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# Mehar approach to solve neutrosophic linear programming problems using possibilistic mean

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Abstract: Khatter (Soft Computing 24 (2020) 16847-16867) pointed out that although several approaches are proposed in the literature to solve single-valued neutrosophic linear programming problems (SVNLPPS) (linear programming problems in which all the parameters except decision variables are either represented by single-valued triangular neutrosophic numbers (SVTNNS) or single-valued trapezoidal neutrosophic numbers (SVTrNNS)). However, all the methods for comparing single-valued neutrosophic numbers (SVNNS), used in existing approaches, are independent from the attitude of the decision maker towards the risk. To fill this gap, Khatter (2020), firstly, proposed a method for comparing two SVNNS by considering the attitude of the decision maker towards the risk. Then, using the proposed comparing method, Khatter (2020) proposed an approach to solve SVNLPPS. In this paper, it is pointed out that a mathematical incorrect result is considered in Khatter's approach. Hence, it is inappropriate to use Khatter's approach. Also, it is pointed out that some mathematical incorrect results are considered in other existing approaches for solving SVNLPPS. Hence, it is inappropriate to use other existing approaches for solving SVNLPPS. Furthermore, to resolve the inappropriateness of Khatter's approach and other existing approaches, a new approach (named as Mehar approach) is proposed to solve SVNLPPS. Finally, correct optimal solution of some existing SVNLPPS is obtained by the proposed Mehar approach.

Keywords: SVNLPPS, SVTNNS, SVTrNNS.

# 1. Introduction

In the last few years, several approaches are proposed in the literature to solve mathematical programming problems under neutrosophic environment (Smarandache 1998). In this section, some recently proposed approaches are discussed in a brief manner.

Hussian et al. (2017) proposed an approach to solve single-valued triangular neutrosophic linear programming problems (SVTNLPPS). In Hussian et al.'s approach (2017), firstly, a

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single-valued triangular neutrosophic linear programming problem (SVTNLPP) is transformed into its equivalent crisp multi-objective linear programming problem (CrMOLPP). Then, the obtained CrMOLPP is transformed into its equivalent crisp linear programming problem (CrLPP). Finally, it is assumed that an optimal solution of the transformed CrLPP also represents an optimal solution of SVTNLPP.

Hussian et al. (2018) proposed an approach to solve single-valued triangular neutrosophic linear fractional programming problems (SVTNLFPPS). In Hussian et al.'s approach (2018), firstly, a single-valued triangular neutrosophic linear fractional programming problem (SVTNLFPP) is transformed into its equivalent crisp multi-objective linear fractional programming problem (CrMOLFPP). Then, the obtained CrMOLFPP is transformed into its equivalent CrMOLPP. After that, the obtained CrMOLPP is transformed into its equivalent CrLPP. Finally, it is assumed that an optimal solution of the transformed CrLPP also represents an optimal solution of SVTNLFPP.

Abdel-Basset et al. (2019a), firstly, proposed a method for comparing two SVTrNNS. Then, using the proposed comparing method, Abdel-Basset et al. (2019a) proposed an approach to solve single-valued trapezoidal neutrosophic linear programming problems (SVTrNLPPS). In Abdel-Basset et al.'s approach (2019a), firstly, a single-valued trapezoidal neutrosophic linear programming problem (SVTrNLPP) is transformed into its equivalent CrLPP. Finally, it is assumed that an optimal solution of the transformed CrLPP also represents an optimal solution of SVTrNLPP.

Singh et al. (2019) pointed out that some mathematical incorrect results are considered in Abdel-Basset et al.'s approach (2019a). Hence, it is inappropriate to use Abdel-Basset et al.'s approach (2019a) in its present form. Singh et al. (2019) also suggested some modifications to resolve the inappropriateness of Abdel-Basset et al.'s approach (2019a).

Abdel-Basset et al. (2019b) proposed an approach to solve SVTNLFPPS. In Abdel-Basset et al.'s approach (2019b), firstly, a SVTNLFPP is transformed into its equivalent CrMOLFPP. Then, the obtained CrMOLFPP is transformed into its equivalent CrMOLPP. After that, the obtained CrMOLPP is transformed into its equivalent CrLPP. Finally, it is assumed that an optimal solution of the transformed CrLPP also represents an optimal solution of SVTNLFPP.

Nafei and Nasseri (2019), firstly, proposed a method for comparing two SVTNNS. Then, using the proposed comparing method, Nafei and Nasseri (2019) proposed an approach to solve single-valued triangular neutrosophic integer programming problems (SVTNIPPS). In Nafei and Nasseri's approach (2019), firstly, a single-valued triangular neutrosophic integer

programming problem (SVTNIPP) is transformed into its equivalent crisp integer programming problem (CrIPP). Finally, it is assumed that an optimal solution of the transformed CrIPP also represents an optimal solution of SVTNIPP.

Das and Dash (2020) pointed out that it is inappropriate to use Hussian et al.'s approach (2017) for solving SVTNLPPS. Das and Dash (2020) also suggested to use Nafei and Nasseri's approach (2019) for solving SVTNLPPS.

