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New Method of Determining Decision Making Unit Congestion under Inter-Temporal Dependence

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Abstract

Congestion is an important concept in data envelopment analysis (DEA). It occurs when the decrease of at least one decision making unit (DMU) input results in the increase of at least one output ineffective on other inputs or outputs. Until present, various methods have been discussed in terms of congestion determination in static DEA although no study has still been performed on congestion calculation in dynamic DEA under inter-temporal dependence. In this paper, congested input levels in multi-period production systems under inter-temporal dependence and further, the inter-temporal dependence of production periods due to the stock capital are studied. The advantage of dynamic DEA congestion method specification lies in the prevention of inappropriate resources allocation during evaluation periods. The validity of the proposed method has further been demonstrated via empirical example for better observation.

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Abstract

Congestion is an important concept in data envelopment analysis (DEA). It occurs when the decrease of at least one decision making unit (DMU) input results in the increase of at least one output ineffective on other inputs or outputs. Until present, various methods have been discussed in terms of congestion determination in static DEA although no study has still been performed on congestion calculation in dynamic DEA under inter-temporal dependence. In this paper, congested input levels in multi-period production systems under inter-temporal dependence and further, the inter-temporal dependence of production periods due to the stock capital are studied. The advantage of dynamic DEA congestion method specification lies in the prevention of inappropriate resources allocation during evaluation periods. The validity of the proposed method has further been demonstrated via empirical example for better observation.

Keywords: Data envelopment analysis (DEA); Congestion; Inter-temporal dependence.

1. Introduction

Data envelopment analysis (DEA) is a non-parametric method based on mathematical programming, applied to evaluate the performance of decision making units. The first DEA model was proposed by charnes et al. (1978); various other models were introduced by other researchers. In static DEA model, the time factor is not considered; if time factor is applied the DEA models can be called dynamic DEA models. During the recent decade, researchers have assessed the efficiency of DMU in dynamic production systems with inter-temporal dependence and have further reviewed the input and output levels. The inter-temporal dependence factors of assessment periods may include: stock capital, lagged output, capital output, and quasi-fixed inputs. In this survey, stock capital was used to determine inter-temporal dependence between input and output levels in multi-period production systems. According to Nemoto and Goto (2003) however, dynamic overall efficiency is obtained by considering quasi-fixed inputs. Also Sueyoshi and Sekitani (2005) determined the return to scale in dynamic DEA by presence of quasi-fixed inputs. Emrouznejad and Thanassoulis (2005) considered the stock capital as a factor of inter-temporal dependence of evaluation periods and achieved the dynamic-technical efficiency of DMU with a development model. The Emrouznejad and Thanassoulis (2005) model was enhanced by Jahanshahloo et al. (2006) and solved the problem of identifying dynamically efficient paths. This case of the inter-temporal dependence has been studied from both theoretical and practical aspects in the inverse DEA problems (Ghobadi et al. 2014, 2019; Jahanshahloo 2015), estimating units cost-efficiency (Soleimani-Chamkhorami and Ghobadi 2021), the problem of merging units (Zeinodinand Ghobadi 2020), and ranking of units (Moonesian et al. (2020)). Nonetheless, in 2009 Kao additionally studied the inter-temporal dependence of assessment periods caused by the intermediate product or quasi-fixed inputs and discovered a mathematical relation between the overall and period efficiencies. Tone et al. (2010) introduced a dynamic non-radial model and divided the carry-over activities between two consecutive terms into four categories: desirable, undesirable, discretionary, and non-discretionary. Ghobadi et al. (2018) proposed a linear programing model to estimate of the technical efficiency of units. The use of the dynamic DEA model in industry was first performed by Fallah et al. (2014). Efficiency evaluation of the regional high-tech industry in china were performed using meta-frontier dynamic DEA by Li et al. (2017).

Mariz et al. (2018) reviewed the articles on dynamic data envelopment analysis models from 1996 to 2016. An application of dynamic DEA to achieve the multi-period R&D efficiency of regional systems has been identified by Chen et al. (2018). Simultaneous estimation of input-output level has been studied by Ghobadi (2020) using ERM-model under inter temporal dependence. Lin et al. (2021) determined efficiency score of each period using a multiplier dynamic DEA model based on directional distance function. Zhou et al. (2021) used uncertain dynamic DEA to obtain supplier's goal setting under sustainable conditions. Numerous other studies on dynamic DEA have also been carried till present but no research on dynamic congestion under the inter-temporal dependence of the input and output levels need yet to be conducted.

