A Hybrid Retrieval Strategy for Case-based Reasoning Using Soft Likelihood Functions

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A hybrid retrieval strategy for case-based reasoning using soft likelihood functions

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Abstract

Case-based reasoning (CBR) is the retrieval of one or more similar cases from an existing case base for the problem to be solved according to the characteristics of the new problem. The core idea of CBR is that similar cases have similar solutions, so whether the CBR system can play a powerful advantage depends on the quality of case retrieval strategy. At present, the commonly used case retrieval algorithm is based on the mean operator method, which is very hard, and a certain local similarity is low will affect the overall result. In order to calculate the global similarity of cases from a new and softer point of view, this paper introduces the soft likelihood functions into case retrieval, combines the soft likelihood functions with KNN, and proposes a hybrid retrieval strategy. The core of the retrieval strategy is to define the global similarity through SLFs, aggregate the local similarity and characteristic similarity together, and also take the attitude characteristics of decision makers into consideration. Through simulation experiments on real data sets, the accuracy rate is more than 81%, which verifies the effectiveness of the retrieval strategy.

Keywords: Case-based reasoning, Retrieval, Soft likelihood function, Ordered weighted average, Attitudinal character.

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1. Introduction

The proposal of case-based reasoning (CBR) can be traced back to the late 1970s [1]. Roger et al. from Yale University in the United States proposed to represent knowledge by means of script, which is regarded as the beginning of CBR research. Since then, CBR has experienced from

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simple basic application research to theoretical perfection [2, 3, 4, 5, 6]. It originated in the fields of cognitive science (CS) and artificial intelligence (AI). Typically, the current problem or situation is referred to as the target case, and the problem or situation that has occurred is referred to as the source case or the historical case. Case-based reasoning refers to recalling previous successful cases which is referred to as the source case or historical case, by comparing the similarities and differences between the source case and target case, and then adjusting the target case solutions based on the successful cases to solve the current problem [7]. In particular, case-based reasoning plays a very important role in the field of application where there is no known standard, no known cycle, and no complete domain theory [8]. CBR can simplify knowledge acquisition, improve problem solving efficiency, improve solving quality, and accumulate knowledge. It provides a method which is quite similar to human solving problems [9].

At present, CBR has been widely used in AI, and it has become a new methodology of problem solving and learning [10]. With the gradual maturity of theories and methods, the applications of CBR have been extended to various fields, including medical treatment [11, 12, 13, 14, 15], planning [16, 17], assessment [18, 19], forecast [20, 21], game [22], recommendation system [23], management [24] and so on [25, 26].

The core idea of CBR is that similar cases have similar solutions. Plenty of scholars developed different case-based reasoning models with the intention of providing a better understanding of case-based reasoning process. One of the representative models is the CBR model introduced by Aamodt and Plaza [27], in which they propose a process for solving a new problem. Before reasoning, we need to choose the appropriate method to build the case base [28]. For a problem to be solved, one or more similar cases are retrieved from the existing case base according to the features of the new problem. Solutions to cases retrieved are employed to generate solutions to the new problem and the solutions will be tested, modified, and evaluated to determine their effectiveness. Solutions that satisfy the user are learned and added to the case bases. The model of the CBR cycle is illustrated in Fig. 1, which is called the 4-R lifecycle model.

From the model of CBR cycle, the CBR reasoning process is mainly divided into four stages: retrieval (R-1), reuse (R-2), revise (R-3) and retain (R-4) [29].

- R-1: RETRIEVE information from the source case base and select potentially available source cases.

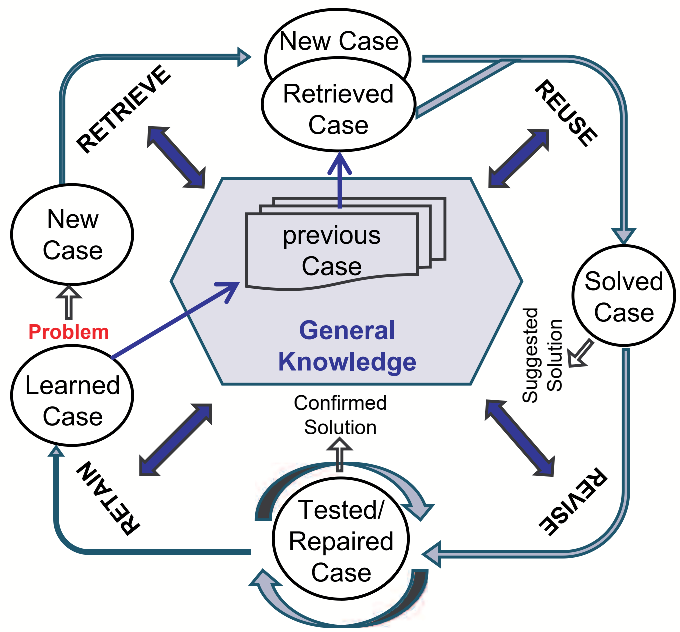

Figure 1: The 4-R lifecycle model of CBR from Aamodt and Plaza [27].

- R-2: REUSE the solutions of the retrieved source cases in new problems or cases.
- R-3: REVISE the proposed solution.
- R-4: RETAIN the solution in favor of subsequent reasoning in the problem.

The 4-R cycle model is summarized as: analyze the features of existing problems, *retrieve* one or more similar cases, try to *reuse* cases, and *retain* new cases in case base according to their importance after the solution is *revised* and applied.

From the 4-R cycle model we can get the fact that whether CBR system could play a strong advantage depends on the quality of case retrieval strategy [30]. The retrieval method directly affects the retrieval speed and accuracy rate [31], and whether the retrieval strategy is reasonable or not directly affects the realization effect of the whole case system. So case retrieval is the key to problem solving. In the aspect of retrieval strategy, there are knowledge guidance strategy [32], template retrieval strategy [33] and nearest neighbor strategy [34, 35].

From the research status of case retrieval at home and abroad [36], the K-nearest neighbor (KNN) retrieval strategy [34] is widely used at present [37]. It calculates the similarity between the target case to be solved and the source case in the case base according to the attribute weight and its eigenvalue [38], and then selects one or some source case solutions with high similarity as the basis of case reuse [39]. In the calculation of similarity, the weight distribution will have a significant influence on the calculation results and the quality of the solution. Attributes that generally play a major role are assigned greater weight; Conversely, less weight is given. KNN generally uses the average weight method. Although it is simple and easy to operate, it is sensitive to noise or irrelevant data, which will affect the reliability of the calculation results. The solution of this problem usually depends on the reasonable allocation of the weight of characteristic attributes, so the allocation of weight has become an important research direction.

On the one hand, although the similarity-based retrieval method has been successfully applied to CBR and received extensive attention, it is not completely consistent with the actual reasoning process. It is easily disturbed by small probability events, and the whole result is easily affected by a certain term. On the other hand, the CBR system was developed for use by decision makers (DMs), which inevitably reflects DMs' personal attitude in different situations. However, the attitude characteristics of DMs are often ignored in similarity calculation, which is unreasonable. Therefore, it is necessary to further study the mechanism of optimal weight allocation in order to improve the quality of problem solving.

