heterogeneous distributions of the degree and the community  
40 size. Different parameters of this benchmark are tunable, which allows for overlapping and  
41 dynamic network settings. These parameters include: size of the network  $N$ , size of the  
42 communities (within  $C_{\min}$  to  $C_{\max}$ ), and mixing parameter, i.e., the overlap among  
43 communities ( $\mu$ ). The combination of these settings helps to analyze the algorithm in detail.

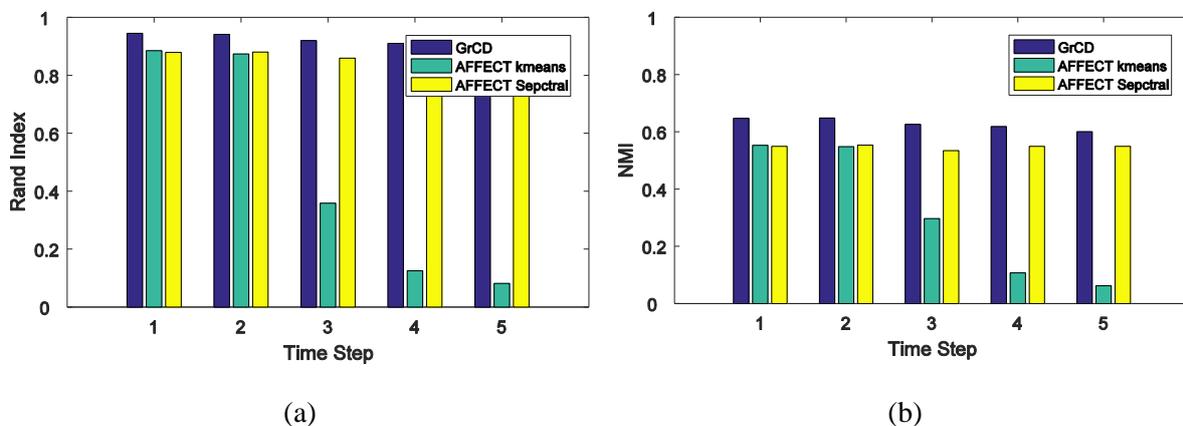
44 In our experiments, 1000 nodes in five time steps undergo different events to evaluate  
45 the performance of the algorithm. The number of nodes may change during different events.  
46 The generated scenarios follow standard settings for producing power-law networks as used in  
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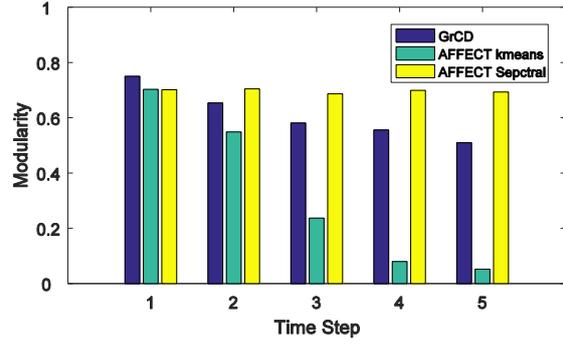
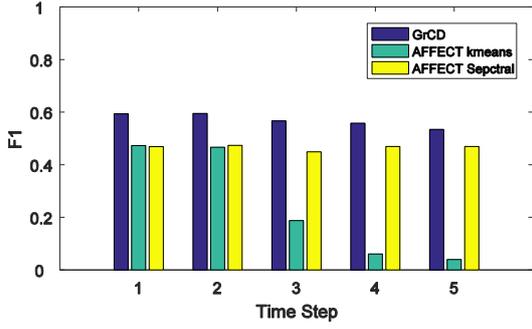
[49]. Events used in the experiments are: 1) birth and death; 2) expansion and contraction; 3) merging and splitting and 4) intermittent communities. The average and maximum degree is set to 20 and 50, respectively, and the minimum and maximum community size is 10 and 50 nodes, respectively. The initial number of nodes is 1000 nodes and the experiments are averaged over repeated runs for consistency.

#### 4.2.1 Birth and death event

In this experiment, 10% of new communities are created by removing nodes from other existing communities, and randomly removing 10% of the existing communities. The results of the experiments are averaged in Table 1. The values of measures obtained in each time step is also illustrated in Figure 5. The number of nodes decreases from 1000 nodes in the beginning to 784 nodes in the last run of the algorithm.

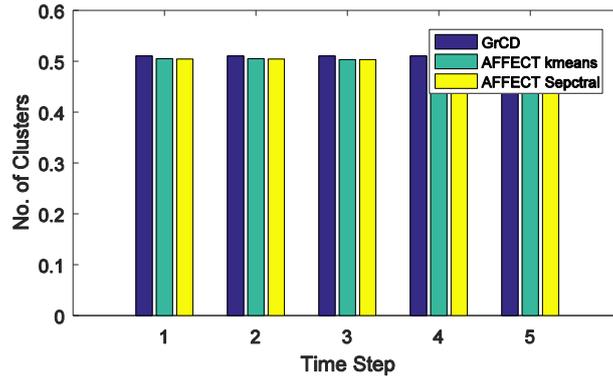
This event is one of the hardest events to capture for all algorithms. As illustrated in Figure 5, the performance is degraded in the NMI, F-measure and modularity measures. AFFEKT k-means encounters serious problems from the third time step, and the performance in all measures deteriorate. Meanwhile, Granular-ARTISON preserves high differences with AFFEKT k-means in all measures by an average of 30%. Specifically, there is a 46% difference in rand index, which measures correct assignment of nodes to their communities. The NMI measure in Granular-ARTISON shows 32% and 8% improvement over AFFEKT k-means and AFFEKT spectral, respectively. The priority is kept on F measures, too. However, the modularity of the AFFEKT spectral is the only case where this algorithm stands higher than Granular-ARTISON. The higher value of the Rand index in Granular-ARTISON, which is the manifestation of the correct assignment of the nodes to their communities, compensates for this shortcoming. The number of clusters assessed by all algorithms is similar.





(c)

(d)



(e)

Figure 5. Performance comparison of Granular-ARTISON in the *birth and death* event of the synthetic data generator with AFFECT k-means and AFFECT spectral in five measures of a) Rand Index, b) NMI, c) F1, d) Modularity and e) Accuracy of the No. of clusters.

