Use of Optimal Control in Studying the Dynamical Behaviors of Fractional Financial Awareness Model

A Mahdy
Taif University College of Science
khaled lotfy (khlotfy_1@yahoo.com)
Zagazig University Faculty of Science
A. El-Bary
Arab Academy for Science Technology and Maritime Transport

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Use of optimal control in studying the dynamical behaviors of fractional financial awareness model

A. M. S. Mahdy[1, 2], Kh. Lotfy[2, 3] and A. A. El-Bary[4, 5]

[1]Department of Mathematics, Faculty of Science, Taif University, Saudi Arabia

[2]Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt

[3]Department of Mathematics, College of Science, Taibah University, Madinah, Saudi Arabia

[4]Arab Academy for Science, Technology and Maritime Transport, P.O. Box 1029, Alexandria, Egypt.

[5]National Committee for Mathematics, Academy of Scientific Research and Technology, Egypt.

amr_mahdy85@yahoo.com, amattaya@tu.edu.sa, khlotfy_1@yahoo.com and aaelbary@aast.edu

Abstract: Around there, we new examination has been done on past investigations of perhaps the main numerical models that portray the worldwide monetary development and that is depicted as a non-straight fragmentary monetary model of mindfulness, where the investigations address the means following: One: The schematic of the model is proposed. Two: The sickness-free balance point (DFE) and the soundness of the harmony point are talked about. Three: The strength of the model is satisfying by drawing the Lyapunov examples. Fourth: The presence of consistently stable arrangements is examined. Five: The Caputo is portrayed as the fragmentary subsidiary. Six: Fragmentary ideal control for NFFMA is examined, by explaining the partial ideal control through drawing when control. Seven: We are utilizing the calculation, summed up Adams–Bashforth–Moulton technique (GABMP) to tackle the is utilized to take the goal of an NFFMA. At last, we show that GABMP is profoundly indistinguishable. The mathematical strategy utilized in this composition to address this model has not been used by any creator before that. Additionally, this model with partial subordinates characterized in this manner has not been concentrated before that. The strategies used are not difficult to impact, regardless of whether logical or mathematical, and give great results.

Keywords: Financial of awareness; Stability; Lyapunov exponents; Fractional optimal control; Hamiltonian.

MSC: 41A28, 65D05, 65H10, 65L20, 65P30, 65P40, 65Z05.

1. Foreword

It is completely perceived that the objective of the statement is to persuade purchasers to purchase items, depending on the overall need of these produces to show that they contrast as an unmistakable brand from different items to help purchasers to buy them [4]. There are numerous approaches to turn the clients consider the items, and administrations offered to them. One of them is publicizing through messages. These messages are through physical media, like papers, magazines TVs, and radios. This mission can be through straightforward media, for example, sites and drawing out messages [39]. It is vital to examine publicizing techniques to build deals to accomplish the most noteworthy benefit for the organization [40]. Consequently, it is a lot helpful to examine and make a fitting dynamic and to address time-sensitive selling and general assessment [41]. There are additionally many methodology models to show up in the connection between promoting that distinguishes tangles from the showcasing and financial administration perspective. Publicizing arrangements are examined over the long run by unique models depicted as differential conditions where sell parcel, arrangements, and all extreme conditions factors are continually evolving. With regard to time. The reason for publicizing is consistently extraordinary for instance, the motivation behind certain commercials is to analyze two, three, or more brands, and for another reason, for example, acquainting a novel item with the shop dependent on these objectives, promoting types are made. Usually, the activity of publicizing is perpetually late on schedule, and it is important to incorporate the memory of various models of an affirmation, so models that depend on the past cases in the current cases have not just their underlying past cases fitting to portray procedures for the announcement.

Hitherto, fractional calculus has gained extraordinary dissemination and importance because of its snappy execution as another model work in a combination of designing and logical spaces ([1]-[58], [63]-[74], for example, viscoelasticity [1] and thermoelasticity ([2], [3], [59]-[62]). The best strategy partial models are leaded as fragmentary differential conditions.

The great object of this composition is to propose a customized concentrate about GABMP for settling NFFMA [4]:

$$\Delta^{\varepsilon} w_1 = -u^{\varepsilon} w_1 - \frac{k^{\varepsilon}}{N} w_1 (N - w_1) + \mu_b^{\varepsilon} N - \mu_d^{\varepsilon} w_1,$$
$$\Delta^{\varepsilon} w_2 = u^{\varepsilon} w_1 + \frac{k^{\varepsilon}}{N} w_1 (N - w_1) - (a^{\varepsilon} + v^{\varepsilon}) w_2 + \delta^{\varepsilon} w_3 - \mu_d^{\varepsilon} w_2, \tag{1.1}$$
$$\Delta^{\varepsilon} w_3 = (a^{\varepsilon} + v^{\varepsilon}) w_2 - \delta^{\varepsilon} w_3 - \mu_d^{\varepsilon} w_3.$$

with given initial case :