Based on the above analysis, inspired by the soft likelihood functions (SLFs) introduced by Yager *et al*. [40], a new case retrieval algorithm using SLFs based ordered weighted average (OWA) (abbreviated as CBR-SLFs) is proposed in this study, which provides a new perspective to retrieve similar cases. The basic idea of case retrieval by the proposed method is as follows: Firstly, calculate the local similarity between different attributes of the target case and the source case; Then, the CBR-SLFs algorithm proposed in this paper is used to calculate the overall similarity, and some potential available source cases with high similarity are obtained; Finally, the source case solution that is closest to the target case is obtained through KNN, and reuse it. This strategy is developed as a flexible computation of likelihood functions of global similarity calculation, and has the advantage of being more robust and practical in case retrieval [41]. Furthermore, SLF-based case retrieval algorithm is developed introducing an attitudinal characteristic to reflect the subjective preference of decision makers, which allows for more flexible choices based on different types of decision makers.

The rest of this article is organized as follows: Section 2 briefly introduces likelihood function in case retrieval, some basic calculations of the OWA aggregation operator and local similarity measurement method for heterogeneous information. Section 3 introduces the application of soft likelihood function in case retrieval ,then takes feature similarity into consideration and gives some examples. Section 4 provides some simulation experiments on benchmark data sets. Finally, section 5 summarizes this paper and puts forward the future research direction.

2. Preliminaries

This section first presents the likelihood functions in case retrieval and OWA aggregation, then introduces local similarity measurement methods for case information.

2.1. Using likelihood functions in case retrieval

In a CBR system, existing knowledge or experience needs to be represented as a case library that typically contains multiple cases. Each case is generally composed of two parts, the description of the problem and the corresponding solution, for the convenience of description, the symbol is given below.

$$C_i = \{D_i, S_i\}, i = 1, 2, ..., n \tag{1}$$

$C = \{C_1, C_2, ..., C_n\}$ is n historical cases in the case base, C_i represents the ith case ($i \in \{1, 2, ..., n\}$) including problem description D_i and corresponding solution S_i. $\mathcal{C}*$ is the target case, and the problem description for the target case is represented as $\mathcal{D}*$. Suppose SIM_i represents the similarity between Ci and the target case. $Sim_j(\mathcal{D}*, D_i)$ represents the similarity of the problem description $\mathcal{D}*$ of the target case and the problem description D_i of the historical case C_i about the characteristic attribute j.

In case reasoning, our goal is to find some order of historical cases in the case base, that is, the similarity between historical cases and target cases, so as to support the selection of source cases with the highest similarity as candidate cases for further revision and use. In other words, the more similar the historical case is, the more willing we are to reuse the case. One way to calculate the similarity of a case is to take the product of the local similarity of different attributes.

$$SIM_i = \prod_{j=1}^{q} sim_{ij} \tag{2}$$

We can see that each additional feature can only reduce the probability that the case C_i is the best candidate case. If any $sim_{ij} = 0$ for $j = 1...q$, then $SIM_i = 0$. More generally, we observe that for any case C_i, as long as there is a low local similarity value, the overall similarity of the case C_i will be greatly reduced. This is a kind of logical anding for a given C_i. The expression of this possibility is too strong, because it requires the premise that all the local similarity of C_i is consistent and high, so that we can think of this suspect's historical case as similar. Therefore, this paper will consider the use of OWA aggregation operator to determine the candidate case similarity of the softer formula. In the following text, we set λ_i as the index function and $\lambda_i(k)$ as the kth probability index of great compatibility of C_i. Here $sim_{i\lambda_i(k)}$ is the kth largest local similarity of the case C_i. We let:

$$Prod_i(j) = \prod_{k=1}^{j} sim_{i\lambda_i(k)} \tag{3}$$

Here $Prod_i(j)$ is the product of the j largest probabilities. We note that $Prod_i(j)$ is monotonically decreasing as a function of j, that is if $j_1 < j_2$, then $Prod_i(j_1) \geq Prod_i(j_2)$. Also we can easily observe that $Prod_i(j) \in [0,1]$ since each $sim_{i\lambda_i(k)} \in [0,1]$.We observe that the likelihood function can now be expressed as $SIM_i = Prod_i(q)$.

2.2. Ordered weight averaging aggregation

Below, we will consider using OWA aggregation operator to provide a class of soft likelihood functions based on $Prod_i(j)$. In order to do this, we need to briefly describe the OWA aggregation operator.

Ordered weight averaging aggregation was first proposed by Yager [42]. An OWA aggregator operator of n dimension is a mapping: $R^n \to R$. $OWA_w(a_1, a_2, \ldots, a_i, \ldots, a_n) = \sum_{j=1}^{n} w_j a_{\lambda(j)}$, where $W = (w_1, w_2, \ldots, w_n)^T$ is the weighted vector associated with the function OWA with $w_j \epsilon [0, 1]$ and $\sum_j w_j = 1$ $(j \epsilon \{1, 2, \ldots, n\})$; $a_{\lambda(j)}$ is the jth largest element in $a_1, a_2, \ldots, a_n$ in order from largest to smallest. Then we called function OWA as ordered weight averaging operator, which is also called OWA operator.

The characteristic of OWA operator is to rearrange the given data $(a_1, a_2, \ldots, a_i, \ldots, a_n)$ into $(a_{\lambda(1)}, a_{\lambda(2)}, \ldots, a_{\lambda(i)}, \ldots, a_{\lambda(n)})$ in order from large to small, and aggregate $(a_{\lambda(1)}, a_{\lambda(2)}, \ldots, a_{\lambda(i)}, \ldots, a_{\lambda(n)})$ by the given weight vector. Furthermore, element a_i has nothing to do with weight w_j, and weight w_j is only related to the jth position in the assembly process, so the weighted vector W is also called the position weighted vector.

Let's notice some special operators[42]:

1. $W^* = (1, 0, \ldots, 0)$, the OWA operator is reduced to the max operator, $OWA(a_1, \ldots, a_n) = a_{\lambda(1)} = max_i(a_i)$.
2. $W_* = (0, 0, \ldots, 1)$, the OWA operator is reduced to the min operator, $OWA(a_1, \ldots, a_n) = a_{\lambda(n)} = min_i(a_i)$.
3. $W_n = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})$, the OWA operator is reduced to a simple arithmetic average operator, $OWA(a_1, \ldots, a_n) = \frac{1}{n}\sum_{i=1}^{n} a_i$.
4. $W_{n-2} = (0, \frac{1}{n-2}, \frac{1}{n-2}, \ldots, \frac{1}{n-2}, 0)$, the OWA operator is reduced to an arithmetic average operator that removes the extremum, $OWA(a_1, \ldots, a_n) = \frac{1}{n-2}(\sum_{i=1}^{n} a_i - max_i(a_i) - min_i(a_i))$
5. $W_k = (0, \ldots, 1, \ldots, 0)$, $OWA(a_1, \ldots, a_n) = a_{\lambda(k)}$.