Das and Edalatpanah (2020) pointed out that a mathematical incorrect result is considered in Nafei and Nasseri's approach (2019). Hence, it is inappropriate to use Nafei and Nasseri's approach (2019). Das and Edalatpanah (2020) also proposed an approach to solve SVTNIPPS. In Das and Edalatpanah's approach (2020), firstly, a SVTNIPP is transformed into its equivalent CrIPP. Finally, it is assumed that an optimal solution of the transformed CrIPP also represents an optimal solution of SVTNIPP.

Khatter (2020) pointed out that although several approaches are proposed in the literature to solve SVNLPPS. However, all the methods for comparing SVNNS, used in existing approaches, are independent from the attitude of the decision maker towards the risk. To fill this gap, Khatter (2020), firstly, proposed a method for comparing two SVNNS by considering the attitude of the decision maker towards the risk. Then, using the proposed comparing method, Khatter (2020) proposed an approach to solve SVNLPPS. In Khatter's approach (2020), a SVNLPP is transformed into its equivalent CrLPP. Finally, it is assumed that an optimal solution of the transformed CrLPP also represents an optimal solution of SVNLPP.

Badr et al. (2020), firstly, proposed a method for comparing two SVTrNNS. Then, using the proposed comparing method, Badr et al. (2020) generalized the crisp two-phase simplex algorithm for solving SVTrNLPPS.

Das et al. (2020) proposed an approach to solve SVTNLFPPS. In this approach, firstly, a SVTNLFPP is split into its equivalent two neutrosophic linear programming problems. Then, the obtained neutrosophic linear programming problems are transformed into their equivalent crisp linear programming problems (CrLPPS). Finally, it is assumed that both optimal solutions of the transformed CrLPPS also represents an optimal solution of SVTNLFPP.

Abdelfattah (2021) proposed an approach to solve SVTNLPPS. In Abdelfattah's approach (2021), firstly, a SVTNLPP is split into two CrLPPS. Then, the obtained CrLPPS are solved independently. Finally, it is assumed that both optimal solutions of the transformed CrLPPS also represents an optimal solution of SVTNLPP.

Kar et al. (2021) proposed a simplex algorithm for solving SVTNLPPS, Badr et al. (2021) proposed a simplex algorithm for solving SVTrNLPPS and Rabie et al. (2021) proposed a two-phase simplex algorithm for solving SVTrNLPPS.

Das et al. (2021) proposed an approach to solve SVTrNLPPS. In this approach, firstly, a SVTrNLPP is transformed into its equivalent CrMOLPP. Then, using a lexicographic approach, the transformed CrMOLPP is solved. Finally, it is assumed that an efficient solution of the transformed CrMOLPP also represents an optimal solution of SVTrNLPP.

ElHadidi et al. (2021a), firstly, proposed a method for comparing two SVTrNNS. Then, using the proposed comparing method, ElHadidi et al. (2021a) proposed an approach to solve SVTrNLPPS. In ElHadidi et al.'s approach (2021a), firstly, a SVTrNLPP is transformed into its equivalent CrLPP. Finally, it is assumed that an optimal solution of the transformed CrLPP also represents an optimal solution of SVTrNLPP.

ElHadidi et al. (2021b) proposed an approach to solve single-valued trapezoidal neutrosophic linear fractional programming problems (SVTrNLFPPS). In ElHadidi et al.'s approach (2021b), firstly, a single-valued trapezoidal neutrosophic linear fractional programming problem (SVTrNLFPP) is transformed into its equivalent CrMOLFPP. Then, the obtained CrMOLFPP is transformed into its equivalent CrMOLPP. After that, the obtained CrMOLPP is transformed into its equivalent CrLPP. Finally, it is assumed that an optimal solution of the transformed CrLPP also represents an optimal solution of SVTrNLFPP.

Das and Edalatpanah (2022) proposed an approach to solve SVTNLFPPS. In Das and Edalatpanah's approach (2022), firstly, a SVTNLFPP is transformed into its equivalent crisp linear fractional programming problem. Then, the obtained crisp linear fractional programming problem is transformed into its equivalent CrLPP. Finally, it is assumed that an optimal solution of the transformed CrLPP also represents an optimal solution of SVTNLFPP.

In this paper, it is shown that some mathematical incorrect results are considered in all existing approaches for solving mathematical programming problems under neutrosophic environment. Hence, it is inappropriate to use existing approaches for solving mathematical programming problems under neutrosophic environment. Also, a new approach (named as Mehar approach) is proposed to solve SVNLPPS.

This paper is organized as follows. In Section 2, some basic concepts related to neutrosophic set theory are reviewed. In Section 3, it is pointed out that it is inappropriate to use existing approaches for solving mathematical programming problems under neutrosophic

environment. In Section 4, a new approach (named as Mehar approach) is proposed to solve SVNLPPS. In Section 5, correct optimal solution of some existing SVNLPPS are obtained by the proposed Mehar approach. Section 6 concludes the paper.

#### 2. Preliminaries

In this section, some basic definitions are reviewed.