Table 1. Comparison of proposed algorithm in synthetic dataset with AFFECT k-means and AFFECT spectral in mean values in five measures of a) Rand Index, b) NMI, c) F1, d) Modularity and e) Accuracy of the No. of clusters.

Network Event	Measure	Granular-ARTISON	AFFECT k-means	AFFECT Spectral
Birth and death event	Rand Index	<b>0.92</b> ( $\pm 0.03$ )	0.46( $\pm 0.39$ )	0.87( $\pm 0.01$ )
	NMI	<b>0.63</b> ( $\pm 0.02$ )	0.31( $\pm 0.23$ )	0.55( $\pm 0.01$ )
	F-measure	<b>0.57</b> ( $\pm 0.03$ )	0.25( $\pm 0.21$ )	0.47( $\pm 0.01$ )
	Modularity	0.61( $\pm 0.09$ )	0.32( $\pm 0.29$ )	<b>0.70</b> ( $\pm 0.01$ )
	No. of cluster accuracy	<b>0.51</b> ( $\pm 0.01$ )	0.50( $\pm 0.01$ )	0.50( $\pm 0.01$ )

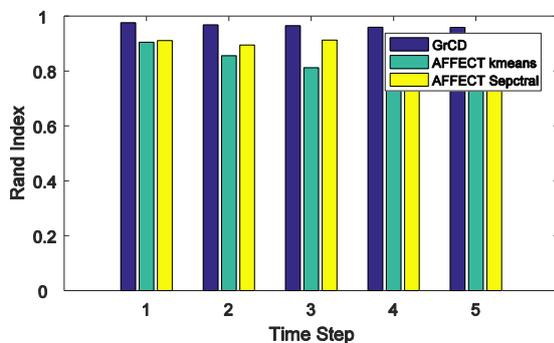
#### 4.2.2 Expansion and contraction event

In this event, 10% of communities are randomly selected and expanded or contracted by 25% of their size. The number of nodes varies from 1000 in the beginning to 970 in the last run. The average values of measures are reported in Table 2, and the results of each time step

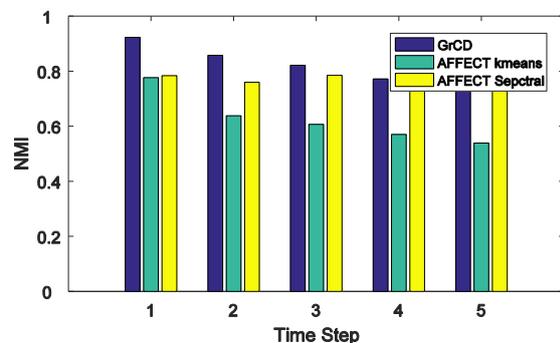
are illustrated in Figure 7. The average value of all measures in Granular-ARTISON is higher than the rest. The Rand index values of all algorithms are close to each other. AFFECT spectral stands in second place and AFFECT k-means in last place. The average values of NMI and F-measure in Granular-ARTISON show 20% and 8% improvement over AFFECT k-means and AFFECT spectral, respectively. Like the birth and death event, the modularity value of AFFECT spectral is slightly higher than Granular-ARTISON. Overall, it appears that the expansion scenario is easier to capture in all algorithms.

**Table 2. Comparison of proposed algorithm in synthetic dataset with AFFECT k-means and AFFECT spectral in five measures of a) Rand Index, b) NMI, c) F1, d) Modularity and e) Accuracy of the No. of clusters.**

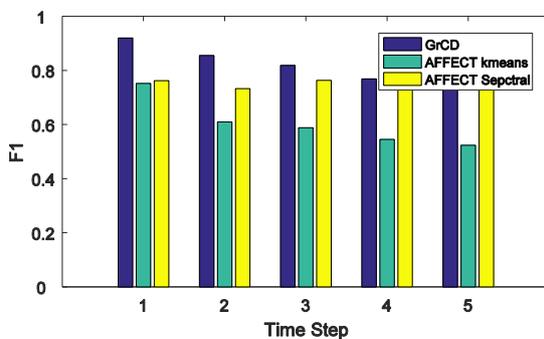
Network Event	Measure	Granular-ARTISON	AFFECT k-means	AFFECT spectral
Expansion and Contraction event	Rand Index	<b>0.97</b> ( $\pm 0.01$ )	0.81( $\pm 0.07$ )	0.91( $\pm 0$ )
	NMI	<b>0.83</b> ( $\pm 0.05$ )	0.63( $\pm 0.05$ )	0.78( $\pm 0.04$ )
	F-measure	<b>0.82</b> ( $\pm 0.07$ )	0.60( $\pm 0.09$ )	0.75( $\pm 0.01$ )
	Modularity	0.64( $\pm 0.07$ )	0.55( $\pm 0.09$ )	<b>0.69</b> ( $\pm 0.01$ )
	No. of cluster accuracy	<b>0.82</b> ( $\pm 0.0$ )	0.62( $\pm 0.02$ )	0.63( $\pm 0.01$ )



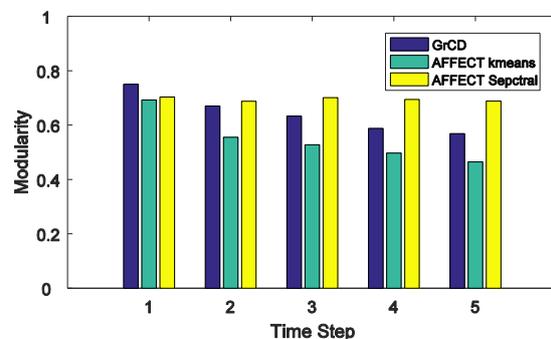
(a)



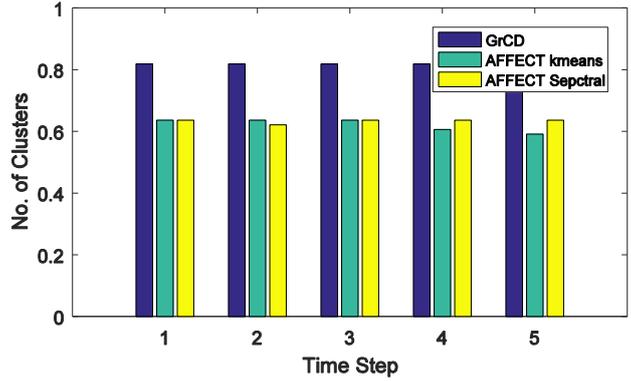
(b)