$$w_1(t) = w_{10}, \quad w_2(t) = w_{20}, \quad w_3(t) = w_{30}. \tag{1.2}$$

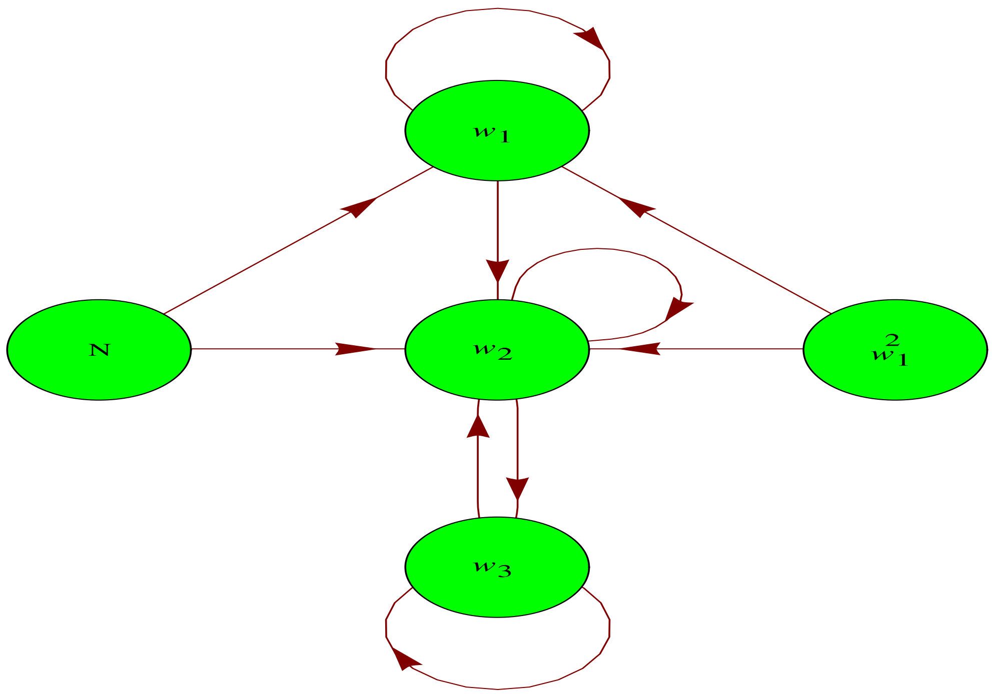

Fig. 1. The suggested graphical of the model.

Definition 1 The Δ^{ε} is Caputo fractional derivative is defined ([5]-[8], [46]):

$$\Delta^{\varepsilon} W(h) = \begin{cases} \dfrac{1}{\Gamma(m-\varepsilon)} \displaystyle\int_0^x \frac{W^{(m)}(\eta)}{(h-\eta)^{\varepsilon-n+1}} d\eta, & 0 \le m-1 < \varepsilon < m,\ 0 < \varepsilon \le 1, \\ W^{(m)}(h), & \varepsilon = m \in N. \end{cases} \tag{1.3}$$

For additional exceptional about the basal definitions and benefits of fractional subsidiaries see ([5]-[8], [46]).

Table 1 (The boundary esteems and their definition)

Parameter	Definition
$N(t)$	Populace all out with time.
$x_1(t)$	Some of a bunch of people who don't have the foggiest idea about the element of the produce.
$x_2(t)$	Some of a bunch of people who think about the item however have not yet purchased it.
$x_3(t)$	Some of the gathering of individuals who have purchased the item.
u	Knowledge, this changs the consumers from the intangibleset $x_1(t)$ into the scope one $x_2(t)$ by informing them about the produce**.**
v	Try advertisement, this carries the consumers from the potential set $x_2(t)$ into the purchased one $x_3(t)$ by encouraging them to buy the produce.
a	Preliminary rate.
k	Associate proportion.
δ	Change proportion.
μ_b^α	Birth proportion.
μ_d^α	Passing rate.
$u\ x_1(t)$	Total number of persons carry to the aware group $x_2(t)$ via declaration.
$(N(t)-x_1(t))$	Connect and report a total of $k(N(t)-x_1(t))$, out of which only a fraction of $x_1(t)/N(t)$ are latterly informed.

The paper is organized into five areas. In segment 2, we study the balance focuses, strength, presence of consistently stable arrangement nonlinear partial monetary models of mindfulness, explain the elements of the model between Lyapunov types, and Poincare maps. Ideal control for NFFMA is examined in Area 3. In area 4, we show a guide to show the action of utilizing (GABMP) to address NFFMA. At long last, appropriate ends are attracted segment 5.

2. Equilibrium and Stability of nonlinear fragmentary monetary models of mindfulness

In this segment, we examine the harmony point and the security of nonlinear fragmentary monetary models of mindfulness (1.1).

2.1. Equilibrium points

We study the harmony points of the nonlinear fragmentary monetary models of mindfulness. The model has one harmony point, more insights concerning balance point and Strength of nonlinear fragmentary models see ([32]-[38]).

Henceforth, we settle the accompanying conditions to decide the harmony point:

$$\begin{aligned}
&\Delta^{\varepsilon} w_1 = -u^{\varepsilon} w_1 - \frac{k^{\varepsilon}}{N} w_1 (N - w_1) + \mu_b^{\varepsilon} N - \mu_d^{\varepsilon} w_1 = 0,\\
&\Delta^{\varepsilon} w_2 = u^{\varepsilon} w_1 + \frac{k^{\varepsilon}}{N} w_1 (N - w_1) - (a^{\varepsilon} + v^{\varepsilon}) w_2 + \delta^{\varepsilon} w_3 - \mu_d^{\varepsilon} w_2 = 0,\\
&\Delta^{\varepsilon} w_3 = (a^{\varepsilon} + v^{\varepsilon}) w_2 - \delta^{\varepsilon} w_3 - \mu_d^{\varepsilon} w_3 = 0.
\end{aligned} \tag{2.1}$$

Equations (2.1) has to win the one equilibrium point.