We can discover that the more weight is assigned to w_j near the top of W(the w_j with a smaller index), the aggregate value is larger; while the more weight is assigned to w_j near the bottom of W(the w_j with a larger index), the aggregate value is smaller. Weighted vector W that can reflect the tendency of the DMs to be optimistic or pessimistic and it determines how OWA is aggregated. Attitudinal character is defined as[43]:

$$AC(W) = \sum_{j=1}^{n} \frac{n-j}{n-1} w_j \tag{4}$$

We can find out that $AC(W) \in [0, 1]$ and $AC(W^*) = 1$, $AC(W_*) = 0$, $AC(W_n) = 0.5$,$AC(W_{n-2}) = 0.5$, $AC(W_k) = \frac{n-k}{n-1}$. The value of attitudinal character $AC(W)$ determines the degree of optimism. In other words, the larger the attitudinal character is, the more optimistic it is and the higher the aggregated value is.

We use a method to get OWA weights–w_j. Assume function f: $[0, 1] \to [0, 1]$ is monotonic; when $x > y$, $f(x) > f(y)$; $f(0) = 0$ and $f(1) = 1$. We obtain:

$$w_j = f(\frac{j}{n}) - f(\frac{j-1}{n}) \tag{5}$$

$w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$; w_j has all the properties of the OWA weights [43].

We call this method of obtaining OWA weights the function method, where w_j and the associated attitudinal character depends not only on the function itself, but also on the cardinality n.

Attitudinal character is defined as [43]:

$$Opt(f) = \int_0^1 f(x)dx \tag{6}$$

When n gets really big, $Opt(f)$ is really just $AC(W)$.

We can find $f(x) = x^m$ for $m \geq 0$, and for this function,

$$\alpha = \int_0^1 x^m dx = \left.\frac{x^{m+1}}{m+1}\right|_0^1 = \frac{1}{m+1} \tag{7}$$

We have $m = \frac{1-\alpha}{\alpha}$, and $\alpha \in [0, 1]$. We can see the larger α is, the more optimistic the attitude of users is. $m = 1$ when $\alpha = 0.5$; $m = 0$ when $\alpha = 1$; $m \to \infty$ when $\alpha \to 0$.

Using the above functional form, the OWA weights is:

$$w_j = f(\frac{j}{n}) - f(\frac{j-1}{n}) = (\frac{j}{n})^m - (\frac{j-1}{n})^m \tag{8}$$

Then for a given α, we can obtain:

$$w_j = (\frac{j}{n})^{\frac{1-\alpha}{\alpha}} - (\frac{j-1}{n})^{\frac{1-\alpha}{\alpha}} \tag{9}$$

Then we shall consider softer formulations for determining candidate similarity by using the OWA aggregation operator.

2.3. Local similarity measurement methods for case information

CBR is very similar to the way humans solve problems. When a new problem is encountered, it uses retrieval method to retrieve and select potentially available source cases from the source case base [44]. CBR can not only give full play to the advantage of the immediacy of computer processing information, but also improve the scientific nature and effectiveness of decision making [45]. In the case-based reasoning system, whether all the follow-up work can play its due role largely depends on the quality of the cases retrieved, so case retrieval is very critical.

The information or data in a case-based reasoning system is usually heterogeneous, and heterogeneity indicates a difference in the type and nature of information or data [46]. The processing of heterogeneous information is a key point in the decision-making process [47, 48]. As case events are usually characterized by risk, complexity and uncertainty [49], plus the imprecision of the environment, decision information is often not always expressed as accurate numbers, including

Boolean values, interval numbers and fuzzy numbers. In addition, because of the fuzziness of human thinking, it is sometimes difficult to express the decision information with quantitative values in the decision-making process, and qualitative language information is also used to evaluate the attributes [50].

Suppose $Sim_j(\mathcal{D}*, D_i)$ represents the similarity between the target case $\mathcal{D}*$ and the historical case D_i about the characteristic attribute j. Heterogeneous decision information contains many types of attribute information such as numerical features, Boolean features, symbolic features with orders, symbolic features without orders, string features, fuzzy features, interval features, and its similarity is calculated as follows [51].

- for numerical features, the similarity between $\mathcal{D}*$ and D_i can be obtained as

$$Sim_j(\mathcal{D}*, D_i) = 1 - \frac{|\mathcal{D}* - D_i|}{\max} \tag{10}$$

- for Boolean features, the similarity between $\mathcal{D}*$ and D_i can be obtained as

$$Sim_j(\mathcal{D}*, D_i) = \begin{cases} 0 & \mathcal{D}* \neq D_i \\ 1 & \mathcal{D}* = D_i \end{cases} \tag{11}$$

- for symbolic features with orders, the similarity between $\mathcal{D}*$ and D_i can be obtained as

$$Sim_j(\mathcal{D}*, D_i) = 1 - \frac{|\mathcal{D}* - D_i|}{g} \tag{12}$$

  where g is the number of value levels.

- for symbolic features without orders, the similarity between $\mathcal{D}*$ and D_i can be obtained as

$$Sim_j(\mathcal{D}*, D_i) = \frac{num(\mathcal{D}* \wedge D_i)}{num(\mathcal{D}* \vee D_i)} \tag{13}$$

- for string features, the similarity between $\mathcal{D}*$ and D_i can be obtained as

$$Sim_j(\mathcal{D}*, D_i) = \frac{t \times l}{\max(len(\mathcal{D}*), len(D_i))} \tag{14}$$

  where t is the matching number, l is the matching length and len is the string length.

- for fuzzy features, the similarity between $\mathcal{D}*$ and D_i can be obtained as

$$\begin{aligned} Sim_j(\mathcal{D}*, D_i) = 1 - \{(n_i - n_i^{'})^2 + \frac{1}{9}[(m_i - m_i^{'})^2 + (r_i - r_i^{'})^2 - (m_i - m_i^{'})(r_i - r_i^{'})] \\ -\frac{1}{2}(n_i - n_i^{'})[(m_i - m_i^{'}) - (r_i - r_i^{'})]\}^{\frac{1}{2}} \end{aligned} \tag{15}$$

$\mathcal{D}*$,D_i are triangular fuzzy number, $\mathcal{D}* = (n_i, m_i, r_i), D_i = (n_i^{'}, m_i^{'}, r_i^{'})$

- for interval features, the similarity between $\mathcal{D}*$ and D_i can be obtained as

$$Sim_j(\mathcal{D}*, D_i) = \frac{len(\mathcal{D}* \bigcap D_i)}{len(\mathcal{D}*) + len(D_i) - len(\mathcal{D}* \bigcap D_i)} \tag{16}$$

where len is the interval length, $\mathcal{D}* \bigcap D_i$ is the overlapping interval.