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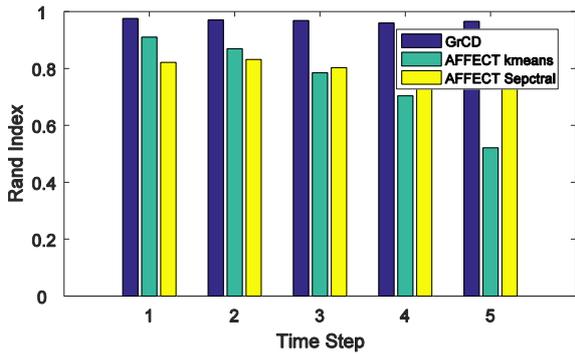
(e)

Figure 6. Performance comparison of Granular-ARTISON in the *expansion and contraction* event of the synthetic data generator with AFFECT k-means and AFFECT spectral in five measures of a) Rand Index, b) NMI, c) F1, d) Modularity and e) Accuracy of the No. of clusters.

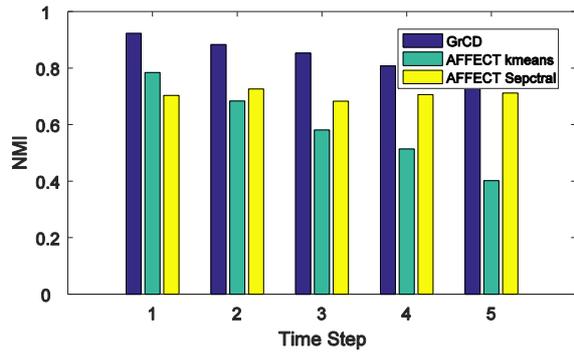
### 4.2.3 Intermittent communities event

In this event, 10% of communities from the first time step hide. The results of this experiment are shown in Figure 7 and the average value and standard deviations are reported in Table 3.

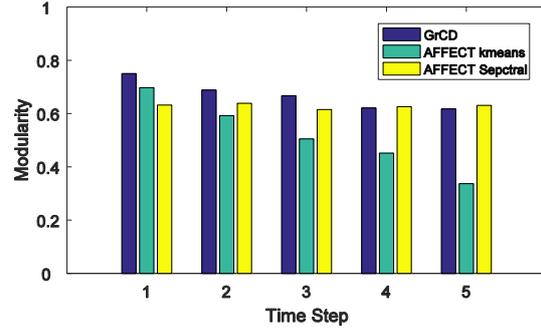
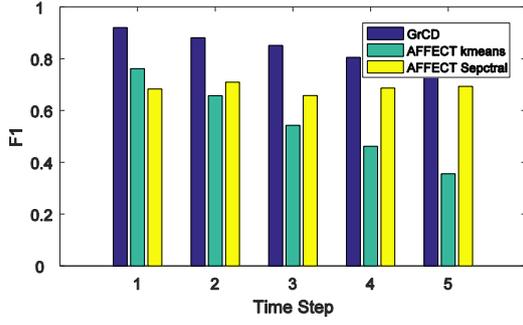
The number of nodes in time steps are as follows: 1000, 892, 917, 909 and 927 nodes. In this experiment, Granular-ARTISON is superior in all measures of the Rand Index, NMI, F-measure and modularity by almost 20% and 10% with respect to AFFECT k-means and AFFECT spectral, respectively. Further, the number of correctly guessed communities is also higher or equal to other algorithms in different steps (Figure 7 (e)).



(a)

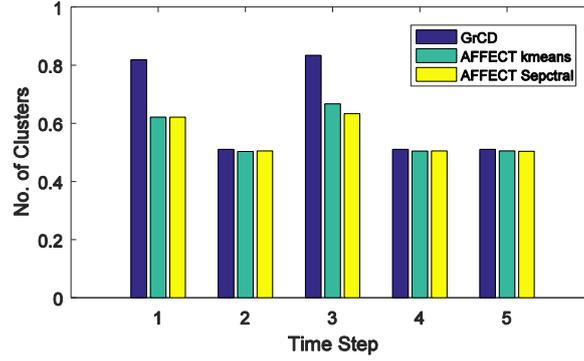


(b)



(c)

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Figure 7. Performance comparison of Granular-ARTISON in the *hiding* event of the synthetic data generator with AFFECT k-means and AFFECT spectral in five measures of a) Rand Index, b) NMI, c) F, d) Modularity and e) Accuracy of the No. of clusters.

Table 3. Comparison of proposed algorithm in synthetic dataset with AFFECT k-means and AFFECT spectral in five measures of a) Rand Index, b) NMI, c) F, d) Modularity and e) Accuracy of the No. of clusters.

Network Event	Measure	Granular-ARTISON	AFFECT k-means	AFFECT Spectral
birth and death	Rand Index	<b>0.97</b> (±0.01)	0.76(±0.15)	0.82(±0.01)
	NMI	<b>0.86</b> (±0.05)	0.59(±0.15)	0.71(±0.02)
	F-measure	<b>0.85</b> (±0.05)	0.56(±0.16)	0.69(±0.02)
	Modularity	<b>0.67</b> (±0.05)	0.52(±0.14)	0.63(±0.01)
	No. of cluster accuracy	<b>0.64</b> (±0.17)	0.56(±0.08)	0.55(±0.07)

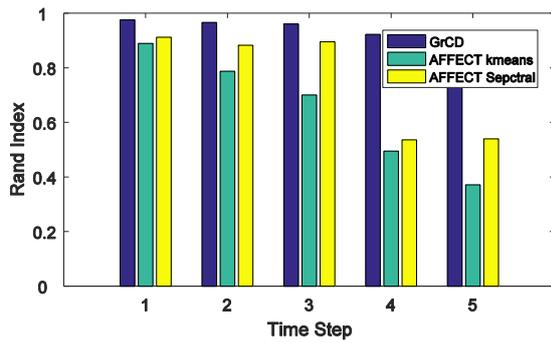
#### 4.2.4 Merging and splitting event

In merging and splitting events, 10% of communities are split, 10% of communities are chosen, and a couple of communities are merged at each time step. The average values of measures are reported in Table 4 and the results of each time step are illustrated in Figure 8. The number of nodes is fixed, but various mergers and splits make the scenarios rather difficult for evolutionary-based algorithms. However, Granular-ARTISON results show its high