Congestion is a significant issue in economy and a futile step in the production systems to reduce output. First survey on congestion was published by Fare and Svensson (1980). Next, in 1986 by using the data envelopment analysis mode, congestion model was described by Fare et al. Congestion was later used by Brocket et al. (1998) in Chinese industrial sector. Cooper et al. (1996) introduced a model based on the slack variable for determining congestion. Cooper et al. (2002) Additionally calculated the congestion using a one-model method. Tone et al. (2004) on the other hand, provided a non-radial model for identifying the weak congestion and also, discerned the strong congestion by application of multiplier model and the upper bound of scale elasticity. Sueyoshi et al. solved the issue associated with the multiple optimal solutions in 2009; the theoretical relation between congestion and negative return to scale was further examined. Other studies on congestion were also carried, such as Wei and Yan (2004) who identified congestion and variant types of returns to scale. In 2009, the same resolved weak congestion using an output-oriented additive model. Kao (2010) however, reduced congestion by merging DMUs. Asgharian et al. (2010) measured congestion by stochastic data envelopment analysis. Mitheon et al. (2013) also studied congestion in two-stage supply chain. Fang (2015) on the other hand, calculated congestion by application of directional distance function and presence of desirable and undesirable outputs. Khoveyni et al. (2017) and Mehdiloozad et al. (2018) identified DMU congestion by present of negative data. The congestion of DMUs in fuzzy data envelopment analysis was determined by khirrollahi et al. (2017). Abrahimzade Adimi et al. (2019) identified the boundary between congested units and non-congested units by introducing a congestion hyperplane. Shabanpour et al. (2019) evaluated the sustainability of supplier using the data envelopment analysis congestion method. Mehdiloozad et al. (2020) specified the congestion using a maximal element of a non-negative polyhedral set and by solving a linear programming model. Khezri et al. (2019) obtained directional congestion in a situation where at least one input or output direction is not known but still did not study dynamic DEA congestion with inter-temporal dependence between input and output levels.

In this paper, congestion calculation method of dynamic DEA under inter-temporal dependence of input and output levels is studied. In addition, congestion amount of congested levels during assessment periods is reviewed i.e. congestion is calculated by multi-period production systems where the stock capital is a factor of the inter-temporal dependence of input and output levels. Also in this research, application of dynamic model to congested paths in accordance with Emrouznejad and Thanassoulis (2005) is further studied. By identification of congested paths, improper allocation of resources is prevented and on top, in order to prove the dependent hypothesis was put into practice in a banking sector as sample.

The remainder of the study is as follows. In section 2, Cooper's method of identifying congestion using non-radial model is presented with a reminder of dynamic model from Emrouznejad and Thanassoulis (2005). In section 3 however, proposed method to calculate the congestion in dynamic DEA under inter-temporal dependence between assessment periods is also studied. A practical example of the proposed model is provided in section 4 and finally in Section 5, the conclusion of the survey is presented.

2. Preliminaries

In this section, a congestion identification model similar to the Cooper et al. (2002) model is presented. In addition, the Emrouznejad and Thanassoulis (2005) model is also mentioned in this section.

2.1 Identification of the congestion

It is assumed that there are n DMUs i.e. $\{DMU_j | j = 1, \dots, n\}$, each DMU has m inputs i.e. $\{x_{ij} | i = 1, \dots, m\}$ and s outputs i.e. $\{y_{rj} | r = 1, \dots, s\}$. The congestion production possibility set under the inputs weak disposability is defined as follow:

$$PPS_{congestion} = \left\{ \binom{x}{y} \epsilon R^{m+s} \middle| \sum_{j=1}^{n} \lambda_j x_{ij} = x,\ \sum_{j=1}^{n} \lambda_j y_{rj} \geq y,\ \sum_{j=1}^{n} \lambda_j = 1,\ \lambda_j \geq 0 \right\},$$

Where λ_j $(j = 1, \dots, n)$ is the intensity variable.

Cooper's non-radial one-model method for calculating the amount of the input congestion is as follows:

$$\begin{array}{lll} Max & \sum_{r=1}^{s} s_{rk}^{+} - \epsilon \sum_{i=1}^{m} s_{ik}^{c} & \\ s.t & \sum_{j=1}^{n} \lambda_j x_{ij} + s_{ik}^{c} = x_{ik} & i = 1, \dots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - s_{rk}^{+} = y_{rk} & r = 1, \dots, s, \\ & \sum_{j=1}^{n} \lambda_j = 1 & \\ & s_{ik}^{c} \geq 0 & i = 1, \dots, m, \\ & s_{rk}^{+} \geq 0 & r = 1, \dots, s, \\ & \lambda_j \geq 0 & j = 1, \dots, n. \end{array} \quad (1)$$