2. 2. Studying the stability

We calculate the Jacobian matrix J for the model (1.1) as follows:

$$J = \begin{bmatrix} -u^{\varepsilon} - \mu_d^{\varepsilon} - k^{\varepsilon} + \frac{2k^{\varepsilon}}{N} w_1 & 0 & 0 \\ u^{\varepsilon} + k^{\varepsilon} - \frac{2k^{\varepsilon}}{N} w_1 & -a^{\varepsilon} - v^{\varepsilon} - \mu_d^{\varepsilon} & \delta^{\varepsilon} \\ 0 & a^{\varepsilon} + v^{\varepsilon} & -\delta^{\varepsilon} - \mu_d^{\varepsilon} \end{bmatrix},$$

at the equilibrium point the matrix Jacobian of (1.1) is approaching by

$$J = \begin{bmatrix} -u^{\varepsilon} - \mu_d^{\varepsilon} - k^{\varepsilon} + 2k^{\varepsilon} G & 0 & 0 \\ u^{\varepsilon} + k^{\varepsilon} - 2k^{\varepsilon} G & -a^{\varepsilon} - v^{\varepsilon} - \mu_d^{\varepsilon} & \delta^{\varepsilon} \\ 0 & a^{\varepsilon} + v^{\varepsilon} & -\delta^{\varepsilon} - \mu_d^{\varepsilon} \end{bmatrix}.$$

So, we get

$$\left|J-\lambda I\right|=\begin{vmatrix}-u^{\varepsilon}-\mu_d^{\varepsilon}-k^{\varepsilon}+2k^{\varepsilon}G-\lambda & 0 & 0\\ u^{\varepsilon}+k^{\varepsilon}-2k^{\varepsilon}G & -a^{\varepsilon}-v^{\varepsilon}-\mu_d^{\varepsilon}-\lambda & \delta^{\varepsilon}\\ 0 & a^{\varepsilon}+v^{\varepsilon} & -\delta^{\varepsilon}-\mu_d^{\varepsilon}-\lambda\end{vmatrix}=0.$$

Then, the eigenvalues approaching by

$$\lambda_1=-\mu_d^{\varepsilon},\quad \lambda_2=-a^{\varepsilon}-\delta^{\varepsilon}-\mu_d^{\varepsilon}-v^{\varepsilon},\quad \lambda_3=-\left(u^{\varepsilon}+\mu_d^{\varepsilon}+k^{\varepsilon}-2k^{\varepsilon}G\right).$$

The solution is stable.

2.3. Clarify Lyapunov exponents and Poincare map

Figures 2-4 explain Lyapunov types in various time spans. Figures 5-10 explain the Poincare guide of the framework for three unique upsides of a, k and delta. All of which included model security. All eigenvalues have negative which implies the steadiness of this fixed point and we ensure its security by plotting its Lyapunov models (LE1, LE2, LE3). The estimation used for choosing Lyapunov models had suggested in [47], see Figures 2-4. From Figure 2-4, we see that each Lyapunov models are negative after a little transient time that derives the structure has consistent and approaches its fixed point.

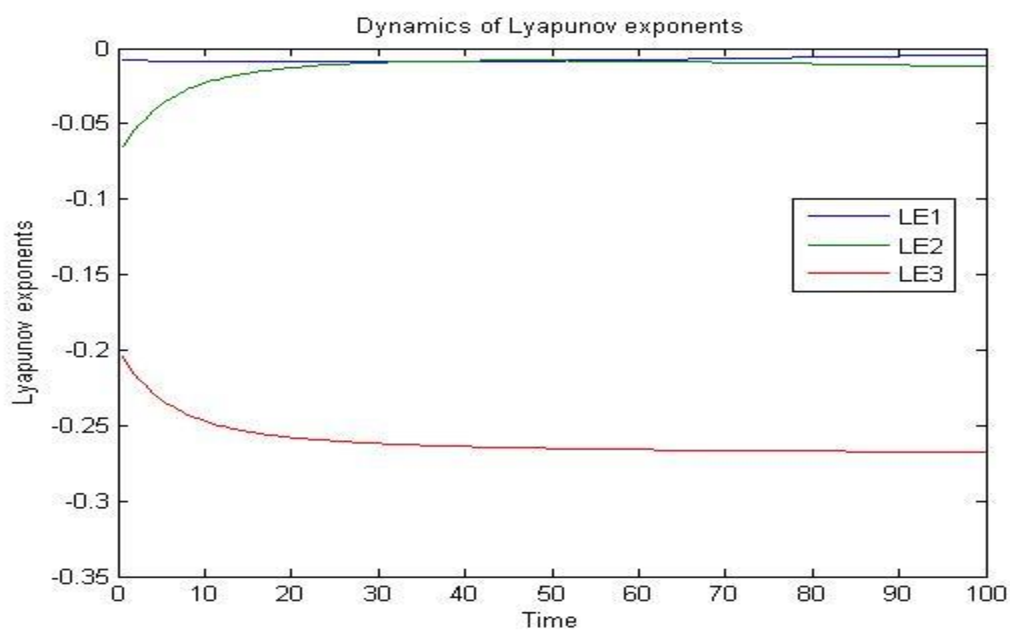

Fig. 2: Perform the Lyapunov exponents' dynamics for the model.