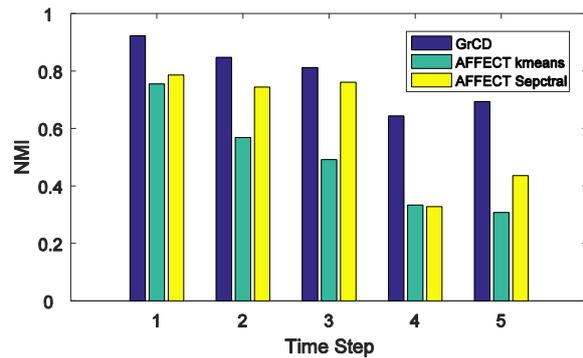
performance compared with the other two algorithms by almost 20% in NMI, F-measure and modularity measures. Finally, the number of communities derived by Granular-ARTISON is closer to ground truth by achieving a mean accuracy of 83% compared to almost 60% of evolutionary-based algorithms.

**Table 4. Comparison of proposed algorithm in synthetic dataset with AFFECT k-means and AFFECT spectral in five measures of a) Rand Index, b) NMI, c) F, d) Modularity and e) Accuracy of the No. of clusters.**

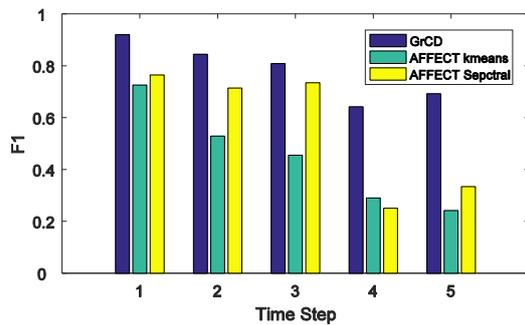
Network Event	Measure	Granular-ARTISON	AFFECT k-means	AFFECT Spectral
Merging and splitting	Rand Index	<b>0.95</b> ( $\pm 0.02$ )	0.65( $\pm 0.21$ )	0.75( $\pm 0.20$ )
	NMI	<b>0.78</b> ( $\pm 0.11$ )	0.49( $\pm 0.16$ )	0.61( $\pm 0.20$ )
	F-measure	<b>0.78</b> ( $\pm 0.11$ )	0.45( $\pm 0.19$ )	0.56( $\pm 0.25$ )
	Modularity	<b>0.60</b> ( $\pm 0.03$ )	0.40( $\pm 0.20$ )	0.56( $\pm 0.17$ )
	No. of clusters accuracy	<b>0.82</b> ( $\pm 0.16$ )	0.61( $\pm 0.03$ )	0.63( $\pm 0.01$ )



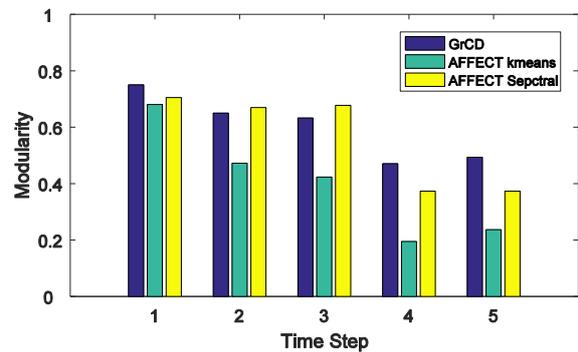
(a)



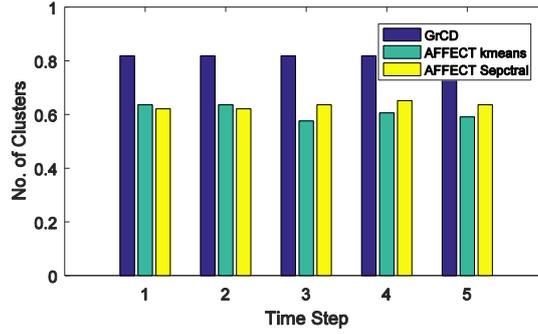
(b)



(c)



(d)



(e)

**Figure 8. Performance comparison of Granular-ARTISON in the merging and splitting event of the synthetic data generator with AFFECT k-means and AFFECT spectral in five measures of a) Rand Index, b) NMI, c) F, d) Modularity and e) Accuracy of the No. of clusters.**

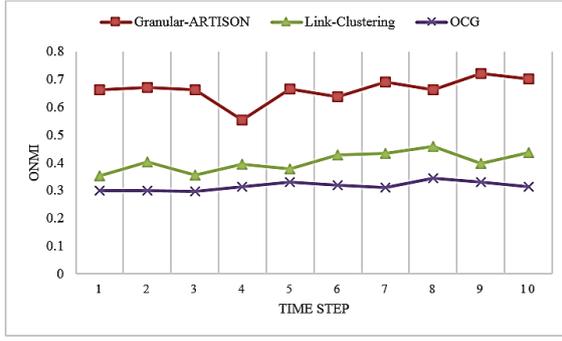
We performed a two-tailed t-test at a 0.05 level of significance tests were performed after ensuring that the data followed a normal distribution (by using the Kolmogorov–Smirnov test). The result of tests performed between the average value of F-measure for Granular-ARTISON and the two other evolutionary algorithms are reported in the Table 5. Results indicate that Granular-ARTISON preserves its difference with other algorithms in all events (less than 0.05). Hence, the good performance of Granular-ARTISON compared to other algorithm can be expected in other datasets, too.