In the model (1) s_{rk}^{+} and s_{ik}^{c} indicate the r^{th} output slack variable of DMU_k and the i^{th} congestion variable of DMU_k respectively. Also, ϵ is a non-Archimedean small positive number. Model (1) is a two-phase process: (a) the maximum output shortage of DMU_k i.e. $s_{rk}^{+^*}$ is specified by output-oriented non-radial BCC model and (b) while keeping $y_{rk} = y_{rk} + s_{rk}^{+^*}$, the minimum of the input surpluses of DMU_k is specified such that DMU_k is located in the congested production possibility set. If the optimal solution of phase 1 and phase 2 is positive, it can be concluded that an increase of input reduces output. In other word, there is congestion in DMU_k. the amount of the inputs congestion is revealed with the following theorem, quite similar to the theorem of Cooper et al. (2002).

Theorem 2.1. DMU_k is congested if and only if in an optimal solution i.e. $(s_{rk}^{+^*}, s_{ik}^{c^*})$ of model (1) the following condition is satisfied:

There is at least one r (r=1,…,s) so that $s_{rk}^{+^*} > 0$ and at least one i (i=1,…,m) so that $s_{ik}^{c^*} > 0$.

In theorem (1) $s_{ik}^{c^*}$ is the amount of the congestion of i^{th} input of the DMU_k and $s_{rk}^{+^*}$ is the amount of the shortfall in r^{th} output due to congestion and inefficiency of the DMU_k.

2.2 Dynamic DEA under inter-temporal dependence

In this section, a special model of dynamic DEA under inter-temporal dependence of Emrouznejad and Thanassoulis (2005) has been reviewed. According to Emrouznejad and Thanassoulis the stock capital change is specific case of the inter-temporal dependence between input and output levels in a dynamic DEA framework. They measured the performance of n DMU set within assessment of window time period $W = \{t|t = \tau, \dots, \tau + T\}$. The initial and terminal time period in assessment window is indicated by $\tau, \tau + T$, respectively. Each DMU is assumed to have three types of input for every time: period-specific input, stock capital inputs, and initial-stock inputs. Moreover, each DMU produces two types of outputs: period-specific outputs and terminal-stock outputs. The set of inputs and outputs of DMU_j is defined as follows:

Period-specific input paths: $x_{ij}^{\tau} \quad i \in I_1, \tau \in W,$

Stock capital input paths: $z_{ij}^{\tau} \quad i \in I_2, \tau \in W,$

Initial-stock inputs: $Z_{ij}^{\tau-1} \quad i \in I_2,$

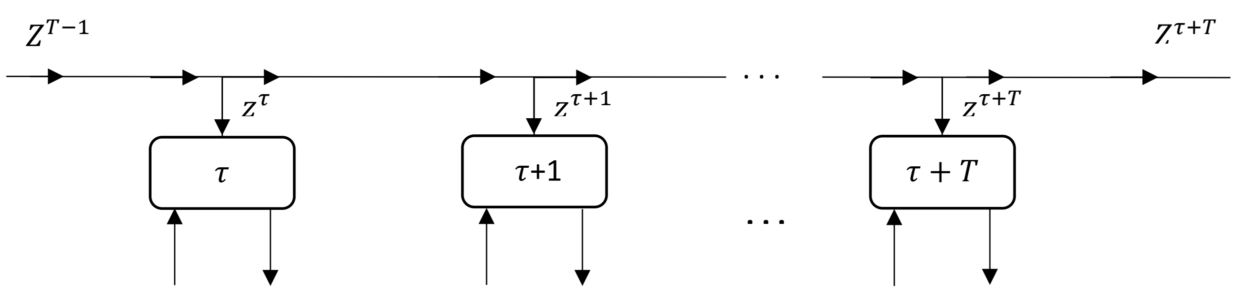
Period-specific output paths: $y_{rj}^{\tau} \quad r \in O, \tau \in W,$

Terminal-stock inputs as output: $Z_{ij}^{\tau+T} \quad i \in I_2.$

Where the set of outputs, $O = \{1, \dots, s\}$ and the set of inputs, $I = \{1, \dots, m\}$ are divided into two subsets: I_1 (period-inputs) and I_2 (capital-inputs), such that $I_1, I_2 \subset I,\ \ I_1 \cup I_2 = I,\ \ I_1 \cap I_2 = \emptyset$. Capital input (z^t) and terminal-stock input $(Z^{\tau+T})$ as output are supplied by initial-stock input $(Z^{\tau-1})$. Jahanshahloo et al. (2015) still modified by Emrouznejad and Thanassoulis (2005) model, adding the following equation.

$$Z^{\tau-1} = \sum_{t=\tau}^{\tau+T} z^t + Z^{\tau+T}.$$

Emrouznejad and Thanassoulis (2005) model of production flow is shown in figure 1.



x^{τ} y^{τ} $x^{\tau+1}$ $y^{\tau+1}$ $x^{\tau+T}$ $y^{\tau+T}$

Fig. 1. Production flow in the assessment window.