3. Case retrieval strategy

In this section, we first propose a new global similarity calculation method based soft likelihood function that integrates the similarity of each attribute, and then, considering the feature similarity, we give a SLFs case retrieval algorithm combining the feature similarity. Our retrieval strategy is to combine case retrieval algorithm based on soft likelihood functions with KNN, so as to improve the performance of case retrieval.

3.1. Case retrieval method based on soft likelihood functions

In the previous section, we have obtained the local attribute similarity between the target case and the historical case under a variety of heterogeneous information environments. The global similarity is then calculated to retrieve the historical cases that are most similar to the target cases from the case base. We apply the soft likelihood functions based on OWA to the case retrieval process, and propose a new global similarity calculation method to improve the previous case retrieval strategy.

Let's consider using soft likelihood functions based OWA as a retrieval strategy for case-based reasoning. For each source case C_i that we denote global similarity as $SIM_{i,W}$, we use the weighting vector W and the $Prod_i(j)$ to calculate it. Here $W = \{w_1, \ldots, w_q\}$, $w_j \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$, and characterizes how we define this softer aggregation function. We define

$$SIM_{i,W} = \sum_{j=1}^{q} w_j Prod_i(j) \tag{17}$$

where as we have already indicated $Prod_i(j) = \prod_{k=1}^{j} sim_{i\lambda_i(k)}$. Here λ_i is an index function so that $\lambda_i(k)$ is an index of the local similarity of attribute with the kth largest probability of compatibility of case C_i.

For each C_i, $Prod_i(j) = Prod_i(j-1)sim_{i\lambda_i(k)}$, as $sim_{i\lambda_i(k)} \leq 1$, so $Prod_i(j)$ is monotonic decreasing in j, and $Prod_i(j) \geq Prod_i(j')$ for $j < j'$. Therefor, the $Prod_i(j)$ for $j = 1 ... q$ using the weighting vector W based on the OWA aggregation is

$$SIM_{i,W} = \sum_{j=1}^{q} w_j Prod_i(j) = OWA_W\{Prod_i(1), \dots, Prod_i(q)\} \tag{18}$$

The actual form of the soft likelihood functions is determined by the weighting vector W which is only related to the location. For some of the special weighting vector,

(1):$W^* = \{w_1 = 1, w_j = 0 | j = 2, \dots, q\}$, we see easily that in this case

$$SIM_{i,W^*} = Prod_i(1) = sim_{i\lambda_i(1)} \tag{19}$$

This is the largest possible value of our soft likelihood function, which is equal to the probability of compatibility associated with the attribute having the largest probability of compatibility of C_i.

(2):$W_* = \{w_q = 1, w_j = 0 | j = 1, \dots, q-1\}$, we see easily that in this case

$$SIM_{i,W_*} = Prod_i(q) = \prod_{j=1}^{q} sim_{ij} \tag{20}$$

This is the classic likelihood function L_i that we introduced earlier. This is the most pessimistic form for determining the compatibility of C_i with the description set D_j, where all of the attribute must be compatible with C_i being the target case.

(3):$W_n = \{w_j = \frac{1}{q} | j = 1, \dots, q\}$, we see easily that in this case

$$SIM_{i,W_n} = \frac{1}{q}\sum_{j=1}^{q} Prod_i(j) = \frac{1}{q}\sum_{j=1}^{q}(\prod_{k=1}^{j} sim_{i\lambda_i(k)}) \tag{21}$$

This is a kind of simple average of the $Prod_i(j)$.

(4):$W_n = \{w_1 = 0, w_j = 0, w_j = \frac{1}{q-2} | j = 2, \dots, q-1\}$, we see easily that in this case

$$\begin{aligned} SIM_{i,W_n} &= \frac{1}{q-2}(\sum_{j=1}^{q} Prod_i(j) - Prod_i(1) - Prod_i(q)) \\ &= \frac{1}{q-2}(\sum_{j=1}^{q}(\prod_{k=1}^{j} sim_{i\lambda_i(k)}) - sim_{i\lambda_i(1)} - \prod_{j=1}^{q} sim_{ij}) \end{aligned} \tag{22}$$

This is a kind of arithmetic average of the $Prod_i(j)$ which removes the extremum.

If we are more optimistic to the likelihood, more of the allocated weight is related to the w_j that has more smaller indices; but if we are more pessimistic to the likelihood, more of the allocated weight is related to the w_j that has more larger indices. Due to $SIM_{i,W}$ is depending on the W, so we discover that the likelihood functions depend on the attitudinal character α which can impact the weighting vector W. If the user is more optimistic, then the α is closer to 1 and the value of SIM_{i,W_N} is larger; while the user is more pessimistic, then the α is closer to 0 and the value of SIM_{i,W_N} is smaller.

As we discussed earlier $w_j = f(\frac{j}{q}) - f(\frac{j-1}{q})$ and $f(x) = x^m$. In addition, we use $m = \frac{1-\alpha}{\alpha}$ to show the desired degree of optimum α. As a result, we can express users' attitude by a softer likelihood function which is more in line with the reality. We can get:

$$SIM_{i,\alpha} = \sum_{j=1}^{q} [(\frac{j}{q})^{\frac{1-\alpha}{\alpha}} - (\frac{j-1}{q})^{\frac{1-\alpha}{\alpha}}] \prod_{k=1}^{j} sim_{i\lambda_i(k)}) \tag{23}$$

Because of the physiological and cognitive limitations of the DMs, he is bounded rational in reality [52]. DMs' reasoning is not only influenced by the information of historical cases, but also implies their personal wisdom, emotion, attitude, cognition, etc. Psychological characteristics have a great influence on the decision-making process of DMs[53]. Therefore, attitude characteristics play an important role in case-based reasoning, and it is necessary to pay attention to DMs' attitude characteristics in case retrieval [54]. On the one hand, the use of attitude characteristics is subjective and highly dependent on users. An optimistic decision-maker and a pessimistic decision-maker tend to make different judgments about the same issue, which needs to be taken into account in the decision-making process. On the other hand, if the description of the target case is accurate and the calculation of similarity is accurate, an optimistic attitude should be adopted. If there is reason to doubt the accuracy of the similarity between the target case and the source case, a pessimistic attitude should be adopted. Therefore, the attitude characteristics of users can be considered as finding a balance between risks and benefits.

Next, we give an example to illustrate our case retrieval algorithm.

Example 1:

Let's have $q = 6$ primary attributes. Local similarity with the 6 attributes between source case and target case is: $C = \{sim_{i1} = 0.7, sim_{i2} = 0.4, sim_{i3} = 0.9, sim_{i4} = 1, sim_{i5} = 0.5, sim_{i6} = 0.8\}$ Then the index function $\lambda_i(k)$ is such that $\lambda_i(1) = 4, \lambda_i(2) = 3, \lambda_i(3) = 6, \lambda_i(4) = 1, \lambda_i(5) = 5, \lambda_i(6) = 2$ to order the probabilities. From these values we can calculate $Prod_i(j) = \prod_{k=1}^{j} sim_{i\lambda_i(k)}$ and these results is in Table 1.