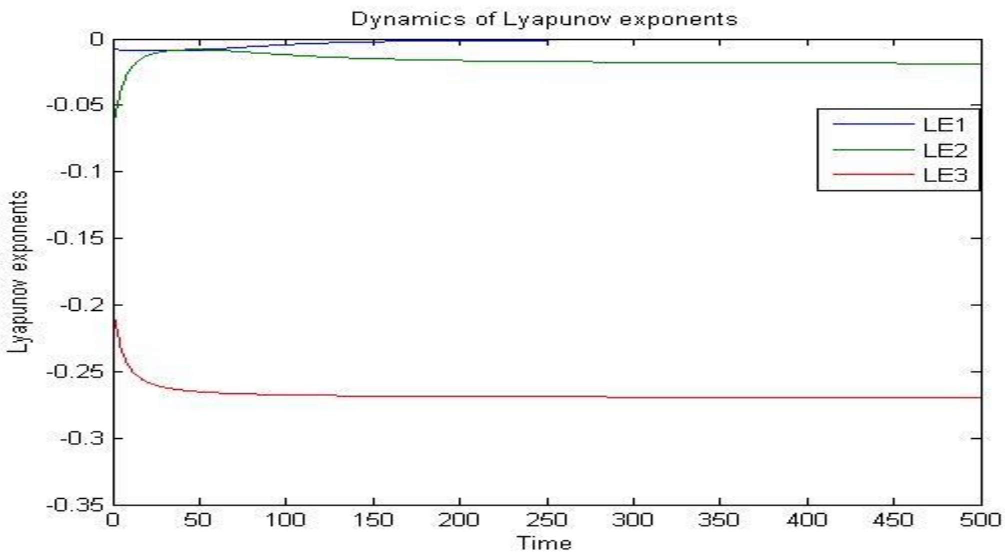

Fig. 3: Perform the Lyapunov exponents' dynamics for the model.

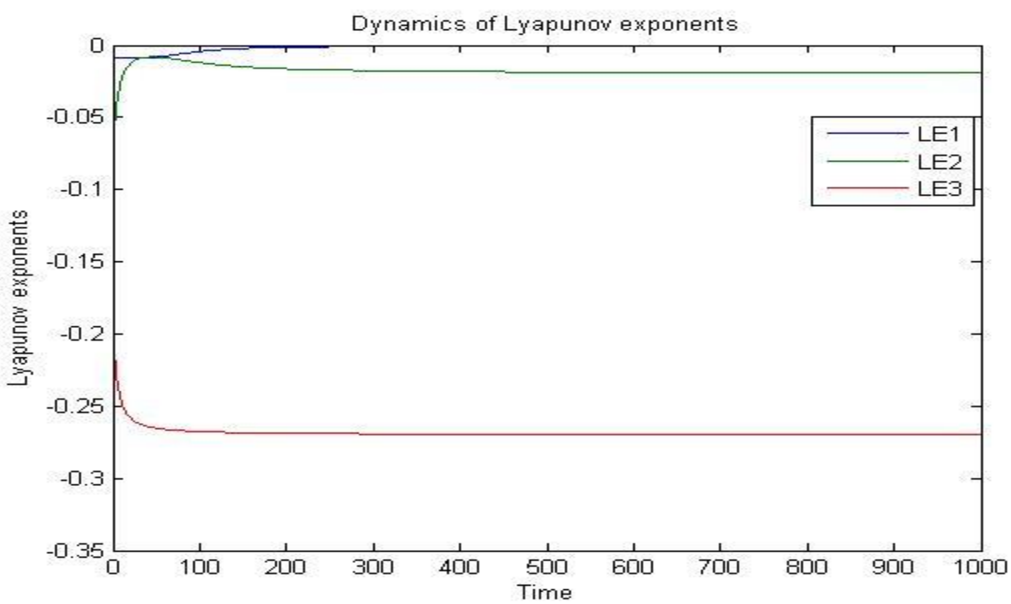

Fig. 4: Perform the Lyapunov exponents' dynamics for the model.

2.4. Existence of Uniformly stable solution:

Let

$$g_1 = -u^{\varepsilon} w_1 - \frac{k^{\varepsilon}}{N} w_1 (N - w_1) + \mu_b^{\varepsilon} N - \mu_d^{\varepsilon} w_1,$$

$$g_2 = u^{\varepsilon} w_1 + \frac{k^{\varepsilon}}{N} w_1 (N - w_1) - (a^{\varepsilon} + \nu^{\varepsilon}) w_2 + \delta^{\varepsilon} w_3 - \mu_d^{\varepsilon} w_2,$$

$$g_3 = (a^{\varepsilon} + \nu^{\varepsilon}) w_2 - \delta^{\varepsilon} w_3 - \mu_d^{\varepsilon} w_3.$$

Let $D=\{w_1,w_2,w_3\in\Re : |w_1,w_2,w_3|\le a,\quad t\in[0,T]\}$,

This implies that every one of the three capacities satisfies the Lipschitz condition concerning the three cases, and afterward every one of the three capacities is ceaseless as for the three cases.

3.1. Optimal control for fractional financial models of awareness

Let us see the case model given in Eqs. (1.1), incontrol accepted, with the set of $\Re^3$ functions for more details in ([42]-[45], [48]):

$$\Omega=\left\{(u(.),v(.))\in\left(L^{\infty}(0,T_f)^2\right)\middle|\ 0\le u(.),v(.)\le 1,\quad \forall t\in[0,T_f]\right\},$$

where T_f has the final time, $u(.)$ and $v(.)$ have controls functions.

The objective function is known as

$$J(u(.),v(.))=\int_0^{T_f}\left[Aw_1(t)+Bu^2(t)+Cv^2(t)\right]dt,\tag{3.1}$$

where A, B, and C illustrate the rule constants.

The premier point in FOCPs is to get the optimal controls $u(.)$ and $v(.)$, which minimize the following objective function:

$$J(u,v)=\int_0^{T_f}\eta[w_1,w_2,w_3,u,v,t]dt,\tag{3.2}$$

subjected to the constraint

$$\Delta^{\varepsilon}w_1=\xi_1,\quad \Delta^{\varepsilon}w_2=\xi_2,\ \Delta^{\varepsilon}w_3=\xi_3,\ \xi_i=\xi(w_1,w_2,w_3,u,v,t),\quad i=1,2,3.\tag{3.3}$$