They have introduced a dynamic production possibility set (PPS) as follows:

$$PPS_D = \left\{ (x^{\tau,\dots,\tau+T}, z^{\tau,\dots,\tau+T}, y^{\tau,\dots,\tau+T}, Z^{\tau-1}, Z^{\tau+T}) \middle| \begin{array}{l} \sum_{j=1}^{n} \lambda_j x_j^t \leq x^t, \sum_{j=1}^{n} \lambda_j z_j^t \leq z^t, \\ \sum_{j=1}^{n} \lambda_j y_j^t \geq y^t, \sum_{j=1}^{n} \lambda_j Z_j^{\tau-1} \leq Z^{\tau-1}, \\ \sum_{j=1}^{n} \lambda_j Z_j^{\tau+T} \geq Z^{\tau+T}, \\ \sum_{j=1}^{n} \lambda_j = 1,\ \lambda_j \geq 0, \forall t \in W. \end{array} \right\}.$$

A linear programing problem for estimating the dynamic relative efficiency of DMU was presented by Emrouznejad and Thanassoulis (2005). Again, this model was modified by Jahanshahloo et al. (2006). (For further details refer to Emrouznejad and Thanassoulis (2005) and Jahanshahloo et al. (2006) hypotheses).

3. Proposed method to calculate the congestion in dynamic DEA under inter-temporal dependence

In this section, first the dynamic congested production possibility set under inputs weak disposability is specified, next, the amount of DMU congestion in dynamic DEA is determined i.e. when the stock capital has affected inputs and outputs levels in multi period production systems.

3.1 The dynamic congested production possibility set (Dynamic congested PPS)

The dynamic congested production possibility set under weak disposability of inputs is defined by:

$$PPS_{DC} = \left\{ (x^{\tau,\dots,\tau+T}, z^{\tau,\dots,\tau+T}, y^{\tau,\dots,\tau+T}, Z^{\tau-1}, Z^{\tau+T}) \middle| \begin{array}{l} \sum_{j=1}^{n} \lambda_j x_j^t = x^t, \sum_{j=1}^{n} \lambda_j z_j^t = z^t, \\ \sum_{j=1}^{n} \lambda_j y_j^t \geq y^t, \sum_{j=1}^{n} \lambda_j Z_j^{\tau-1} = Z^{\tau-1}, \\ \sum_{j=1}^{n} \lambda_j Z_j^{\tau+T} = Z^{\tau+T}, \\ \sum_{j=1}^{n} \lambda_j = 1,\ \lambda_j \geq 0, \forall t \in W. \end{array} \right\}.$$

The postulates of observations, convexity, and strong disposability of period-outputs satisfy the dynamic congested production possibility set. Terminal stock as output depend on stock capital inputs. The postulate of inputs weak disposability satisfied stock capital inputs and by so doing terminal stock as output satisfied the outputs weak disposability axiom.

Since congestion is a frontier concept, model (2) is proposed to obtain an efficient unit or projection of an inefficient unit in the dynamic congested production possibility set. In other words, dynamic output-oriented model under weak disposability of inputs is presented in model (2). Unit under evaluation in model (2) is DMU_k.

$$
\begin{array}{lll}
Max & \sum_{t=\tau}^{\tau+T}\sum_{r=1}^{s} s_r^{+t} & \\
s.t. & \sum_{j=1}^{n}\lambda_j x_{ij}^t = x_{ik}^t & i\in I_1,\ t\in W, \\
& \sum_{j=1}^{n}\lambda_j z_{ij}^t = z_{ik}^t & i\in I_2,\ t\in W, \\
& \sum_{j=1}^{n}\lambda_j Z_{ij}^{\tau-1} = Z_{ik}^{\tau-1} & i\in I_2, \\
& \sum_{j=1}^{n}\lambda_j y_{ij}^t - s_r^{+t} = y_{ik}^t & r=1,\dots,s,\ t\in W, \\
& \sum_{j=1}^{n}\lambda_j Z_{ij}^{\tau+T} = Z_{ik}^{\tau+T} & i\in I_2, \\
& \sum_{j=1}^{n}\lambda_j = 1, & \\
& Z_{ij}^{\tau-1} = \sum_{t=\tau}^{\tau+T} z_{ij}^t + Z_{ij}^{\tau+T} & i\in I_2,\ j=1,\dots,n, \\
& \lambda_j \geq 0 & j=1,\dots,n, \\
& s_r^{+t} \geq 0 & r=1,\dots,s,\ t\in W.
\end{array} \qquad (2)
$$

Where s_r^{+t} indicates the amount of rth the output slack in the time period t and also λ_j displays the amount of jth the intensity variable.