**Table 5. T-test results of average F-measure between Granular-ARTISON and the other evolutionary algorithm on four dynamic events.**

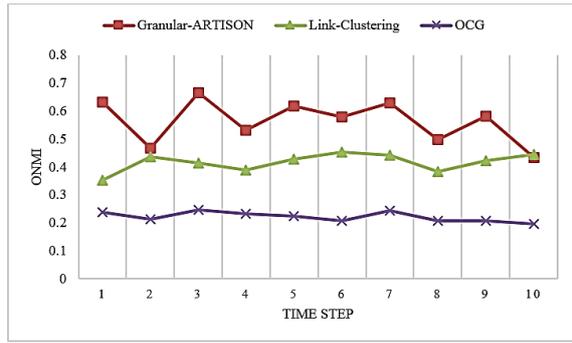
	Birth and death	Expansion	Hiding	Merge and split
<b>Granular-ARTISON and AFFECT k-means</b>	0.018344	8.11461E-05	0.00418	0.00128
<b>Granular-ARTISON and AFFECT spectral</b>	0.000879	0.084382	0.00191	0.02585

#### 4.2.5 Overlapping scenarios

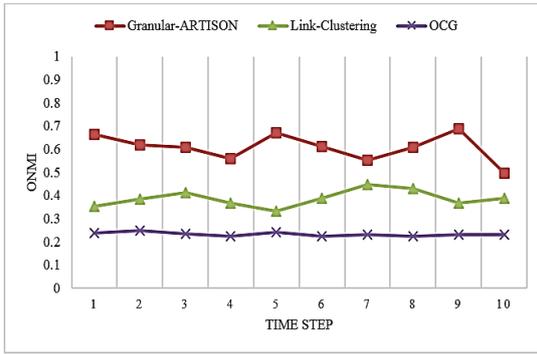
When Granular-ARTISON is working in the overlapping case, the upper bound members of each community are considered as its overlapping members. In this case, the performance of the algorithm is benchmarked against the other overlapping algorithms. As the results in Figure 9 illustrates, Granular-ARTISON shows much better performance than the other two overlapping algorithms in overlapping-NMI measure.



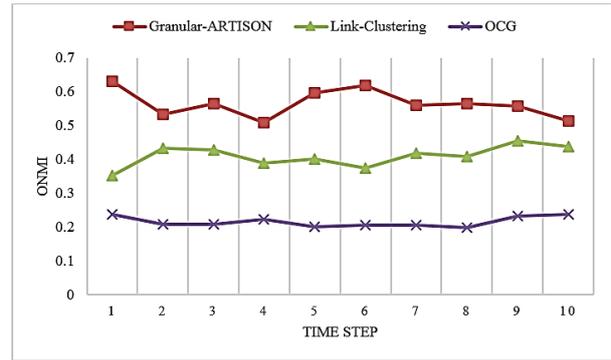
(a): birth and death event



(b): expansion and contraction event



(c): hiding event



(d): merge and split event

**Figure 9. Performance evaluation of Granular-ARTISON in overlapping mode in the four scenarios of a) birth and death, 2) expansion and contraction, 3) hide and 4) merge and split versus Link-Clustering and OCG algorithms in overlapping-NMI measure**

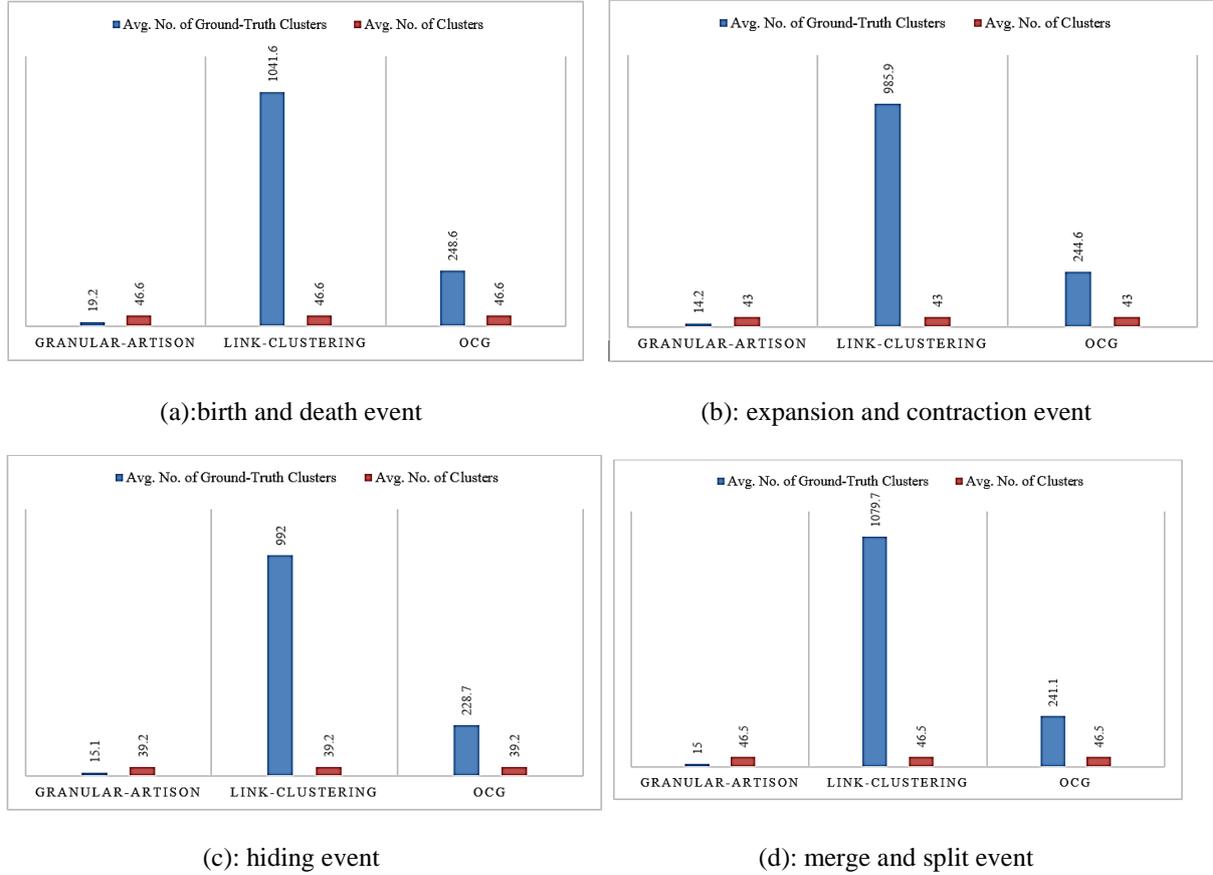
In the best case of birth and death event, the algorithm on average shows 24% and 34% higher value than Link-Clustering and OCG algorithms, respectively. In all events Link-Clustering algorithm stands in the second place and the lowest overlapping-NMI value is for OCG algorithm. The average and deviation values are reported in Table 6.