**Definition 1** (Wang et al. 2010) Let X be a universal set. Then, the set  $\tilde{A} = \{\langle x, T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x) \rangle : x \in X\}$ , defined over the universal set X, is said to be a single-valued neutrosophic set, where  $T_{\tilde{A}} : X \to [0,1], I_{\tilde{A}} : X \to [0,1]$  and  $F_{\tilde{A}} : X \to [0,1]$  represents the truth, indeterminacy and falsity membership functions respectively. Also,  $0 \le T_{\tilde{A}}(x) + I_{\tilde{A}}(x) + F_{\tilde{A}}(x) \le 3 \ \forall x \in \tilde{A}$ .

**Definition 2** (Deli and Subas 2014) A single-valued neutrosophic set  $\tilde{A} = (a_{\tilde{A}}^1, a_{\tilde{A}}^2, a_{\tilde{A}}^3; w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}})$ , where  $0 \le w_{\tilde{A}} \le 1, 0 \le u_{\tilde{A}} \le 1, 0 \le y_{\tilde{A}} \le 1, 0 \le w_{\tilde{A}} + u_{\tilde{A}} + y_{\tilde{A}} \le 3$ , is said to be single-valued triangular neutrosophic number (SVTNN) if its membership functions are defined as

$$T_{\tilde{A}}(x) = \begin{cases} w_{\tilde{A}}\left(\frac{x-a_{\tilde{A}}^{1}}{a_{\tilde{A}}^{2}-a_{\tilde{A}}^{1}}\right), & a_{\tilde{A}}^{1} \leq x < a_{\tilde{A}}^{2}, \\ w_{\tilde{A}}, & x = a_{\tilde{A}}^{2}, \\ w_{\tilde{A}}\left(\frac{a_{\tilde{A}}^{3}-x}{a_{\tilde{A}}^{3}-a_{\tilde{A}}^{2}}\right), & a_{\tilde{A}}^{2} < x \leq a_{\tilde{A}}^{3}, \\ 0, & \text{otherwise} \end{cases}$$

$$I_{\tilde{A}}(x) = \begin{cases} \frac{a_{\tilde{A}}^{2}-x+u_{\tilde{A}}(x-a_{\tilde{A}}^{1})}{a_{\tilde{A}}^{2}-a_{\tilde{A}}^{1}}, & a_{\tilde{A}}^{1} \leq x < a_{\tilde{A}}^{2}, \\ u_{\tilde{A}}, & x = a_{\tilde{A}}^{2}, \\ \frac{x-a_{\tilde{A}}^{2}+u_{\tilde{A}}(a_{\tilde{A}}^{3}-x)}{a_{\tilde{A}}^{2}-a_{\tilde{A}}^{2}}, & a_{\tilde{A}}^{2} < x \leq a_{\tilde{A}}^{3}, \\ 1, & \text{otherwise} \end{cases}$$

$$F_{\tilde{A}}(x) = \begin{cases} \frac{a_{\tilde{A}}^{2}-x+y_{\tilde{A}}(x-a_{\tilde{A}}^{1})}{a_{\tilde{A}}^{2}-a_{\tilde{A}}^{2}}, & a_{\tilde{A}}^{2} < x \leq a_{\tilde{A}}^{3}, \\ 1, & \text{otherwise} \end{cases}$$

$$F_{\tilde{A}}(x) = \begin{cases} \frac{a_{\tilde{A}}^{2}-x+y_{\tilde{A}}(x-a_{\tilde{A}}^{1})}{a_{\tilde{A}}^{2}-a_{\tilde{A}}^{1}}, & a_{\tilde{A}}^{1} \leq x < a_{\tilde{A}}^{2}, \\ y_{\tilde{A}}, & x = a_{\tilde{A}}^{2}, \\ \frac{x-a_{\tilde{A}}^{2}+y_{\tilde{A}}(a_{\tilde{A}}^{3}-x)}{a_{\tilde{A}}^{3}-a_{\tilde{A}}^{2}}, & a_{\tilde{A}}^{2} < x \leq a_{\tilde{A}}^{3}, \\ 1, & \text{otherwise} \end{cases}$$

**Definition 3** (Deli and Subas 2014) A single-valued neutrosophic set  $\tilde{A} = (a_{\tilde{A}}^1, a_{\tilde{A}}^2, a_{\tilde{A}}^3, a_{\tilde{A}}^4; w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}})$ , where  $0 \le w_{\tilde{A}} \le 1, 0 \le u_{\tilde{A}} \le 1, 0 \le y_{\tilde{A}} \le 1, 0 \le w_{\tilde{A}} + u_{\tilde{A}} + y_{\tilde{A}} \le 3$ , is said to be single-valued trapezoidal neutrosophic number (SVTrNN) if its membership functions are defined as