The next starting conditions are fulfilled:

$$w_1(0)=w_{10},\quad w_2(0)=w_{20},\quad w_3(0)=w_{30}.\tag{3.4}$$

To realize the FOCP, let us think a revised objective (cost) function as directs:

$$\overline{J}=\int_0^{T_f}\left[H(w_1,w_2,w_3,u,v,t)-\sum_{i=1}^{3}\lambda_i\xi_i(w_1,w_2,w_3,u,v,t)\right]dt,\tag{3.5}$$

the Hamiltonian at the goal functional (3.5) and the control financial models of awareness (1.1) is given as follows:

$$H\left(w_1,w_2,w_3,u,v,t\right)=\eta\left(w_1,w_2,w_3,u,v,t\right)+\sum_{i=1}^{3}\lambda_i\xi_i\left(w_1,w_2,w_3,u,v,t\right), \tag{3.6}$$

$$\begin{aligned} H &= Aw_1+Bu^2+Cv^2+\lambda_1\left[-u^\varepsilon w_1-\frac{k^\varepsilon}{N}w_1\left(N-w_1\right)+\mu_b^\varepsilon N-\mu_d^\varepsilon w_1\right] \\ &\quad+\lambda_2\left[u^\varepsilon w_1+\frac{k^\varepsilon}{N}w_1\left(N-w_1\right)-\left(a^\varepsilon+v^\varepsilon\right)w_2+\delta^\varepsilon w_3-\mu_d^\varepsilon w_2\right] \\ &\quad+\lambda_3\left[\left(a^\varepsilon+v^\varepsilon\right)w_2-\delta^\varepsilon w_3-\mu_d^\varepsilon w_3\right]. \end{aligned} \tag{3.7}$$

From (3.5) and (3.7), we can deduce the necessary and sufficient conditions for FOPC as follows

$$\Delta^\varepsilon\lambda_1=\frac{\partial H}{\partial w_1},\quad \Delta^\varepsilon\lambda_2=\frac{\partial H}{\partial w_2},\quad \Delta^\varepsilon\lambda_3=\frac{\partial H}{\partial w_3}, \tag{3.8}$$

$$\frac{\partial H}{\partial u}=0,\qquad \frac{\partial H}{\partial v}=0. \tag{3.9}$$

$$\Delta^\varepsilon w_1=\frac{\partial H}{\partial\lambda_1},\quad \Delta^\varepsilon w_2=\frac{\partial H}{\partial\lambda_2},\quad \Delta^\varepsilon w_3=\frac{\partial H}{\partial\lambda_3}, \tag{3.10}$$

$$\lambda_j,(T_f)=0. \tag{3.11}$$

where λ_j, $j=1,2,3$ have Lagrange multipliers. Eqs. (3.9) and (3.10) appear the necessary conditions in terms of a Hamiltonian for the FOPC.
We arrive at the following theorem:

Theorem 1.
If u and v are optimal controls with the uniform state w_1^*, w_2^* and w_3^*, consequently there be adjoint variables λ_i^*, $i=1,2,3,$ fulfilled the next:

(i) Co-state equations (adjoint equations)

Laying the cases in the content hypothesis and applying conditions (3.8) ([42]-[45]), we obtain the accompanying three conditions, which can be composed as follows:-

$$\Delta^\varepsilon\lambda_1^*=A+\lambda_1^*\left(-u^\varepsilon-k^\varepsilon+\frac{2k^\varepsilon}{N}w_1-\mu_d^\varepsilon\right)+\lambda_2^*\left(u^\varepsilon+k^\varepsilon-\frac{2k^\varepsilon}{N}w_1\right), \tag{3.12}$$

$$\Delta^\varepsilon\lambda_2^*=\lambda_2^*\left(-a^\varepsilon-v^\varepsilon-\mu_d^\varepsilon\right)+\lambda_3^*\left(a^\varepsilon+v^\varepsilon\right), \tag{3.13}$$

$$\Delta^\varepsilon\lambda_3^*=\lambda_2^*\left(\delta^\varepsilon\right)+\lambda_3^*\left(-\delta^\varepsilon-\mu_d^\varepsilon\right). \tag{3.14}$$

(ii) Transversality conditions:

$$\lambda_i^*(T_f)=0,\quad i=1,2,3. \tag{3.15}$$

(iii) Optimality conditions

$$H\left(w_1^*,w_2^*,w_3^*,u^*,v^*,\lambda^*\right)=\min_{0\le u^*,v^*\le 1} H\left(w_1^*,w_2^*,w_3^*,u^*,v^*,\lambda^*\right), \tag{3.16}$$

As well, the control functions u^*, v^* are offered by

$$\frac{\partial H}{\partial u}=0\Rightarrow \quad u^{\varepsilon-2}=\frac{2B}{\alpha w_1^*\left(\lambda_1^*-\lambda_2^*\right)} \qquad \Rightarrow u^{\varepsilon}=\frac{2Bu^2}{\alpha w_1^*\left(\lambda_1^*-\lambda_2^*\right)}, \tag{3.17}$$

$$\frac{\partial H}{\partial v}=0 \qquad \Rightarrow v^{\varepsilon-2}=\frac{2C}{\alpha w_2^*\left(\lambda_2^*-\lambda_3^*\right)} \qquad \Rightarrow v^{\varepsilon}=\frac{2Cv^2}{\alpha w_2^*\left(\lambda_2^*-\lambda_3^*\right)}, \tag{3.18}$$

$$u^*=\min\left\{1, \quad \max\left\{0,\frac{\alpha w_1^*\left(\lambda_1^*-\lambda_2^*\right)}{2B}\right\}\right\}, \tag{3.19}$$

$$v^*=\min\left\{1, \quad \max\left\{0,\frac{\alpha w_2^*\left(\lambda_2^*-\lambda_3^*\right)}{2C}\right\}\right\}. \tag{3.20}$$

Proof. The co-state system Eqs. (3-12)-(3-14) are found from Eq.(3-10) where the Hamiltonian H^* is given by

$$H^*=A_1w_1^*+B\,u^{*2}+C\,v^{*2}+\lambda_1^*\Delta^\varepsilon w_1^*+\lambda_2^*\Delta^\varepsilon w_2^*+\lambda_3^*\Delta^\varepsilon w_3^*. \tag{3-21}$$

Moreover, the case in Eq. (3-11) else fulfilled, and the optimal control in Eqs. (3-19)–(3-20) can be derived from Eq. (3-9).