**Table 6. Average value and standard deviation value of Granular-ARTISON in overlapping mode in the four scenarios of a) birth and death, 2) expansion and contraction, 3) hide and 4) merge and split versus Link-Clustering and OCG algorithms in overlapping-NMI measure**

	Birth and Death	Expansion and Contraction	Hiding	Merge and Split
<b>Granular-ARTISON</b>	<b>0.66(±0.04)</b>	<b>0.56(±0.07)</b>	<b>0.60(±0.03)</b>	<b>0.56(±0.03)</b>
<b>Link-Clustering</b>	0.40(±0.03)	0.51(±0.04)	0.38(±0.03)	0.40(±0.03)
<b>OCG</b>	0.31(±0.01)	0.22 (±0.01)	0.23 (±0.01)	0.21(±0.01)

Another measure used to assess the accuracy of the algorithms in the number of clusters found is to investigate the number of clusters found by each algorithm and compare the value

to the ground truth value. Figure 10 illustrates the results of this experiment. Each experiment is executed for 50 times and average values are reported.



**Figure 10. Performance evaluation of Granular-ARTISON in overlapping mode in the four scenarios of a) birth and death, 2) expansion and contraction, 3) hide and 4) merge and split versus Link-Clustering and OCG algorithms in the accuracy of the average number of clusters**

The high difference of the number of clusters found compared to ground truth value in both Link-Clustering and OCG value are verified in these experiments. This problem is reported in different papers [15] and demands more attention. Obviously, Granular-ARTISON shows the best performance in all scenarios of this experiment.

**Table 7. Average value and standard deviation value of Granular-ARTISON in overlapping mode in the four scenarios of a) birth and death, 2) expansion and contraction, 3) hide and 4) merge and split versus Link-Clustering and OCG algorithms in the accuracy of the average number of clusters**

	Birth and Death	Expansion and Contraction	Hiding	Merge and Split
<b>Ground-Truth</b>	46.6( $\pm 2.53$ )	43.0( $\pm 2.98$ )	39.2( $\pm 1.08$ )	46.5( $\pm 2.98$ )
<b>Granular-ARTISON</b>	<b>19.2(<math>\pm 2.69</math>)</b>	<b>14.2(<math>\pm 3.23</math>)</b>	<b>15.1(<math>\pm 3.50</math>)</b>	<b>15.0(<math>\pm 2.63</math>)</b>
<b>Link-Clustering</b>	1041.6( $\pm 148.93$ )	985.9( $\pm 115.16$ )	999.0( $\pm 132.27$ )	1079.7( $\pm 157.95$ )

<b>OCG</b>	248.6( $\pm 13.25$ )	244.65( $\pm 17.95$ )	228.7 ( $\pm 2.51$ )	241.1( $\pm 13.25$ )
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### 4.3 Real dataset evaluation

In this section, we evaluate the performance of Granular-ARTISON on several well-known benchmarks used heavily for evaluation of community detection algorithms. Networks studied are Zachary Karate club [41], Doubtful Sound Dolphins [42], and the Risk Game network [45]. These benchmarks assess the performance of the algorithms in static settings. For the purpose of the evaluation in dynamic real networks, we use the weighted blog dataset [44] provided by NEC laboratory gathered during 15 months of monitoring. The dataset was recently used in numerous studies of dynamic community detection [50,44]. For a comparison with the state-of-the-art algorithms in different category of community detection, two other algorithms, i.e., Louvain [46] and SLPA [18] /LabelRankT [12], are used. Louvain is a modularity-based algorithm, and SLPA and LabelRankT are placed in the label propagation-based category. In SLPA, the probability of observing a label is interpreted as the membership strength. This algorithm can determine the number of communities automatically and is applicable to both weighted and directed versions. All algorithms can deal with weighted networks. Further, LabelRankT, which is proposed by the same author of the SLPA algorithm, is specifically designed for dynamic settings and is added for comparison with the dynamic NEC dataset. Table 8 gives a summary of the statistics on the datasets used for the evaluation of our algorithm in this paper.

Table 8. Statistics of the datasets used for experiments

Network dataset	# Nodes	# Edges	average degree
<b>Karate</b>	34	78	4.5
<b>Risk</b>	42	165	5.5
<b>Dolphins</b>	62	165	5.1
<b>NEC Blog</b>	407	148,681	8.3

#### 4.3.1 Zachary Karate club

The well-known Karate club network shows the friendship networks of Football College. The experiments on this network split the network perfectly into two partitions without any mismatch in different measures of Rand Index, F-measure and NMI. The modularity of

Granular-ARTISON is also higher than other algorithms. Finally, the number of clusters is best assessed in Granular-ARTISON and AFFECT spectral in common.

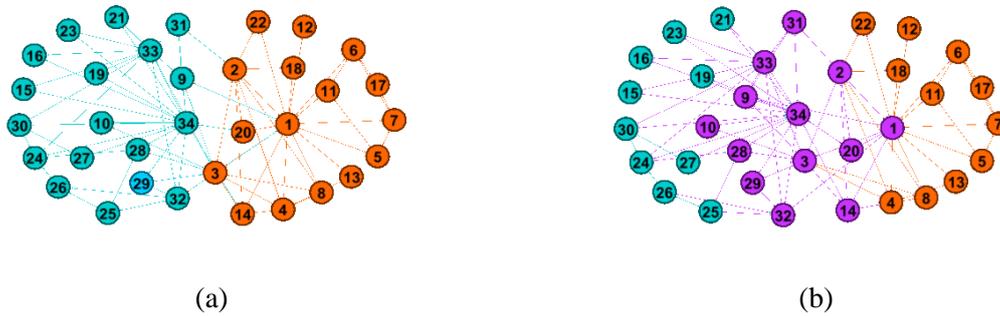


Figure 11. Community structure of Zachary club football network discovered by Granular-ARTISON in two cases of a) disjoint clusters and b) overlapping clusters (purple items are common in overlapping regions of two communities).