$$T_{\tilde{A}}(x) = \begin{cases} w_{\tilde{A}} \left(\frac{x-a_{\tilde{A}}^{1}}{a_{\tilde{A}}^{2}-a_{\tilde{A}}^{1}}\right), & a_{\tilde{A}}^{1} \leq x < a_{\tilde{A}}^{2}, \\ w_{\tilde{A}}, & a_{\tilde{A}}^{2} \leq x \leq a_{\tilde{A}}^{3}, \\ w_{\tilde{A}} \left(\frac{a_{\tilde{A}}^{4}-x}{a_{\tilde{A}}^{4}-a_{\tilde{A}}^{3}}\right), & a_{\tilde{A}}^{3} < x \leq a_{\tilde{A}}^{4}, \\ 0, & \text{otherwise} \end{cases}$$

$$I_{\tilde{A}}(x) = \begin{cases} \frac{a_{\tilde{A}}^{2}-x+u_{\tilde{A}}(x-a_{\tilde{A}}^{1})}{a_{\tilde{A}}^{2}-a_{\tilde{A}}^{1}}, & a_{\tilde{A}}^{1} \leq x < a_{\tilde{A}}^{2}, \\ u_{\tilde{A}}, & a_{\tilde{A}}^{2} \leq x \leq a_{\tilde{A}}^{3}, \\ \frac{x-a_{\tilde{A}}^{3}+u_{\tilde{A}}(a_{\tilde{A}}^{4}-x)}{a_{\tilde{A}}^{4}-a_{\tilde{A}}^{3}}, & a_{\tilde{A}}^{3} < x \leq a_{\tilde{A}}^{4}, \\ 1, & \text{otherwise} \end{cases}$$

$$F_{\tilde{A}}(x) = \begin{cases} \frac{a_{\tilde{A}}^{2}-x+y_{\tilde{A}}(x-a_{\tilde{A}}^{1})}{a_{\tilde{A}}^{2}-a_{\tilde{A}}^{1}}, & a_{\tilde{A}}^{3} \leq x \leq a_{\tilde{A}}^{3}, \\ \frac{x-a_{\tilde{A}}^{3}+y_{\tilde{A}}(a_{\tilde{A}}^{4}-x)}{a_{\tilde{A}}^{2}-a_{\tilde{A}}^{1}}, & a_{\tilde{A}}^{2} \leq x \leq a_{\tilde{A}}^{3}, \\ \frac{x-a_{\tilde{A}}^{3}+y_{\tilde{A}}(a_{\tilde{A}}^{4}-x)}{a_{\tilde{A}}^{4}-a_{\tilde{A}}^{3}}, & a_{\tilde{A}}^{3} < x \leq a_{\tilde{A}}^{4}, \\ 1, & \text{otherwise} \end{cases}$$

**Definition 4** (Deli and Subas 2014) Let  $\tilde{A}_1 = \left(a_{\tilde{A}_1}^1, a_{\tilde{A}_1}^2, a_{\tilde{A}_1}^3; w_{\tilde{A}_1}, u_{\tilde{A}_1}, y_{\tilde{A}_1}\right)$  and  $\tilde{A}_2 = \left(a_{\tilde{A}_2}^1, a_{\tilde{A}_2}^2, a_{\tilde{A}_2}^3; w_{\tilde{A}_2}, u_{\tilde{A}_2}, y_{\tilde{A}_2}\right)$  be two SVTNNS. Then,  $\tilde{A}_1 \oplus \tilde{A}_2 = \left(a_{\tilde{A}_1}^1 + a_{\tilde{A}_2}^1, a_{\tilde{A}_1}^2 + a_{\tilde{A}_2}^2, a_{\tilde{A}_1}^3 + a_{\tilde{A}_2}^2; min\left(w_{\tilde{A}_1}, w_{\tilde{A}_2}\right), max\left(u_{\tilde{A}_1}, u_{\tilde{A}_2}\right), max\left(y_{\tilde{A}_1}, y_{\tilde{A}_2}\right)\right)$ 

**Definition 5** (Deli and Subas 2014) Let  $\tilde{A}_1 = (a_{\tilde{A}_1}^1, a_{\tilde{A}_1}^2, a_{\tilde{A}_1}^3, a_{\tilde{A}_1}^4; w_{\tilde{A}_1}, u_{\tilde{A}_1}, y_{\tilde{A}_1})$  and  $\tilde{A}_2 = (a_{\tilde{A}_2}^1, a_{\tilde{A}_2}^2, a_{\tilde{A}_2}^3, a_{\tilde{A}_1}^4; w_{\tilde{A}_2}, u_{\tilde{A}_2}, y_{\tilde{A}_2})$  be two SVTrNNS. Then,  $\tilde{A}_1 \oplus \tilde{A}_2 = (a_{\tilde{A}_1}^1 + a_{\tilde{A}_2}^1, a_{\tilde{A}_1}^2 + a_{\tilde{A}_2}^2, a_{\tilde{A}_1}^3 + a_{\tilde{A}_2}^3, a_{\tilde{A}_1}^4 + a_{\tilde{A}_2}^4; \min(w_{\tilde{A}_1}, w_{\tilde{A}_2}), \max(u_{\tilde{A}_1}, u_{\tilde{A}_2}), \max(y_{\tilde{A}_1}, y_{\tilde{A}_2}))$ 