Letting u^* and v^* in (1-1), the next case system will be found as:

$$\begin{aligned}
\Delta^\varepsilon w_1^* &= -u^{*\varepsilon}w_1^*-\frac{k^\varepsilon}{N}w_1^*\left(N-w_1^*\right)+\mu_b^\varepsilon N-\mu_d^\varepsilon w_1^*,\\
\Delta^\varepsilon w_2^* &= u^{*\varepsilon}w_1^*+\frac{k^\varepsilon}{N}w_1^*\left(N-w_1^*\right)-\left(a^\varepsilon+v^{*\varepsilon}\right)w_2^*+\delta^\varepsilon w_3^*-\mu_d^\varepsilon w_2^*,\\
\Delta^\varepsilon w_3^* &= \left(a^\varepsilon+v^{*\varepsilon}\right)w_2^*-\delta^\varepsilon w_3^*-\mu_d^\varepsilon w_3^*.
\end{aligned} \tag{3-22}$$

For extra properties of fractional optimal control see the references ([42]-[45]).

4. Applications

Here **GABMP** is approaching in this here [72, 73]. In this style, the **GABMM** is derived for obtaining the numerical solution of the FODEs. Put

$$D^\varepsilon z(t)=g\left[t,z(t)\right], \quad 0\le\varepsilon\le T, \tag{4.1}$$

$$z^{(r)}(0)=z_0^r, \quad r=0,1,\ldots,[\varepsilon]-1, \tag{4.2}$$

be a general case of **FODEs**. We gain the solution *z(t)* in think of implementation of fractional integral on **(4.1).**

$$z(t)=\sum_{r=0}^{[\varepsilon]-1}\frac{z_0^{(r)}}{r!}t^r+\int_0^t\frac{(t-\omega)}{\Gamma(\varepsilon)}g(\omega,g(\omega))d\omega. \quad \textbf{(4.3)}$$

By setting $k=\frac{t_n}{m}$, $t_n=nh$, $n=0,1,\ldots,m$, **eq.(4.3)** will be described as next for integer positive m.

$$z_n(t_{n+1})=\sum_{r=0}^{[\varepsilon]-1}\frac{z_0^r}{r!}t_{n+1}^r+\frac{k^\varepsilon}{\Gamma(\varepsilon+2)}g\left(t_{n+1},z_k^p(t_{n+1})\right)+\frac{k^\varepsilon}{\Gamma(\varepsilon+2)}\sum_{j=0}^{n}a_{j,n+1}g\left(t_j,z_j(t_j)\right), \quad \textbf{(4.4)}$$

$a_{j,n+1}=\left\{n^{\varepsilon+1}-(n+1)^\varepsilon(n-\varepsilon)\right\},\ if\ \ j=0,\ \left\{(n-j+2)^{\varepsilon+1}-2(n-j+1)^\varepsilon+(n-j)^{\varepsilon+1}\right\},$ **if**

$if\ \ 0<j\le n, 1,\ \ if\ \ j=n+1.$

In which the predicted value $z_k^p(t_{n+1})$ may be derived as

$$z_k^p(t_{n+1})=\sum_{r=0}^{[\varepsilon]-1}\frac{z_0^r}{r!}t_{n+1}^r+\frac{1}{\Gamma(\varepsilon)}\sum_{j=0}^{n}b_{j,n+1}g\left(t_j,z_j(t_j)\right), \quad \textbf{(4.5)}$$

in which

$$b_{j,n+1}=\frac{k^\varepsilon\left[(n-j+1)^\varepsilon-(n-j)^\varepsilon\right]}{\varepsilon}.$$

The estimated error is

$\max_{j=0,1,\ldots,m}\left|z(t_j)-z_k(t_j)\right|=o(k^p),$ **in which** $p=\min\{1+\varepsilon,2\}.$

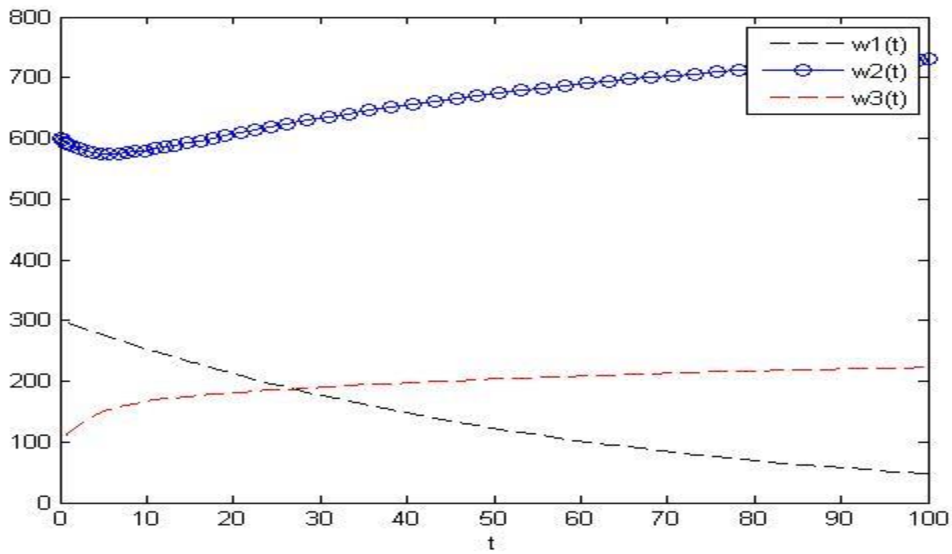

Figure 5. The approximate solution of x_1, x_2, x_3 using **GABMP** after control at $\varepsilon=1$.