Table 9. Comparison of Granular-ARTISON in Zachary club football dataset with AFFECT k-means, AFFECT spectral, Louvain and SLPA in mean value of five measures: Rand Index, NMI, F, modularity and the accuracy of the assessed number of communities

	Granular-ARTISON	AFFECT k-means	AFFECT Spectral	Louvain	SLPA
<b>Rand Index</b>	<b>1</b>	0.84	0.94	0.66	0.68
<b>NMI</b>	<b>1</b>	0.65	0.84	0.55	0.45
<b>F-measure</b>	<b>1</b>	0.69	0.84	0.49	0.44
<b>Modularity</b>	<b>0.37</b>	0.26	0.36	0.32	0.22
<b>No. of clusters accuracy</b>	<b>1</b>	0.2	1	0.25	0.75

### 4.3.2 Dolphins

The other social network studied to test the accuracy of the algorithm is a social network of frequent associations among 62 dolphins in a community in Doubtful Sound [42]. In this network, dolphins are represented as vertices, and a link is attached between two nodes if the corresponding dolphins are observed together more often than expected by chance over a period of seven years from 1994 to 2001. The groups of dolphins are mainly divided into the male ones and female ones.

Our result is shown to be completely the same as the ground truth. The two communities are marked by purple and blue, respectively (shown in Figure 12).

Table8. Comparison of Granular-ARTISON in *dolphins' network* dataset with AFFECT k-means, AFFECT spectral, Louvain and SLPA in mean value of five measures: Rand Index, NMI, F, modularity and the accuracy of the assessed number of communities

	Granular-ARTISON	AFFECT k-means	AFFECT Spectral	Louvain	SLPA
<b>Rand Index</b>	0.74	0.69	0.74	0.73	<b>0.94</b>
<b>NMI</b>	<b>0.65</b>	0.49	0.71	0.56	0.55
<b>F measure</b>	<b>0.64</b>	0.44	0.66	0.56	0.55
<b>Modularity</b>	0.38	0.34	0.38	0.24	<b>0.40</b>
<b>No. of clusters accuracy</b>	<b>0.88</b>	0.75	0.75	0.75	0.75

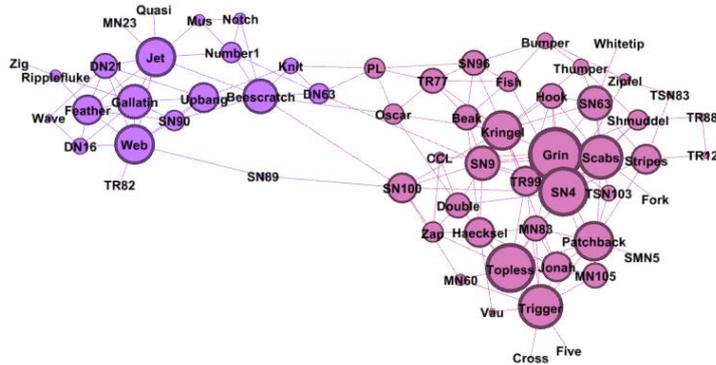


Figure 12. Community structure of Dolphin network discovered by Granular-ARTISON

### 4.3.3 Risk game network

The risk game network is recently used for assessing community detection accuracy of the algorithms [36]. It is a popular strategy game played on a board depicting a political map of the Earth, divided into forty-two territories which are grouped into six continents.

Table8. Comparison of Granular-ARTISON in *risk map* dataset with AFFECT k-means, AFFECT spectral, Louvain and SLPA in mean value of five measures: Rand Index, NMI, F, modularity and the accuracy of the assessed number of communities.

	Granular-ARTISON	AFFECT k-means	AFFECT Spectral	Louvain	SLPA
<b>Rand Index</b>	<b>0.87</b>	0.70	0.63	0.85	0.73
<b>NMI</b>	<b>0.71</b>	0.52	0.71	0.56	0.55
<b>F1 measure</b>	0.75	0.53	0.49	0.72	<b>0.90</b>
<b>Modularity</b>	0.60	0.43	0.39	0.43	<b>0.62</b>
<b>No. of clusters accuracy</b>	0.92	0.75	0.67	0.58	<b>1</b>

### 4.3.4 NEC Blog Dataset

The temporal dataset is gathered over a 15-month period by an in-house blog crawler of NEC laboratory [44]. It contains information on 407 blogs which contribute to 148,681 links to each other representing the interaction among individuals, e.g., hyperlinks in blogs. These interactions provide dynamic communities over time. Ground truth information is available for this dataset. The results of the experiment are illustrated in Figure 13.