**Definition 6** (Basumatary and Said 2020) Let  $\tilde{A} = (a_{\tilde{A}}^1, a_{\tilde{A}}^2, a_{\tilde{A}}^3; w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}})$  be a SVTNN and k be a real number. Then,

$$k\tilde{A} = \begin{cases} \left(ka_{\tilde{A}}^{1}, ka_{\tilde{A}}^{2}, ka_{\tilde{A}}^{3}; w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}}\right), & \text{if } k \ge 0; \\ \left(ka_{\tilde{A}}^{3}, ka_{\tilde{A}}^{2}, ka_{\tilde{A}}^{1}; w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}}\right), & \text{if } k < 0. \end{cases}$$

**Definition 7** (Basumatary and Said 2020) Let  $\tilde{A} = (a_{\tilde{A}}^1, a_{\tilde{A}}^2, a_{\tilde{A}}^3, a_{\tilde{A}}^4; w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}})$  be a SVTrNN and k be a real number. Then,

$$k\tilde{A} = \begin{cases} \left(ka_{\tilde{A}}^{1}, ka_{\tilde{A}}^{2}, ka_{\tilde{A}}^{3}, ka_{\tilde{A}}^{4}; w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}}\right), & \text{if } k \ge 0; \\ \left(ka_{\tilde{A}}^{4}, ka_{\tilde{A}}^{3}, ka_{\tilde{A}}^{2}, ka_{\tilde{A}}^{1}; w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}}\right), & \text{if } k < 0. \end{cases}$$

**Definition 8** (Khatter 2020) Let  $\tilde{A}_1 = (a_{\tilde{A}_1}^1, a_{\tilde{A}_1}^2, a_{\tilde{A}_1}^3; w_{\tilde{A}_1}, u_{\tilde{A}_1}, y_{\tilde{A}_1})$  and  $\tilde{A}_2 = (a_{\tilde{A}_2}^1, a_{\tilde{A}_2}^2, a_{\tilde{A}_2}^3; w_{\tilde{A}_2}, u_{\tilde{A}_2}, y_{\tilde{A}_2})$  be two SVTNNS. Then, (i)  $\tilde{A}_1 < \tilde{A}_2$  if  $V(\tilde{A}_1) < V(\tilde{A}_2)$ , (ii)  $\tilde{A}_1 > \tilde{A}_2$  if  $V(\tilde{A}_1) > V(\tilde{A}_2)$ ,

(iii) 
$$\tilde{A}_1 \approx \tilde{A}_2$$
 if  $V(\tilde{A}_1) = V(\tilde{A}_2)$ .

where,

(a) 
$$V(\tilde{A}_{i}) = \lambda \left(\frac{a_{\tilde{A}_{i}}^{1} + 4a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}}{6}\right) w_{\tilde{A}_{i}}^{2} + (1 - \lambda) \left(\frac{\left[2\left(a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) - \left(a_{\tilde{A}_{i}}^{1} - 2a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right)u_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 4a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right)u_{\tilde{A}_{i}}^{2}\right]}{6} + \frac{\left[2\left(a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) - \left(a_{\tilde{A}_{i}}^{1} - 2a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right)y_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 4a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right)y_{\tilde{A}_{i}}^{2}\right]}{6}\right), \lambda \in [0, 1]; i = 1, 2.$$

- (b)  $\lambda$  reflects the attitude of the decision maker towards the risk.
- (c)  $\lambda \in [0, 0.5)$  indicates that the expert is risk taker and gives preference to uncertainty.
- (d)  $\lambda = 0.5$  indicates that the expert is neutral about deciding the parameters of SVTNLPP problem.
- (e) λ ∈ (0.5, 1] indicates that the expert is risk aversive about deciding the parameters of SVTNLPP problem and gives preference to certainty.

**Definition 9** (Khatter 2020) Let  $\tilde{A}_1 = (a_{\tilde{A}_1}^1, a_{\tilde{A}_1}^2, a_{\tilde{A}_1}^3, a_{\tilde{A}_1}^4; w_{\tilde{A}_1}, u_{\tilde{A}_1}, y_{\tilde{A}_1})$  and  $\tilde{A}_2 = (a_{\tilde{A}_2}^1, a_{\tilde{A}_2}^2, a_{\tilde{A}_2}^3, a_{\tilde{A}_2}^4; w_{\tilde{A}_2}, u_{\tilde{A}_2}, y_{\tilde{A}_2})$  be two SVTrNNS. Then,

(i) 
$$\tilde{A}_1 \prec \tilde{A}_2$$
 if  $V(\tilde{A}_1) < V(\tilde{A}_2)$ ,  
(ii)  $\tilde{A}_1 > \tilde{A}_2$  if  $V(\tilde{A}_1) > V(\tilde{A}_2)$ ,

(iii) 
$$\tilde{A}_1 \approx \tilde{A}_2$$
 if  $V(\tilde{A}_1) = V(\tilde{A}_2)$ .