Here, efficient and congested units from static DEA to dynamic DEA can be defined in details.

Definition 3.1. (Dynamically efficient) Suppose $(s_r^{+t^*}, \lambda_j^*)$ is an optimal solution of model (2) then the assessment path of DMU_k is dynamically efficient if and only if for each r (r=1,…,s) and for each t $(t=\tau,\dots,\tau+T)$, $s_r^{+t^*}=0$, otherwise, projection of the DMU_k i.e. $(x_k^{\tau,\dots,\tau+T}, z_k^{\tau,\dots,\tau+T}, y_k^{\tau,\dots,\tau+T} + s_k^{(\tau,\dots,\tau+T)^*}, Z_k^{\tau-1}, Z_k^{\tau+T})$ is dynamically efficient.

Definition 3.2. (Congested path) The efficient assessment path $(x_k^{\tau,\dots,\tau+T}, z_k^{\tau,\dots,\tau+T}, y_k^{\tau,\dots,\tau+T}, Z_k^{\tau-1}, Z_k^{\tau+T})$ of DMU_k in model (2) is called congested path when an increase (decrease) of at least one input in least one time period is associated with a decrease (increase) of at least one output in least one time period, without negative impact on other inputs and outputs.

3.2 Determination of the amount of DMU congestion in dynamic DEA

In this section, the amount of DMU congestion and the congested paths in dynamic DEA is determined by application of Emrouznejad and Thanassoulis (2005) model where the inter-temporal dependence factor of evaluation periods is stock capital. In model (3) however, in order to calculate the congested path, first the maximum increase period outputs of DMU_k is specified. Next, the minimum decrease of period and capital inputs is obtained such that DMU_k is located in the dynamic congested production possibility set. In addition, the terminal –stock inputs as outputs is considered fixed.

$$
\begin{array}{lll}
Max & \sum_{t=\tau}^{\tau+T}\sum_{r=1}^{s} s_r^{+t} - \epsilon\left(\sum_{t=\tau}^{\tau+T}\sum_{i\in I_1} s_i^{-t} + \sum_{t=\tau}^{\tau+T}\sum_{i\in I_2}\delta_i^{-t} + \sum_{i\in I_2}\gamma_i^{-}\right) & \\
s.t. & \sum_{j=1}^{n}\lambda_j x_{ij}^t + s_i^{-t} = x_{ik}^t & i\in I_1, t\in W, \\
& \sum_{j=1}^{n}\lambda_j z_{ij}^t + \delta_i^{-t} = z_{ik}^t & i\in I_2, t\in W, \\
& \sum_{j=1}^{n}\lambda_j Z_{ij}^{\tau-1} + \gamma_i^{\tau-1} = Z_{ik}^{\tau-1} & i\in I_2, \\
& \sum_{j=1}^{n}\lambda_j y_{ij}^t - s_r^{+t} = y_{ik}^t & r=1,\dots,s,\ t\in W, \\
& \sum_{j=1}^{n}\lambda_j Z_{ij}^{\tau+T} = Z_{ik}^{\tau+T} & i\in I_2,
\end{array}
$$

$$\sum_{t=\tau}^{\tau+T} \delta_i^{-t} = \gamma_i^{\tau-1} \qquad i \in I_2, \qquad (3)$$
$$\sum_{j=1}^{n} \lambda_j = 1,$$
$$\lambda_j \geq 0 \qquad j = 1, \dots, n,$$
$$s_i^{-t} \geq 0 \qquad i \in I_1,\ t \in W,$$
$$\delta_i^{-t} \geq 0 \qquad i \in I_2,\ t \in W,$$
$$\gamma_i^{-t} \geq 0 \qquad i \in I_2,$$
$$s_r^{+t} \geq 0 \qquad r = 1, \dots, s,\ t \in W.$$

Nonetheless in the model (3) s_i^{-t}, δ_i^{-t}, and s_r^{+t} are the period-input, capital-input, and period-output the slack variables in the period t respectively. Also $\gamma_i^{\tau-1}$ is the slack variable of initial-stock input. In order to, the projection of the DMU_k satisfies in the equation $Z^{\tau-1} = \sum_{t=\tau}^{\tau+T} z^t + Z^{\tau+T}$, the constraint $\sum_{t=\tau}^{\tau+T} \delta_i^{-t} = \gamma_i^{\tau-1}$, is applied in model (3).