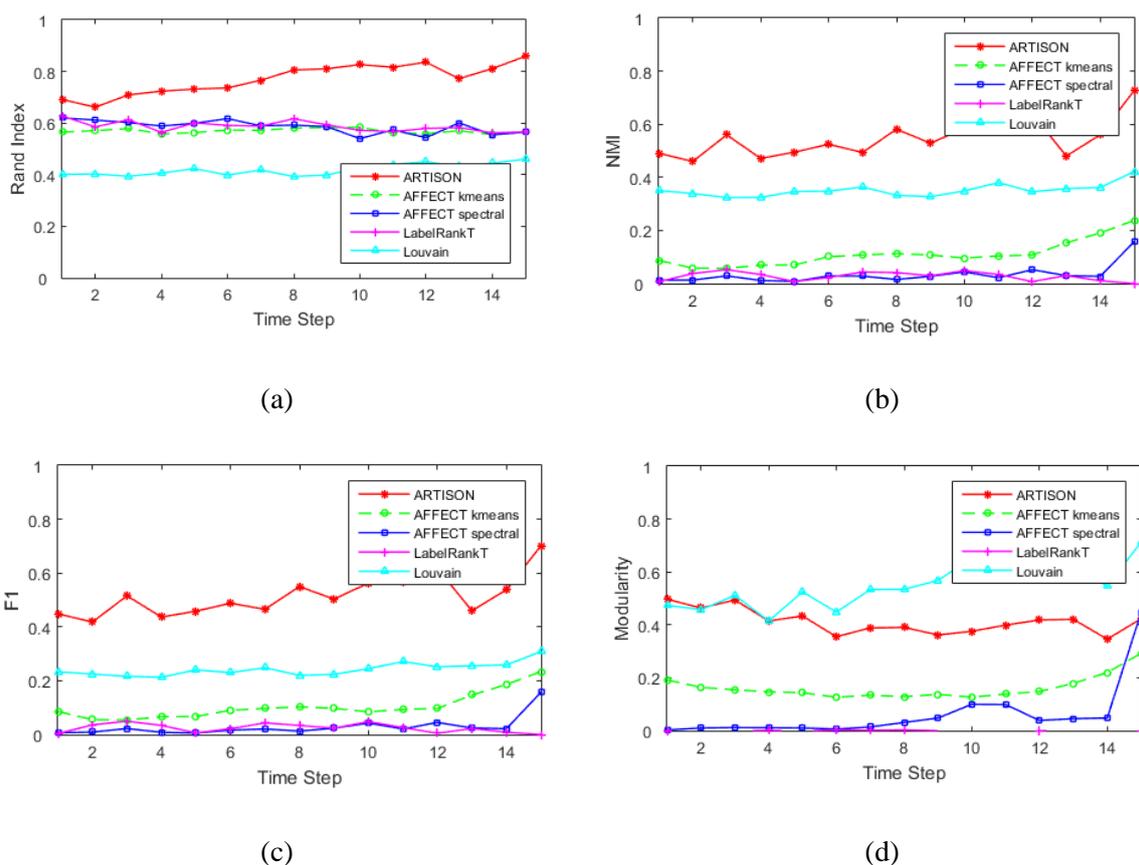


Figure 13. Performance comparison of Granular-ARTISON in NEC blog dataset with AFFECT k-means, AFFECT spectral, Louvain and LabelRankT in four measures of a) Rand Index, b) NMI, c) F-measure and d) modularity

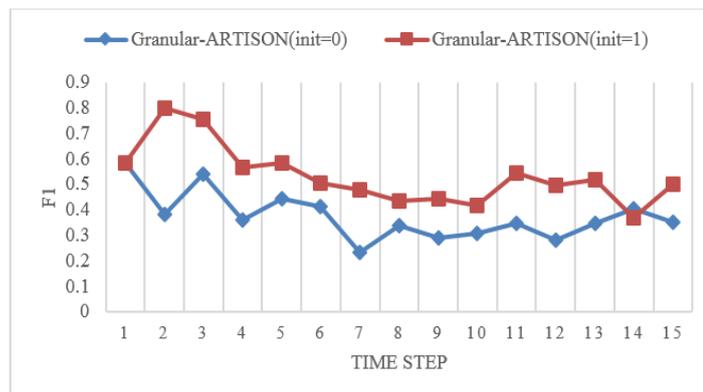
The results of this experiment on this large real network show a very promising result. In addition to being superior in almost all the measures, Granular-ARTISON preserves the highest difference in the values of the measures compared with other algorithms. Louvain is the only algorithm which has a higher value of modularity, which is expected from this algorithm. The comparison among the four other algorithms inferior to Granular-ARTISON is as follows: interestingly, AFFECT k-means shows a better result than AFFECT spectral. The results indicate that AFFECT k-means in most measures is ranked second, and Louvain is in third

place. AFFECT spectral in NMI, F and modularity index is in fourth place. The LabelRankT algorithm works worse than all others in this dataset. For NMI and F-measures, Granular-ARTISON always outperforms the dynamic k-means based algorithms. In particular, in the best-tuned measure for accuracy, which penalizes both false negative and false positive results; i.e., F-measure, Granular-ARTISON shows its superiority with regard to other algorithms by 40% in the worst case and 60% in the best case. The average and slight variations in the results in this set of experiments are reported in Table 10.

**Table 10. Comparison of the proposed algorithm in NEC dataset with AFFECT K-MEANS, AFFECT spectral, Louvain and fast modularity in mean value of five measures: Rand Index, NMI, Precision and F and Modularity**

	<b>Granular-ARTISON</b>	<b>AFFECT k-means</b>	<b>AFFECT Spectral</b>	<b>Louvain</b>	<b>LabelRankT</b>
<b>Rand Index</b>	<b>0.87(±0.04)</b>	0.57(±0.01)	0.59(±0.03)	0.42(±0.02)	0.59(±0.02)
<b>NMI</b>	<b>0.62(±0.08)</b>	0.62(±0.14)	0.12(±0.05)	0.35(±0.02)	0.03(±0.03)
<b>F measure</b>	<b>0.61(±0.08)</b>	0.11(±0.05)	0.03(±0.04)	0.24(±0.02)	0.02 (±0.02)
<b>Modularity</b>	<b>0.29(±0.05)</b>	0.17(±0.05)	0.07(±0.05)	0.56(±0.10)	0.0 (±0.0)

Finally, we experiment the effect of using incremental community discovery in the proposed algorithm. As illustrated in Figure 14 and Table 11, when the community structure in each time step is initialized using the results derived from the previous time step, the algorithm shows higher performance and the changes of the results in the consequent time steps are smoother (on average 16% improvement is obtained).



**Figure 14. Performance evaluation of Granular-ARTISON in F1 measure where two scenarios is considered: a) no initialization of the previous state of the network is considered (blue line) and b) the algorithm is initialized in each time step by the network results obtained in the previous time step (red line).**

Table 11. Average value of F-measure in two set of experiments evaluating the forget factor presence and incremental initialization influence in Granular-ARTISON algorithm

	incremental initialization	no incremental initialization
Average F-measure	0.53( $\pm$ 0.09)	0.37( $\pm$ 0.03)

## 5 Conclusion and future works

We have proposed a dynamic community detection algorithm called Granular-ARTISON based on granulation concepts and our previous human-inspired community detection algorithm. Granular-ARTISON works using local information and functions in dynamic/static and overlapping/non-overlapping contexts. These features are particularly important in real social networks and very few algorithms deal with all of them simultaneously. Like the incremental mining algorithms, Granular-ARTISON exploits the previously discovered community prototypes for new community discovery to preserve temporal smoothness. However, the granulation mechanism in both lower and higher levels of the algorithm make our representative-based algorithms unique in different aspects. Specifically, the algorithm can deal with both low and abrupt changes in the network while being able to determine the correct number of communities automatically. We used extensive evaluation to show the effectiveness of our Granular-ARTISON model against state-of-the-art algorithms on both real and synthetic datasets. In almost all cases, the results of the experiments confirmed the superiority of Granular-ARTISON in different measures. Meanwhile, several improvements are being researched for this algorithm. Designing proper meet operations among macro granules to split larger communities into smaller ones due to shrinkage and deriving quantified membership degree for communities is under investigation. Finally, since the algorithm works totally by local information, the experiments for running the algorithm in the streaming mode are under study.

## Conflict of Interest

No conflict of interest is associated with this article.

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