The value of α is different for different users and we can calculate some typical $SIM_{i,\alpha}$. For $q = 6$, $w_j = (\frac{j}{6})^{\frac{1-\alpha}{\alpha}} - (\frac{j-1}{6})^{\frac{1-\alpha}{\alpha}}$ and $SIM_{i,\alpha} = \sum_{j=1}^{6} w_j Prod_i(j)$.

Table 1: Probability products

Ordered probability	$Prod_i(j)$
$sim_{i\lambda_i(1)} = sim_{i4} = 1$	$Prod_i(1) = 1$
$sim_{i\lambda_i(2)} = sim_{i3} = 0.9$	$Prod_i(2) = 1 \times 0.9 = 0.9$
$sim_{i\lambda_i(3)} = sim_{i6} = 0.8$	$Prod_i(3) = 0.9 \times 0.8 = 0.72$
$sim_{i\lambda_i(4)} = sim_{i1} = 0.7$	$Prod_i(4) = 0.72 \times 0.7 = 0.504$
$sim_{i\lambda_i(5)} = sim_{i5} = 0.5$	$Prod_i(5) = 0.504 \times 0.5 = 0.252$
$sim_{i\lambda_i(6)} = sim_{i2} = 0.4$	$Prod_i(6) = 0.252 \times 0.4 = 0.1008$

(1) $\alpha = 0.8$: This is a very optimistic attitudinal character. $m = \frac{1-\alpha}{\alpha} = 0.25$ and $w_j = (\frac{j}{6})^{0.25} - (\frac{j-1}{6})^{0.25}$. The results are given in Table 2.

Table 2: The numerical example of $\alpha = 0.8$

	$(\frac{j}{6})^{0.25}$	$(\frac{j-1}{6})^{0.25}$	w_j	$Prod_i(j)$	$w_j Prod_i(j)$
$j=1$	0.6389	0	0.6389	1	0.6389
$j=2$	0.7598	0.6389	0.1209	0.9	0.1088
$j=3$	0.8409	0.7598	0.0811	0.72	0.0584
$j=4$	0.9036	0.8409	0.0627	0.504	0.0316
$j=5$	0.9554	0.9036	0.0518	0.252	0.0131
$j=6$	1	0.9554	0.0446	0.1008	0.0045
			$\sum_j w_j = 1$		$\sum_j w_j Prod_i(j) = 0.8553$

So $SIM_{i,\alpha} = 0.8553$ when $\alpha = 0.8$.

(2) $\alpha = 0.5$: This is a very neutral attitudinal character. $m = \frac{1-\alpha}{\alpha} = 1$ and $w_j = (\frac{j}{6}) - (\frac{j-1}{6}) = \frac{1}{6}$. We can get: $SIM_{i,\alpha} = \frac{1}{6}\sum_{j=1}^{6} Prod_i(j) = \frac{1}{6}(1 + 0.9 + 0.72 + 0.504 + 0.252 + 0.1008) = 0.579$. So $SIM_{i,\alpha} = 0.579$ when $\alpha = 0.5$.

(3) $\alpha = 0.2$: This is a very pessimistic attitudinal character. $m = \frac{1-\alpha}{\alpha} = 4$ and $w_j = (\frac{j}{6})^4 - (\frac{j-1}{6})^4$. The results are given in Table 3. So $SIM_{i,\alpha} = 0.2393$ when $\alpha = 0.8$.

We can find from these examples that as α increases, so does $SIM_{i,\alpha}$. We see that the order of C_i basically depends on the order of sim_{ij} via the indexing function $\lambda_i(k)$. Here for a given case C_i, the smaller the probability of a piece of attribute the lower it is in the ordering.

Table 3: The numerical example of $\alpha = 0.2$

	$(\frac{j}{6})^4$	$(\frac{j-1}{6})^4$	w_j	$Prod_i(j)$	$w_j Prod_i(j)$
$j=1$	0.0008	0	0.0008	1	0.0008
$j=2$	0.0123	0.0008	0.0116	0.9	0.0104
$j=3$	0.0625	0.0123	0.0502	0.72	0.0361
$j=4$	0.1975	0.0625	0.1350	0.504	0.0681
$j=5$	0.4823	0.1975	0.2847	0.252	0.0718
$j=6$	1	0.4823	0.5177	0.1008	0.0522
			$\sum_j w_j = 1$		$\sum_j w_j Prod_i(j) = 0.2393$

3.2. SLFs case retrieval algorithm combined with feature similarity

When CBR is carried out, the attributes of the target case and the attributes of the source case in the case base are not necessarily the same [55], that is, we need to consider the feature similarity [56]. To solve the global similarity, both local similarity and feature similarity should be taken into consideration. In case retrieval, feature similarity is represented by different reliability of each attribute [57]. Therefore, the reliability of each attribute should be taken into consideration in the case retrieval algorithm of SLFs.

The reliability of each attribute is represented by $R_{ij} = \{r_{i1}, r_{i2}, ...r_{iq}\}$, $R_{ij} \epsilon [0, 1]$, and $r_{ij}(j \epsilon 1, 2, ..., q)$ represents the reliability of attribute j of the historical case i. In a case search, the reliability of each attribute does not change. So in this case, the value of r_{ij} depends only on j, not on i. Next we give a description of SLFs case retrieval algorithm considering reliability [40].

First of all, let's calculate the total reliability as $R_i = \sum_{j=1}^{q} R_{ij}$, and then we use this to calculate the associated normalized reliability $r_{ij} = \frac{Rij}{R_i}$. Obviously, $\sum_{j=1}^{q} r_{ij} = 1$.

We need to consider the products of the probability and the normalized reliability associated with target case C_i and then calculate the soft likelihood in the face of reliability associated with each sim_{ij}. We define an index function σ_i and $\sigma_i(k)$ is the index of the kth largest of these products. So $sim_{i\sigma_i(k)} \times r_{i\sigma_i(k)}$ is the kth largest of the $sim \times r$ where $sim_{i\sigma_i(k)}$ is the probability corresponding to the kth largest of the $sim \times r$ products associated with C_i and $r_{i\sigma_i(k)}$ is its associated reliability.

For a given C_i, the order of the local similarity is based on the product of the probability of compatibility of the local similarity of each attribute and the reliability of each attribute. The smaller this product the lower the piece of local similarity in the ordering. Either a small probability of compatibility or a small reliability can lead to a lower ordering. If all the attribute have the same reliability, then the index $\sigma_i(k)$ is simply based on the probabilities. We have:

$$Prod_i(j) = \prod_{k=1}^{j} sim_{i\sigma_i(k)} \tag{24}$$

where $Prod_i(j)$ is the product of the first j ordered probabilities and σ_i induces the order.

$$N_{ij} = \sum_{k=1}^{j} r_{i\sigma_i(k)} \tag{25}$$

where N_{ij} is the sum of the normalized reliability associated with the j largest $sim \times r$ products for the target case C_i.