where,

$$\begin{split} V(\tilde{A}_{i}) &= \lambda \left( \frac{a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4}}{6} \right) w_{\tilde{A}_{i}}^{2} + (1 - \lambda) \left( \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) u_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) u_{\tilde{A}_{i}}^{2} \right] \\ & - \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) v_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) v_{\tilde{A}_{i}}^{2} \right] \\ & - \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) v_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) v_{\tilde{A}_{i}}^{2} \right] \\ & - \frac{1}{6} \lambda_{i}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) v_{\tilde{A}_{i}}^{2} - \left(a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) v_{\tilde{A}_{i}}^{2} \right] \\ & - \frac{1}{6} \lambda_{i}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) v_{\tilde{A}_{i}}^{2} - \left(a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) v_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) v_{\tilde{A}_{i}}^{2} \right] \\ & - \frac{1}{6} \lambda_{i}^{2} + 2a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{$$

#### 3. Inappropriateness of existing approaches

In this section,

- (i) A mathematical incorrect result, considered in Singh et al.'s approach (2019) and Khatter's approach (2020), is pointed out. It can be easily verified that the same mathematical incorrect result is also considered in the existing approaches (Bera and Mahapatra 2019, Emam et al. 2019, Badr et al. 2020, Nafei et al. 2020, Basumatary and Said 2020, Das and Dash 2020, Das and Edalatpanah 2020, Stephen and Helen 2020, Badr et al. 2021, Rabie et al. 2021, SN and Ulaganathan 2021, ElHadidi et al. 2021a, Wang et al. 2021, Das and Edalatpanah 2022).
- (ii) A mathematical incorrect result, considered in Abdelfattah's approach (2021), is pointed out. It can be easily verified that the same mathematical incorrect result is also considered in the existing approach (Das et al. 2020).
- (iii) A mathematical incorrect result, considered in Das et al.'s approach (2021), is pointed out. It can be easily verified that the same mathematical incorrect result is also considered in the existing approaches (Hussian et al. 2018, Abdel-Basset et al. 2019b, ElHadidi et al. 2021b).
- (iv) A mathematical incorrect result, considered in Kar et al.'s approach (2021), is pointed out.

#### 3.1 Inappropriateness of Singh et al.'s approach

In Singh et al.'s approach (2019), firstly, the SVTrNLPP ( $P_1$ ) is transformed into the CrLPP ( $P_2$ ). Then, the CrLPP ( $P_2$ ) is transformed into the CrLPP ( $P_3$ ). After that, the CrLPP ( $P_3$ ) is transformed into the CrLPP ( $P_4$ ). Finally, it is assumed that an optimal solution of the CrLPP ( $P_4$ ) also represents an optimal solution of the SVTrNLPP ( $P_1$ ).

# SVTrNLPP (P<sub>1</sub>)

Maximize/Minimize  $\left(\sum_{j=1}^{n} \left(c_{\tilde{c}_{j}}^{1}, c_{\tilde{c}_{j}}^{2}, c_{\tilde{c}_{j}}^{3}, c_{\tilde{c}_{j}}^{4}; w_{\tilde{c}_{j}}, u_{\tilde{c}_{j}}, y_{\tilde{c}_{j}}\right) x_{j}\right)$ Subject to

$$\begin{split} \sum_{j=1}^{n} \left( a_{\tilde{a}_{ij}}^{1}, a_{\tilde{a}_{ij}}^{2}, a_{\tilde{a}_{ij}}^{3}, a_{\tilde{a}_{ij}}^{4}; w_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}}, y_{\tilde{a}_{ij}} \right) x_{j} (\leqslant, \approx, \geqslant) \left( b_{\tilde{b}_{i}}^{1}, b_{\tilde{b}_{i}}^{2}, b_{\tilde{b}_{i}}^{3}, b_{\tilde{b}_{i}}^{4}; w_{\tilde{b}_{i}}, u_{\tilde{b}_{i}}, y_{\tilde{b}_{i}} \right); i = 1, 2, ..., m, \\ x_{j} \ge 0; \ j = 1, 2, ..., n, \\ \text{where,} \end{split}$$

- (i) *m*: number of constraints.
- (ii) *n*: number of variables.

(iii) 
$$\left(c_{\tilde{c}_{j}}^{1}, c_{\tilde{c}_{j}}^{2}, c_{\tilde{c}_{j}}^{3}, c_{\tilde{c}_{j}}^{4}; w_{\tilde{c}_{j}}, y_{\tilde{c}_{j}}\right)$$
 is a SVTrNN for each  $j = 1, 2, ..., n$ .  
(iv)  $\left(b_{\tilde{b}_{i}}^{1}, b_{\tilde{b}_{i}}^{2}, b_{\tilde{b}_{i}}^{3}, b_{\tilde{b}_{i}}^{4}; w_{\tilde{b}_{i}}, u_{\tilde{b}_{i}}, y_{\tilde{b}_{i}}\right)$  is a SVTrNN for each  $i = 1, 2, ..., m$ .  
(v)  $\left(a_{\tilde{a}_{ij}}^{1}, a_{\tilde{a}_{ij}}^{2}, a_{\tilde{a}_{ij}}^{3}, a_{\tilde{a}_{ij}}^{4}; w_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}}, y_{\tilde{a}_{ij}}\right)$  is a SVTrNN for each  $i = 1, 2, ..., m; j = 1, 2, ..., m$ .