Consequently, on the basis of the following theorem, the congested paths and the amount of DMU congestion at each evaluation period are identified.

Theorem 3.1. The assessment path $(x_k^{\tau,\dots,\tau+T}, z_k^{\tau,\dots,\tau+T}, y_k^{\tau,\dots,\tau+T}, Z_k^{\tau-1}, Z_k^{\tau+T})$ of DMU_k is congested if and only if for an optimal solution i.e. $(s_i^{-t^*}, \delta_i^{-t^*}, \gamma_i^{(\tau-1)^*}, s_r^{+t^*}, \lambda_j^*)$ in model (3) the following condition exists:

There is at least one r $(r = 1, \dots, s)$ and at least one t $(t = \tau, \dots, \tau + T)$ such that $s_r^{+t^*} > 0$, and at least one of the following two cases is true:
Case 1. There is at least one $i \in I_1$, and at least one t so that $s_i^{-t^*} > 0$,
Case 2. There is at least one $i \in I_2$, and at least one t so that $\delta_i^{-t^*} > 0$.

Proof. Model (3) is a two-phase process. In the first phase, the maximum increase of DMU_k period-output is obtained by dynamic output-oriented non-radial model. Now, assume that $s_r^{+t^*}$ is an optimal solution in the first phase. while keeping $y_{rk}^t = y_{rk}^t + s_{rk}^{+t^*}$ in the second phase, minimum decrease of period and capital inputs is determined such that DMU_k is located in the congested production possibility set. Still, if the optimal solution in phases 1 and 2 is positive, i.e. there is at least one r and at least one t and that $s_r^{+t^*} > 0$, and also that at least one of the two cases in 1 and 2 holds true $(s_i^{-t^*} > 0 \vee \delta_i^{-t^*} > 0)$, it can be conclude that an assessment path $(x_{ik}^{\tau,\dots,\tau+T} - s_{ik}^{(\tau,\dots,\tau+T)^*}, z_{ik}^{\tau,\dots,\tau+T} - \delta_{ik}^{(\tau,\dots,\tau+T)^*}, y_{rk}^{\tau,\dots,\tau+T} + s_{rk}^{(\tau,\dots,\tau+T)^*}, Z_{ik}^{\tau-1} - \gamma_i^{(\tau-1)^*}, Z_{ik}^{\tau+T})$ is found such that, a decrease in at least one of the inputs in at least one of the time periods is associated with an increase in at least one of outputs in at least one of the time periods of DMU_k. Certainly this is to the significance that DMU_k has congestion.

Remark 1. In optimal solution of model (3), $s_i^{-t^*}$ & $\delta_i^{-t^*}$ are the amount of the period i^{th} input congestion and the amount of the capital i^{th} input congestion in the time period t, respectively.

4. Empirical example

In this section, the applicability of proposed method was studied via banking centers empirically. The data set is based on Ghobadi et al. (2019). Twenty Iran commercial bank branches were studied for period of three months. Each branch was provided two period-inputs, a capital input,

and three period-outputs; period-inputs are employee score and deferred claims. Moreover, if any branch appealed for grants, financial assistance from central bank (capital stock) was provided. According to the bank rating the maximum amount of central bank grants is fixed during assessment window called initial capital ($Z^{\tau-1}$). The initial bank capital is divided between: a) time periods in assessment window and b) remainder of initial capital called terminal stock ($Z^{\tau+T}$), used as output for final period of assessment window. Also, for each bank correlation is always created in term of:
Initial capital= (sum of period capitals) + terminal capital
The period outputs are loans, deposit, and profit. Data set is shown in the table 1.

Table 1: The data set of 20 bank branches in the 3-month period.