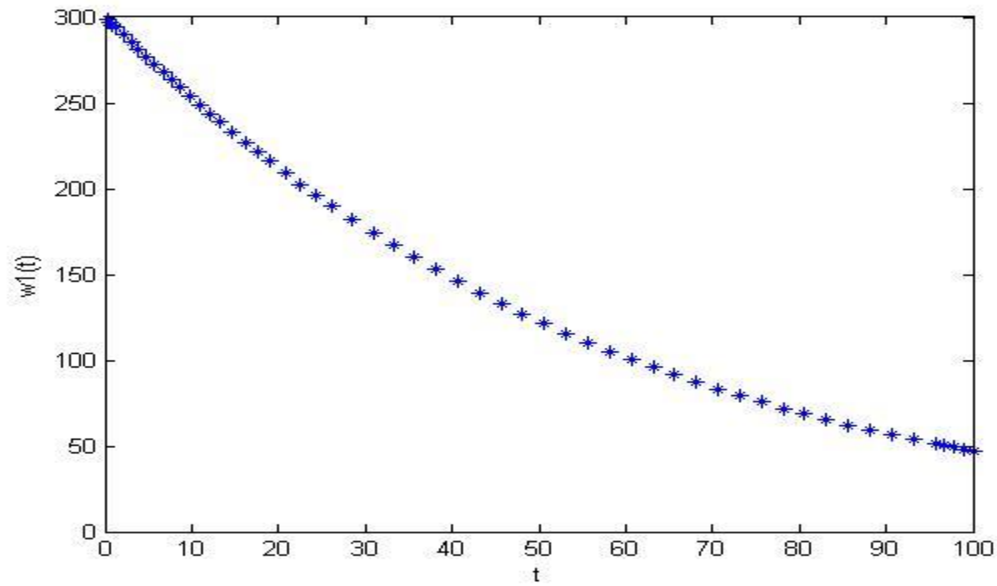

Figure 6. The approximate solution of w_1 using **GABMP** after control at $\varepsilon = 0.85$.

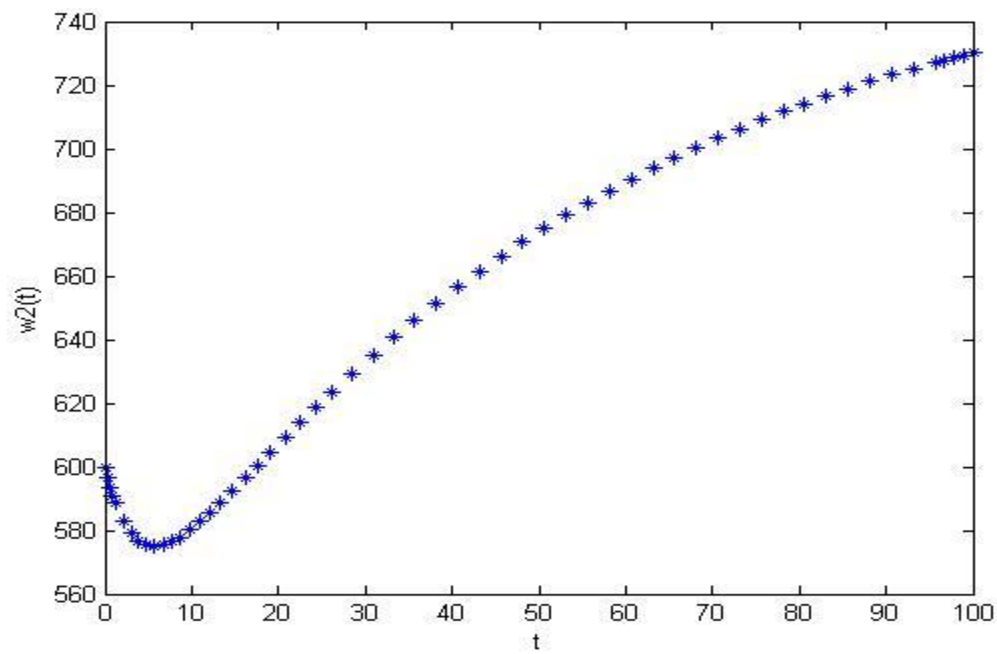

Figure 7. The approximate solution of w_2 using **GABMP** after control at $\varepsilon = 0.85$.

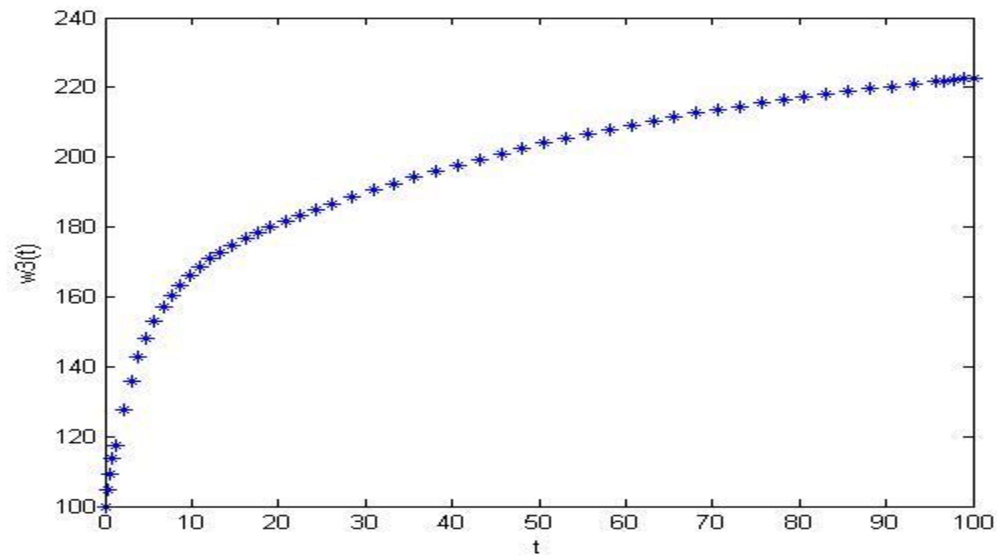

Figure 8. The approximate solution of w_3 using **GABMP** after control at $\varepsilon = 0.85$.