# $CrLPP(P_2)$

Maximize/Minimize  $\left( R\left( \sum_{j=1}^{n} \left( c_{\tilde{c}_{j}}^{1}, c_{\tilde{c}_{j}}^{2}, c_{\tilde{c}_{j}}^{3}, c_{\tilde{c}_{j}}^{4}; w_{\tilde{c}_{j}}, u_{\tilde{c}_{j}}, y_{\tilde{c}_{j}} \right) x_{j} \right) \right)$ 

Subject to

$$R\left(\sum_{j=1}^{n} \left(a_{\tilde{a}_{ij}}^{1}, a_{\tilde{a}_{ij}}^{2}, a_{\tilde{a}_{ij}}^{3}, a_{\tilde{a}_{ij}}^{4}; w_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}}, y_{\tilde{a}_{ij}}\right) x_{j}\right) (\leq , =, \geq) R\left(b_{\tilde{b}_{i}}^{1}, b_{\tilde{b}_{i}}^{2}, b_{\tilde{b}_{i}}^{3}, b_{\tilde{b}_{i}}^{4}; w_{\tilde{b}_{i}}, u_{\tilde{b}_{i}}, y_{\tilde{b}_{i}}\right); i$$
$$= 1, 2, \dots, m,$$

 $x_j \ge 0; \ j = 1, 2, ..., n,$ 

where,

(i) 
$$R(\tilde{A}) = \left(\frac{a_{\tilde{A}}^1 + 2(a_{\tilde{A}}^2 + a_{\tilde{A}}^3) + a_{\tilde{A}}^4}{2}\right) + (w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}})$$
, if the problem is of maximization.  
(ii)  $R(\tilde{A}) = \left(\frac{a_{\tilde{A}}^1 - 3(a_{\tilde{A}}^2 + a_{\tilde{A}}^3) + a_{\tilde{A}}^4}{2}\right) + (w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}})$ , if the problem is of minimization.

# $CrLPP(P_3)$

 $\begin{aligned} \text{Maximize/Minimize} \left( \sum_{j=1}^{n} R\left( c_{\tilde{c}_{j}}^{1}, c_{\tilde{c}_{j}}^{2}, c_{\tilde{c}_{j}}^{3}, c_{\tilde{c}_{j}}^{4}; w_{\tilde{c}_{j}}, u_{\tilde{c}_{j}}, y_{\tilde{c}_{j}} \right) x_{j} - \sum_{j=1}^{n} w_{\tilde{c}_{j}x_{j}} + \sum_{j=1}^{n} u_{\tilde{c}_{j}x_{j}} + \\ \sum_{j=1}^{n} y_{\tilde{c}_{j}x_{j}} + \min_{1 \le j \le n} \left( w_{\tilde{c}_{j}x_{j}} \right) - \max_{1 \le j \le n} \left( u_{\tilde{c}_{j}x_{j}} \right) - \max_{1 \le j \le n} \left( y_{\tilde{c}_{j}x_{j}} \right) \end{aligned}$ 

Subject to

Constraints of the CrLPP  $(P_2)$ .

# $CrLPP(P_4)$

 $\begin{aligned} \text{Maximize/Minimize} \left( \sum_{j=1}^{n} R\left( c_{\tilde{c}_{j}}^{1}, c_{\tilde{c}_{j}}^{2}, c_{\tilde{c}_{j}}^{3}, c_{\tilde{c}_{j}}^{4}; w_{\tilde{c}_{j}}, u_{\tilde{c}_{j}}, y_{\tilde{c}_{j}} \right) x_{j} - \sum_{j=1}^{n} w_{\tilde{c}_{j}x_{j}} + \sum_{j=1}^{n} u_{\tilde{c}_{j}x_{j}} + \\ \sum_{j=1}^{n} y_{\tilde{c}_{j}x_{j}} + \min_{1 \le j \le n} \left( w_{\tilde{c}_{j}x_{j}} \right) - \max_{1 \le j \le n} \left( u_{\tilde{c}_{j}x_{j}} \right) - \max_{1 \le j \le n} \left( y_{\tilde{c}_{j}x_{j}} \right) \end{aligned}$ 

Subject to

$$\sum_{j=1}^{n} \left( R\left(a_{\tilde{a}_{ij}}^{1}, a_{\tilde{a}_{ij}}^{2}, a_{\tilde{a}_{ij}}^{3}, a_{\tilde{a}_{ij}}^{4}; w_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}}, y_{\tilde{a}_{ij}}\right) x_{j} \right) (\leq, =, \geq) R\left(b_{\tilde{b}_{i}}^{1}, b_{\tilde{b}_{i}}^{2}, b_{\tilde{b}_{i}}^{3}, b_{\tilde{b}_{i}}^{4}; w_{\tilde{b}_{i}}, u_{\tilde{b}_{i}}, y_{\tilde{b}_{i}}\right); i$$
$$= 1, 2, \dots, m,$$
$$x_{j} \geq 0; \quad j = 1, 2, \dots, n.$$