The data set in period t=1						
Bank	Employee score (x_1)	Deferred claims (x_2)	Capital input (z_1)	Loans (y_1)	Deposit (y_2)	Profit (y_3)
1	19.83	4603910	25415944	75097467	80023776	2211465
2	7.08	9547	631223	25258238	40413775	767801
3	4.01	136115	39950793	60530507	48420589	1770276
4	14.92	1035215	2823106	65413851	101707340	3898218
5	5.33	1030194	3536597	45667593	67411796	1291753
6	12.84	2664633	56976524	157015854	163070472	4334957
7	15.72	1086083	260385226	462186659	397280289	168494852
8	10.94	225665	22036925	105618280	130611124	4072227
9	13.08	7480348	310092	54863663	90586108	2335849
10	15.97	3486536	163475721	274163028	229181190	5869352
11	4.53	1531195	1978878	27052607	40188125	557744
12	3.88	106162	2484718	32767317	48484615	1044161
13	14.6	430201	55740403	164983122	175915348	667562
14	12.19	288733	45231323	142754970	155702255	4650278
15	9.93	79572	3673025	48940586	72499127	2145795
16	3.2	23274	10685654	30547469	32161835	923161
17	19.25	655170	45300766	178762561	209467807	9683107
18	6.18	70840	63930945	88958994	67461848	4552123
19	6	5771009	25973395	49562263	44092348	695511
20	7.86	604842	5532201	40493880	54972684	1539726
The data set in period t=2						
Bank	Employee score (x_1)	Deferred claims (x_2)	Capital input (z_1)	Loans (y_1)	Deposit (y_2)	Profit (y_3)
1	9.38	4258676	33423425	77540480	75205729	2372091
2	5.66	9547	2635237	28099162	40501342	767801
3	4.76	136115	42532282	63386263	49871113	1779213
4	7.69	1035215	6186940	69347301	100617551	3917817
5	3.67	837568	7640404	51678971	69103984	137823
6	11	1217053	73818634	165665373	158442184	4786804

7	11.04	406113	237525938	457271922	408482255	17214726
8	7.7	22915	21522051	111799242	141070488	4089487
9	10.44	7479093	3651731	63460385	96511315	2467889
10	12.24	3674116	193337240	300159210	241263111	6667268
11	3.22	1512675	4830066	28785809	37491612	559513
12	4.07	106162	3853539	37246308	52947228	1059440
13	11.19	430201	50458213	174059672	195596771	6683103
14	9.09	288733	63179340	147933524	144033211	5174718
15	5.14	79572	2663053	56425243	87184148	2518669
16	3.37	23274	16251577	32096496	29011554	925370
17	12.02	654770	36316407	186258838	234294279	10570787
18	6.33	68785	61785788	92482607	72931204	4761568
19	5.64	152093	29507223	51036143	43318395	695511
20	4.1	604842	2316070	44447422	68159592	1563397

The data set in period t=3

Bank	Employee score (x_1)	Deferred claims (x_2)	Capital input (z_1)	Loans (y_1)	Deposit (y_2)	Profit (y_3)
1	9.39	4164067	34302366	79100108	76524475	2608693
2	5.67	9547	6014493	31026240	39083163	767801
3	4.76	136115	39988455	64986239	53471773	1936626
4	7.7	1033215	9851458	73820219	100647643	3924818
5	3.68	786287	13105122	58194638	70522315	1400253
6	11.01	1267053	70323601	174946921	174602767	4898347
7	11.06	395522	229207401	465979022	426766238	17976626
8	7.71	225915	17014036	116557642	156249562	4304301
9	10.46	7477837	10980081	79367959	107494554	2469807
10	12.25	3657016	213492432	308891608	239236387	7726739
11	3.22	1512675	8979634	31425356	35479015	579594
12	4.07	106162	5806457	40643483	54711936	1069796
13	11.2	430201	60083361	175858151	186753718	6685865
14	9.1	277182	91013826	153347359	128483906	5174718
15	5.15	79572	6205102	60613709	86298673	2764479
16	3.37	23274	14326858	33842858	33071151	966104
17	12.04	654770	39962610	215702229	274686310	10576311
18	6.33	68211	61511021	93642456	74504425	5066401
19	5.65	148329	31628113	54119292	45692855	698963
20	4.1	604842	6803359	47543624	63980358	1571893

Initial and terminal capitals

Bank	Initial capital (Z^0)	Terminal stock (Z^3)
1	117244609	24102874
2	26566357	17285404
3	158665116	36793586
4	29209796	10348292

5	39252881	14970758
6	267062245	65943486
7	1037209839	310091274
8	87622951	27049939
9	30043956	15102052
10	729364697	159059304
11	34213206	18424628
12	34626288	22481574
13	218101708	51819731
14	282355290	82930801
15	27056255	14515075
16	51708898	10444809
17	234220827	112641044
18	254370473	67142719
19	134860848	47752117
20	25822016	11170386

By application of proposed model (3), the dynamic congestion of twenty branches in assessment window was studied. Results of model (3) demonstrate that s_i^{-t}, δ_i^{-t}, and s_r^{+t} for all banks except the first as being zero. Therefore, it can be concluded that bank 1 has a dynamic congested path. Still, in order to remove congested path from bank 1 to attain efficient path during evaluation period, the following results were obtained in view of table 2:

1. The amounts of 5.76, 0.97, and 0.97 of bank 1 employees score were reduced in the first, second, and third periods, respectively.
2. The excess amounts of deferred claims in bank 1 were 3293267.40 and 3159264.23 in the first, and third periods, respectively. There were no additional deferred claims in the second period.
3. The bank 1 needed less capital stock. Therefore, the amounts of 8905429.16, 10993792.12, and 10571478.49 of bank 1 capital stock were decreased in the first, second, and third periods.
4. Shortfall of output (loans) due to congestion and inefficiency were 14226309.81, 16965193.49, and 20954224.32 in first, second, and third periods, respectively.
5. for elimination of output shortage (deposit) caused by congestion and inefficiency, the amounts of 37894332.85, 41986953.64, and 45701903.51 were added to deposits in the first, second, and third periods.
6.shortage of outputs (profits) due to congestion and inefficiency were 1799012.76, 1751931.59, and 1565536.16 in the first, second, and third periods respectively.

Consequently, to determine the amount of congestion on congested path in assessment window indicates that reducing of employee score, deferred claims, and capital stock leads to increased loan, deposit, and profit. In other word, if the first bank hires employees with lower score and reduces deferred claims and also uses less capital stock, the outputs of the bank 1 shall increase in all time periods.

Table 2: The input congestion and the output shortage due to congestion and inefficiency in the assessment window.

The results of the model (3) in the period t=1						
Bank branches	The input congestion (x_1)	The input congestion (x_2)	The input congestion (z_1)	The output shortage (y_1)	The output shortage (y_2)	The output shortage (y_1)
1	5.76	3293267.40	8905429.16	14226309.81	37894332.85	1799012.76
The results of the model (3) in the period t=2						
1	0.97	0	10993792.12	16965193.49	41986953.64	1751931.59
The results of the model (3) in the period t=3						
1	0.97	3159264.23	10571478.49	20954224.32	45701903.51	1565536.16

5. Conclusion

So far, no method has been proposed to identify the congestion under inter-temporal dependence. In this research, a method for identifying congestion in dynamic data envelopment analysis under inter-temporal dependence is introduced whereby the stock capital is a factor of inter-temporal dependence on the input and output levels. By application of this method, unit that has congested path is identified. In addition, the amount of the excess input and the output shortage amount of congested assessment path needed to eliminate the congested assessment path and create efficient assessment path for time period is determined. Our future research is to study dynamic DEA congestion and existence of special data such as: fuzzy, interval, integer, and etc. Therefore, to determine congestion in DEA dynamic by considering quasi-fixed inputs can be a future research.

Conflict of Interest:
Author **Tahereh Shahsavan** declares that he has no conflict of interest.
Author **Masoud Sanei** declares that he has no conflict of interest.
Author **Ghasem Tohidi** declares that he has no conflict of interest.
Author **Farhad Hosseinzadeh Lotfi** declares that he has no conflict of interest.
Author **Saeid Ghobadi** declares that he has no conflict of interest.

Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

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Figures

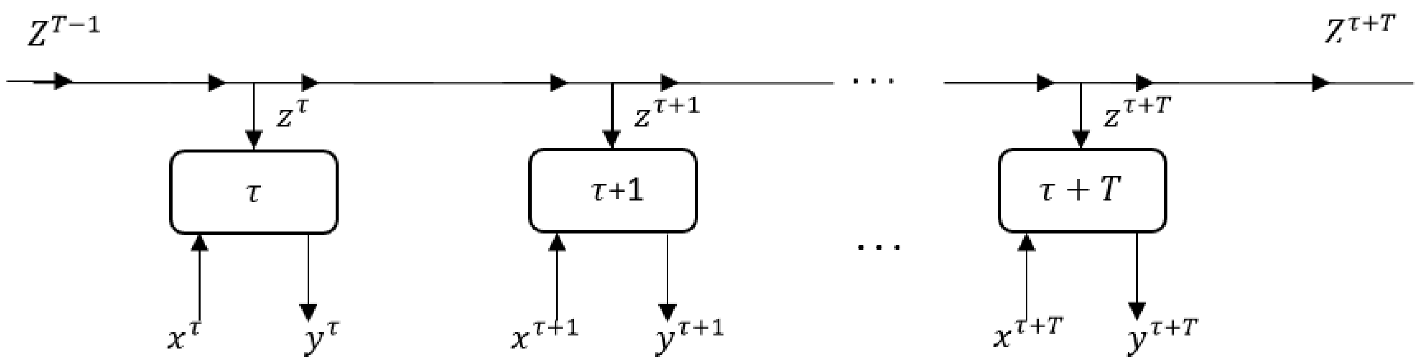

Figure 1

Production flow in the assessment window.