We define f(x) as the weight generating function used to implement the desired soft likelihood function, then for $j = 1...q$ we calculate the OWA weights associated with C_i:

$$w_{ij} = f(S_{ij}) - f(S_{i(j-i)}) \tag{26}$$

Then the soft likelihood function for target case C_i in the face of reliability is:

$$SIM_{i,f} = \sum_{j=1}^{q} w_{ij} Prod_i(j) \tag{27}$$

If the reliability of $r_{i\sigma_i(k)}$ is 0, $S_{ij} = S_{i(j-1)}$ and $w_{ij} = S_{ij} - S_{i(j-1)} = 0$. If all the reliability are $r_{ij} = \frac{1}{q}$, $S_{ij} = \frac{j}{q}$ and $w_{ij} = f(\frac{j}{q}) - f(\frac{j-1}{q})$. This is the same situation as not considering reliability.

When $f(x) = x^m$ and $m = \frac{1-\alpha}{\alpha}$, we get $f(x) = x^{\frac{1-\alpha}{\alpha}}$ and the weight is

$$w_{ij} = S_{ij}^{\frac{1-\alpha}{\alpha}} - S_{i(j-1)}^{\frac{1-\alpha}{\alpha}} \tag{28}$$

Now we can improve our previous illustrative computations for the case where there are non-equal degrees of importance associated with the attribute. Next, we give an example to illustrate our case retrieval algorithm.

Example 2:

Let's have $q = 6$ primary attributes. Local similarity with the 6 attributes between source case and target case is(the same as Example 1): $C = \{sim_{i1} = 0.7, sim_{i2} = 0.4, sim_{i3} = 0.9, sim_{i4} = 1, sim_{i5} = 0.5, sim_{i6} = 0.8\}$. The associated non-normalized evidence reliability is: $R = \{R_{i1} =$

Table 4: Probability reliability

j	Reliability r_{ij}	Probability reliability $sim_{ij} \times r_{ij}$	Index order
1	0.244	0.171	1
2	0.171	0.068	6
3	0.097	0.088	5
4	0.122	0.122	2
5	0.220	0.110	4
6	0.146	0.117	3

$1, R_{i2} = 0.7, R_{i3} = 0.4, R_{i4} = 0.5, R_{i5} = 0.9, R_{i6} = 0.6\}$. The normalized reliability is: $r_{ij} = \frac{R_{ij}}{\sum_{k=1}^{q} R_{ik}} = \frac{R_{ij}}{4.1}$

We calculate the probability-reliability products as shown in Table 4.

Then the index function $\sigma_i(k)$ is: $\{\sigma_i(1) = 1, \sigma_i(2) = 4, \sigma_i(3) = 6, \sigma_i(4) = 5, \sigma_i(5) = 3, \sigma_i(6) = 2\}$.

We can calculate $Prod_i(j) = \prod_{k=1}^{j} sim_{i\sigma_i(k)} = Prod_i(j-1) sim_{i\sigma_i(j)}$ as shown in Table 5.

Table 5: Probability products

Ordered probability	$Prod_i(j)$
$sim_{i\sigma_i(1)} = sim_{i4} = 0.7$	$Prod_i(1) = 0.7$
$sim_{i\sigma_i(2)} = sim_{i1} = 1$	$Prod_i(2) = 0.7 \times 1 = 0.7$
$sim_{i\sigma_i(3)} = sim_{i6} = 0.8$	$Prod_i(3) = 0.7 \times 0.8 = 0.56$
$sim_{i\sigma_i(4)} = sim_{i5} = 0.5$	$Prod_i(4) = 0.56 \times 0.5 = 0.28$
$sim_{i\sigma_i(5)} = sim_{i3} = 0.9$	$Prod_i(5) = 0.28 \times 0.9 = 0.252$
$sim_{i\sigma_i(6)} = sim_{i2} = 0.4$	$Prod_i(6) = 0.252 \times 0.4 = 0.1008$

We can use $N_{ij} = \sum_{k=1}^{j} r_{i\sigma_i(k)} = N_i(j-1) + r_{i\sigma_i(j)}$ calculate the normalized reliability based on the index σ_i as shown in Table 6.

For different α, we can use $SIM_{i,\alpha} = \sum_{j=1}^{q} w_{ij} Prod_i(j)$ to calculate the $SIM_{i,\alpha}$ with different reliability associated with the attribute and $w_{ij} = S_{ij}^{\frac{1-\alpha}{\alpha}} - S_{i(j-1)}^{\frac{1-\alpha}{\alpha}}$. Now we calculate some typical $SIM_{i,\alpha}$.

(1) $\alpha = 0.8$: This is a very optimistic attitudinal character. $m = \frac{1-\alpha}{\alpha} = 0.25$. We can get Table 7. So $SIM_{i,\alpha} = 0.617$ when $\alpha = 0.8$.

Table 6: Sum of normalized probabilities

$r_{i\sigma_i(j)}$	N_{ij}
$r_{i\sigma_i(1)} = 0.244$	$N_{i1} = 0.244$
$r_{i\sigma_i(2)} = 0.122$	$N_{i2} = 0.366$
$r_{i\sigma_i(3)} = 0.146$	$N_{i3} = 0.512$
$r_{i\sigma_i(4)} = 0.220$	$N_{i4} = 0.732$
$r_{i\sigma_i(5)} = 0.097$	$N_{i5} = 0.829$
$r_{i\sigma_i(6)} = 0.171$	$N_{i6} = 1$

Table 7: The numerical example of $\alpha = 0.8$

	$N_{ij}^{0.25}$	$N_{i(j-1)}^{0.25}$	w_{ij}	$Prod_i(j)$	$w_{ij}Prod_i(j)$
$j = 1$	0.703	0	0.703	0.7	0.492
$j = 2$	0.778	0.703	0.075	0.7	0.052
$j = 3$	0.846	0.778	0.068	0.56	0.038
$j = 4$	0.925	0.846	0.079	0.28	0.022
$j = 5$	0.954	0.925	0.029	0.252	0.0074
$j = 6$	1	0.954	0.046	0.1008	0.0046
			$\sum_j w_{ij} = 1$		$\sum_j w_{ij}Prod_i(j) = 0.617$

(2) $\alpha = 0.5$: This is a very neutral attitudinal character. $m = \frac{1-\alpha}{\alpha} = 1$. We can get Table 8. So $SIM_{i,\alpha} = 0.441$ when $\alpha = 0.5$.

Table 8: The numerical example of $\alpha = 0.5$

	N_{ij}	$N_{i(j-1)}$	w_{ij}	$Prod_i(j)$	$w_{ij} Prod_i(j)$
$j = 1$	0.244	0	0.244	0.7	0.171
$j = 2$	0.366	0.244	0.122	0.7	0.085
$j = 3$	0.512	0.366	0.146	0.56	0.082
$j = 4$	0.732	0.512	0.220	0.28	0.062
$j = 5$	0.829	0.732	0.097	0.252	0.024
$j = 6$	1	0.829	0.171	0.1008	0.017
			$\sum_j w_{ij} = 1$		$\sum_j w_{ij} Prod_i(j) = 0.441$

(3) $\alpha = 0.2$: This is a very pessimistic attitudinal character. $m = \frac{1-\alpha}{\alpha} = 4$. We can get Table 9. So $SIM_{i,\alpha} = 0.202$ when $\alpha = 0.2$.