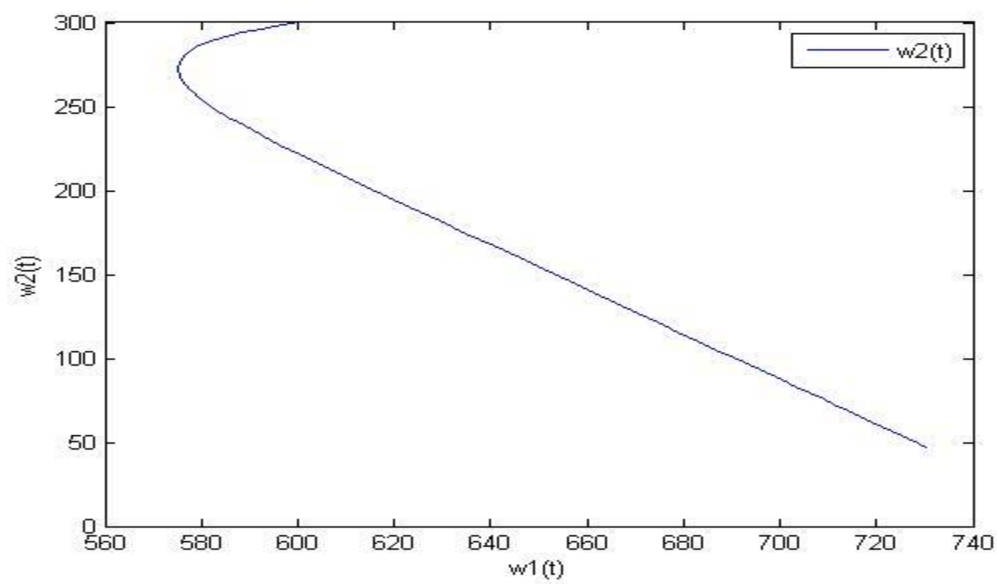

Figure 9. $w_1(t)$ Vs $w_2(t)$ using **GABMP** after control at $\varepsilon = 0.85$.

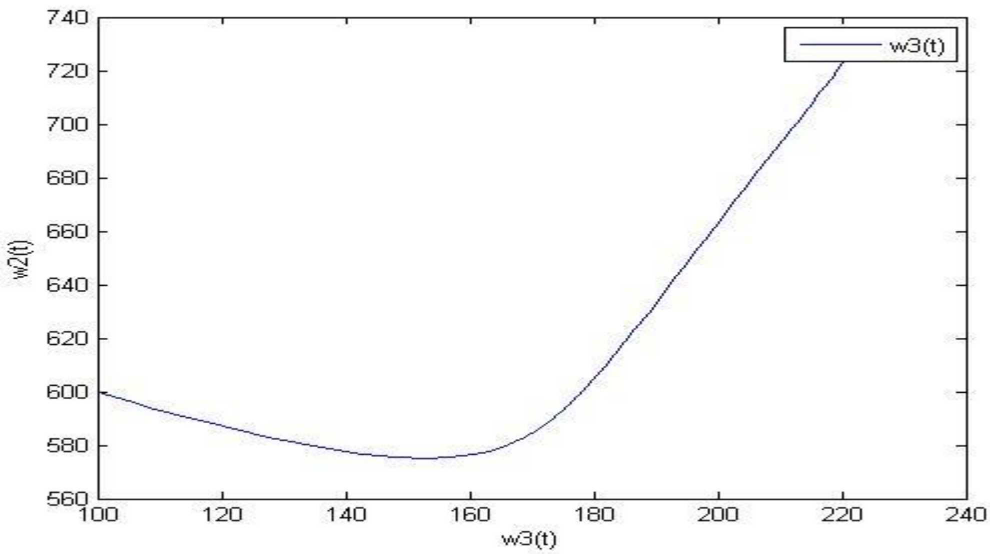

Figure 10. $w_2(t)$ Vs $w_3(t)$ using **GABMP** after control at $\varepsilon = 0.85$.

In figure 5-10, show the approximate solutions of NFFMA by using **GABMP** at $\varepsilon = 1$, illustrates the phase spaces.show the 0.85

It is no doubt that the activity of this way is greatly increased by the calculation of further terms $w_1(t)$, $w_2(t)$ and $w_3(t)$ by using **GABMP**.

5. Conclusions

In this paper, the graphical of the model is recommended. The sickness-free harmony point (DFE) and the steadiness of the balance point are explaining. The steadiness of the model is fulfilling by drawing the Lyapunov types and Poincare map. The presence of consistently stable arrangements is addressing. The Caputo is portraying as the fractional subordinate. Fragmentary ideal control for NFFMA is examining, through explaining the partial ideal control through drawing when control. GABMP is utilizing to take the goal of an NFFMA. We are showing that GABMP is exceptionally indistinguishable. At last, a novel examination has been done on past investigations of quite possibly the most driving numerical models that name the worldwide financial development and that is depicted as an NFFMA, where the explored at the upper.

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Conflicts of Interest: No irreconcilable situations concerning the paper, initiation, or arrival of this article.

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