Table 9: The numerical example of $\alpha = 0.2$

	N_{ij}^4	$N_{i(j-1)}^4$	w_{ij}	$Prod_i(j)$	$w_{ij} Prod_i(j)$
$j = 1$	0.0035	0	0.0035	0.7	0.0025
$j = 2$	0.018	0.0035	0.0144	0.7	0.0101
$j = 3$	0.069	0.018	0.051	0.56	0.028
$j = 4$	0.287	0.069	0.218	0.28	0.061
$j = 5$	0.472	0.287	0.185	0.252	0.047
$j = 6$	1	0.472	0.528	0.1008	0.053
			$\sum_j w_{ij} = 1$		$\sum_j w_{ij} Prod_i(j) = 0.202$

In the above we clearly see the increasing nature of the soft likelihood value from Table 10 as the degree of optimism α increases.

Finally let us try to develop a more formal understanding of the mechanics underlying this approach for calculating the soft-likelihood associated with a decision maker. Again using the index function for the probability-reliability product we have $Prod(j) = \prod_{k=1}^{j} sim_{\sigma(k)}$ and $N_j = \sum_{k=1}^{j} r_{\sigma(k)}$.

Table 10: As optimism α increases.

Optimism α	Soft likelihood value
0.2	0.202
0.5	0.441
0.8	0.617

We then calculate the soft likelihood value SIM_f using an OWA aggregation based on the weight generating function f: $[0,1] \longrightarrow [1,0]$. Here then $SIM_f = \sum_{j=1}^{q} w_j Prod(j)$, where $w_j = f(S_j) - f(S_{(j-i)})$.

We note the characterizing parameter is $\alpha = \int_0^1 f(y)dy$. α ranges from 1, most optimistic; to $\alpha = 0$, most pessimistic. Here to capture the more general behavior, we will use $\alpha = 0.5$, a neutral case.

The retrieval strategy we proposed is to combine the case retrieval algorithm based on soft likelihood functions developed above with KNN, replacing the traditional KNN strategy combined with the ordinary mean algorithm or the weight average method, so as to improve the accuracy of case retrieval in CBR.

4. Experimental verification

In this section, we describe and simulate the algorithm proposed in this paper to evaluate the effectiveness of the proposed case retrieval method. We selected 10 classification data sets from UCI resource base for classification experiment. The abbreviations of names, number of samples, number of attributes, number of categories and other information of each data set are shown in Table 11. Detailed descriptions of each dataset are omitted here.

In this paper, our main contribution is to develop a case retrieval algorithm based on soft likelihood functions, and apply the proposed CBR-SLFs method to KNN to obtain a new CBR retrieval strategy. In order to make a fair and detailed comparison, it is necessary to compare its performance with traditional retrieval strategies. At present, the retrieval strategy based on KNN generally uses average-based method when calculating the similarity between the target case and the source case in the case base.

The experimental process is as follows. Firstly, the data set is divided into training set and test

Table 11: The general information of the used data sets.

Data set	Instance	Class	Attribute	Area
Iris	150	3	4	Life
Balance	625	3	4	Social
Survival	306	2	3	Life
Endgame	958	2	9	Game
Blood	748	2	4	Business
Banknote	1372	2	4	Computer
Breast	116	2	9	Life
Vertebral	310	2	6	N/A
User	403	4	5	Computer
Wholesale	440	2	7	Business

set by using the 10-fold cross validation method. The training set is used as the historical case base, and each case in the test set is used as the target case. Based on the case base, different retrieval strategies are used to find solutions for each target case. If the obtained solution is consistent with the corresponding solution of the test set, the retrieval strategy is considered to be effective. The effectiveness of the retrieval strategy is represented by the ratio of the number of cases with effective solutions to the number of elements in the test set. For all data sets, the process is repeated 100 times and a simple average is reported.

In order to verify the effect of case retrieval strategy of CBR-SLFs proposed in this paper on CBR classification accuracy, the following 5 case retrieval algorithms were used for comparative experiments:

(1) The KNN retrieval strategy based on mean operator is used to investigate the performance of case retrieval, denoted as KNN-Mean;

(2) The KNN retrieval strategy based on trim mean operator is used to investigate the performance of case retrieval, denoted as KNN-Trim;

(3) The KNN retrieval strategy based on weighted average operator is used to investigate the performance of case retrieval, denoted as KNN-Weight;

(4) The KNN retrieval strategy based on SLFs operator proposed in this paper is used to investigate the performance of case retrieval, denoted as KNN-SLF;

(5) The KNN retrieval strategy based on SLFs operator considering attribute reliability proposed in this paper is used to investigate the performance of case retrieval, denoted as KNN-RESLF.

Note that since the data set used in the experiment does not provide the degree of reliability of the feature, we use a random method to generate the degree of reliability of the attribute.

For the KNN, we study the case of k values between 5 and 20. As can be seen from Fig. 2, different K values have little influence on the efficiency of the retrieval strategy. The efficiency of the retrieval strategy is basically flat but fluctuates slightly, indicating that the retrieval strategy is not very sensitive to K. In the comparison test, take $k = 11$.

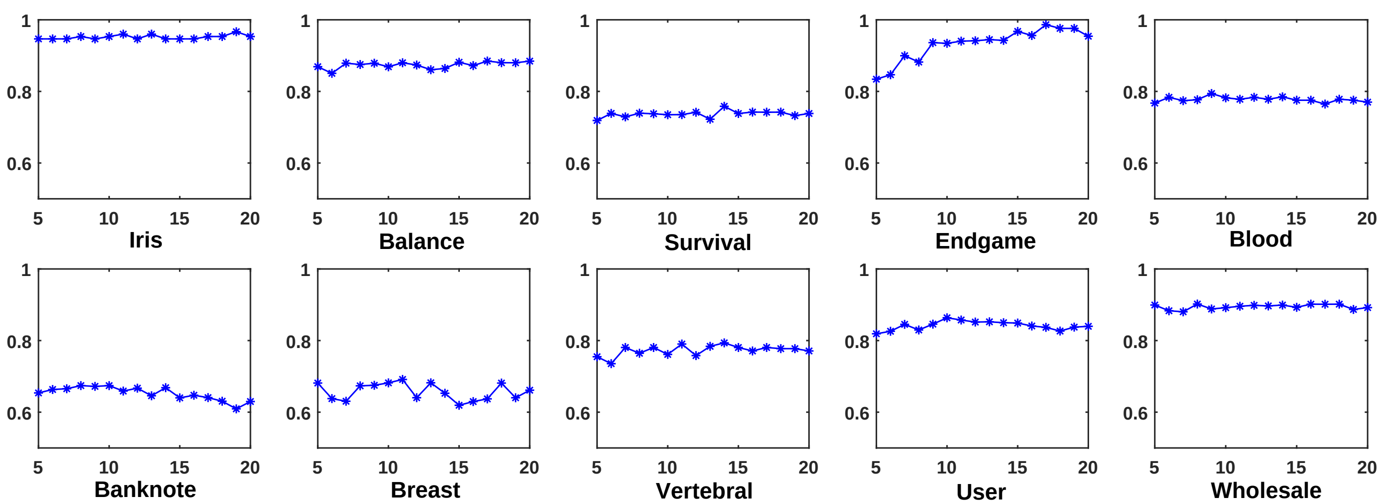

Figure 2: The performance of the retrieval strategy with KNN-RESLF algorithm under different K.

The soft likelihood functions involve the DMs' attitude parameter α. Fig. 3 shows the influence of the value of α from 0...1, that is, the DMs' attitude from negative to positive, on the correctness of the retrieval strategy. It can be seen that the selection of parameters and different data set types will have impact on the retrieval effect, and the value of α needs to be determined according to the characteristics of the actual decision maker and the field in which the case is located. In the comparison test, take the DMs' attitude is neutral, i.e., $\alpha = 0.5$.

We obtained the accuracy of different retrieval strategies in each data set, as shown in Table 12. In order to compare the performance of different retrieval strategies more clearly, we average the accuracy of each retrieval strategy in all data sets to represent the performance of the 5 retrieval

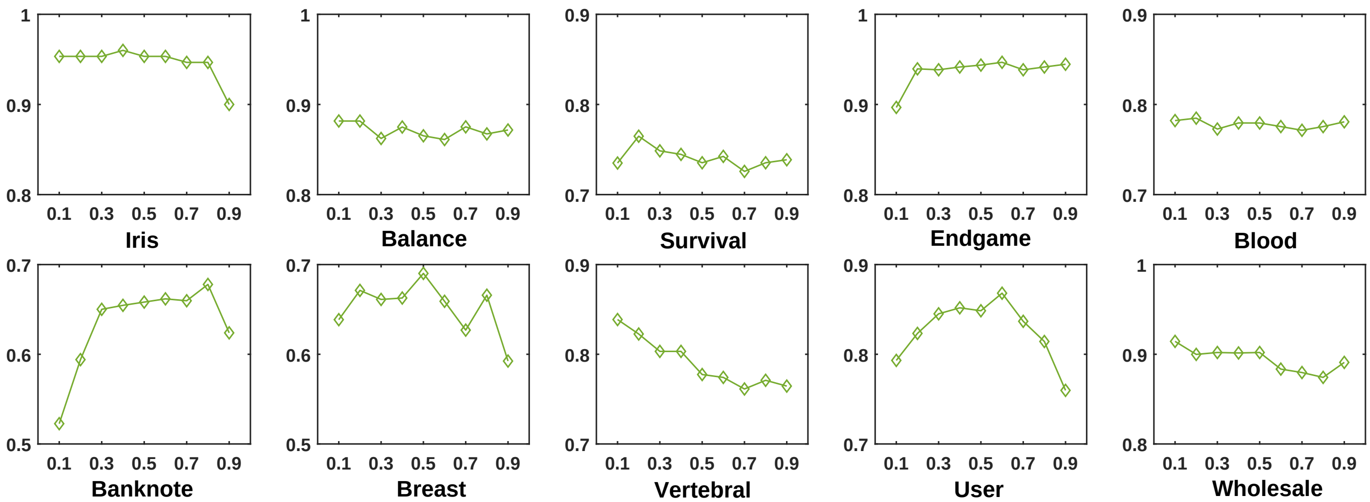

Figure 3: The performance of the retrieval strategy with KNN-RESLF algorithm under different α.

Table 12: The performance of case-based reasoning with different retrieval strategy.

Data set	Knn-Mean	Knn-Trim	KNN-Weight	Knn-SLF	Knn-RESLF
Iris	0.9545	0.9555	0.9541	0.9548	0.9526
Balance	0.8104	0.7625	0.8313	0.8716	0.8725
Survival	0.7273	0.7264	0.7334	0.7417	0.7429
Endgame	0.8907	0.8584	0.9015	0.9373	0.9417
Blood	0.7579	0.7589	0.7579	0.7753	0.7757
Banknote	0.5698	0.6024	0.5728	0.6505	0.6507
Breast	0.6634	0.6760	0.6798	0.6781	0.6726
Vertebral	0.8126	0.7836	0.8086	0.8242	0.8284
User	0.8258	0.8282	0.8234	0.8457	0.8462
Wholesale	0.8634	0.8702	0.9037	0.8967	0.8934
Average	0.7876	0.7822	0.7967	0.8176	0.8177

strategies. As can be seen from Table 12:

(1) The retrieval strategy trim mean-based algorithm is the worst in almost all data sets;

(2) The retrieval strategies of KNN-SLF and KNN-RESLF are better than other retrieval strategies;

(3) The ranking of average retrieval efficiency based on all data sets can be obtained by various retrieval strategies: $KNN-RESLF \approx KNN-SLF > KNN-Weight > KNN-Mean > KNN-Trim$.

The above analysis can illustrate the advantages of the retrieval strategy based on SLFs proposed in this paper. In the experiment, the performance of the retrieval strategy of KNN-SLF is very similar to that of KNN-RESLF. But in practical application, the reliability degree of each attribute is not random, but according to the importance of the attribute itself or given by experts. The accuracy of KNN retrieval strategy based on SLFs operator considering attribute reliability may be higher in practical application.

5. Conclusion

We introduce the soft likelihood functions based on OWA operator into case-based reasoning, and propose a retrieval strategy based on case-based reasoning process. It can reduce the interference of small probability events, and also consider the attitude characteristics of decision makers, which is more in line with the actual decision-making process. By defining the global similarity, including local similarity and feature similarity, the case that is most similar to the target case can be retrieved. Local similarity defines the similarity between different types of characteristic variables, and feature similarity indicates the degree of similarity of different features. By aggregating local similarity and feature similarity by CBR-SLFs, the global similarity between two cases can be obtained, which can be used as the basis for case retrieval. Experimental results on real data sets show that the proposed retrieval strategy based on SLFs is superior to the traditional KNN method.

However, this paper also has some limitations, this study only on UCI data set to verify the proposed method, lack of practical application. Moreover, in the experimental verification of this paper, the reliability degree of attributes is generated by random method, which is very brief. In practice, this step is usually completed by decision makers or experts.

In the future research, the CBR-SLFs retrieval strategy will be further improved. Firstly, the theoretical and experimental studies on the relevant parameters of the algorithm can be further improved to improve the adaptability and reliability of the method. Secondly, this study only covers a limited number of feature types. Considering the various data types that may exist in the actual CBR process, we will enrich the feature types more comprehensively in the following research. Next, the attributes of a case are not completely unrelated. We can combine the characteristics of specific research problems to study the interaction between attributes. And in the future, CBR can be applied to solve complex problems in practice, such as disease diagnosis and image recognition and so on.

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