It is pertinent to mention that Singh et al. (2019) have used the relation  $R(\tilde{A}_1 \oplus \tilde{A}_2) = R(\tilde{A}_1) + R(\tilde{A}_2)$  to transform the CrLPP ( $P_3$ ) into the CrLPP ( $P_4$ ). While, the following example clearly indicates that  $R(\tilde{A}_1 \oplus \tilde{A}_2) \neq R(\tilde{A}_1) + R(\tilde{A}_2)$  i.e., the CrLPP ( $P_4$ ) is not equivalent to the CrLPP ( $P_3$ ). Hence, it is inappropriate to use Singh et al.'s approach (2020).

Let  $\tilde{A}_1 = (10,20,30,40; 0.8,0.5,0.3)$  and  $\tilde{A}_2 = (30,50,70,90; 0.7,0.3,0.2)$  be two SVTrNNS. Then, using Definition 5, discussed in Section 2,

$$\tilde{A}_1 \oplus \tilde{A}_2 = (10 + 30,20 + 50,30 + 70,40 + 90; \min(0.8,0.7), \max(0.5,0.3), \max(0.3,0.2))$$
  
= (40,70,100,130; 0.7,0.5,0.3)

Therefore, using the existing expression (Abdel-Basset et al. 2019a),

$$R(\tilde{A}_{i}) = \left(\frac{a_{\tilde{A}_{i}}^{1} + 2\left(a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) + a_{\tilde{A}_{i}}^{4}}{2}\right) + \left(w_{\tilde{A}_{i}} - u_{\tilde{A}_{i}} - y_{\tilde{A}_{i}}\right),$$

$$R(\tilde{A}_{1} \oplus \tilde{A}_{2}) = R(40,70,100,130; 0.7,0.5,0.3) = 254.9.$$

$$R(\tilde{A}_{1}) = R(10,20,30,40; 0.8,0.5,0.3) = 75.$$

$$R(\tilde{A}_{2}) = R(30,50,70,90; 0.7,0.3,0.2) = 180.2.$$
Hence,

$$R(\tilde{A}_1) + R(\tilde{A}_2) = 255.2.$$
 (2)

It is obvious from (1) and (2) that  $R(\tilde{A}_1 \oplus \tilde{A}_2) \neq R(\tilde{A}_1) + R(\tilde{A}_2)$ .

# 3.2 Inappropriateness of Khatter's approach

In Khatter's approach (2019), firstly, the SVTrNLPP ( $P_1$ ) is transformed into the CrLPP ( $P_5$ ). Then, the CrLPP ( $P_5$ ) is transformed into the CrLPP ( $P_6$ ). Finally, it is assumed that an optimal solution of the CrLPP ( $P_6$ ) also represent an optimal solution of the SVTrNLPP ( $P_1$ ).

# $CrLPP(P_5)$

Maximize/Minimize  $\left(V\left(\sum_{j=1}^{n}\left(c_{\tilde{c}_{j}}^{1},c_{\tilde{c}_{j}}^{2},c_{\tilde{c}_{j}}^{3},c_{\tilde{c}_{j}}^{4};w_{\tilde{c}_{j}},u_{\tilde{c}_{j}},y_{\tilde{c}_{j}}\right)x_{j}\right)\right)$ 

Subject to

$$V\left(\sum_{j=1}^{n} \left(a_{\tilde{a}_{ij}}^{1}, a_{\tilde{a}_{ij}}^{2}, a_{\tilde{a}_{ij}}^{3}, a_{\tilde{a}_{ij}}^{4}; w_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}}, y_{\tilde{a}_{ij}}\right) x_{j}\right) (\leq =, \geq) V\left(b_{\tilde{b}_{i}}^{1}, b_{\tilde{b}_{i}}^{2}, b_{\tilde{b}_{i}}^{3}, b_{\tilde{b}_{i}}^{4}; w_{\tilde{b}_{i}}, u_{\tilde{b}_{i}}, y_{\tilde{b}_{i}}\right); i$$
$$= 1, 2, \dots, m,$$

 $x_j \ge 0; \ j = 1, 2, ..., n,$ where,

$$\begin{split} V(\tilde{A}_{i}) &= \lambda \left( \frac{a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4}}{6} \right) w_{\tilde{A}_{i}}^{2} + (1 - \lambda) \left( \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) u_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) u_{\tilde{A}_{i}}^{2} \right] \\ & - \left[ \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}}^{2} \right] \\ & - \left[ \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}}^{2} \right] \\ & - \left[ \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}}^{2} \right]}{6} \right), \lambda \in [0, 1]$$

#### $CrLPP(P_6)$

Maximize/Minimize  $\left(\sum_{j=1}^{n} V\left(c_{\tilde{c}_{j}}^{1}, c_{\tilde{c}_{j}}^{2}, c_{\tilde{c}_{j}}^{3}, c_{\tilde{c}_{j}}^{4}; w_{\tilde{c}_{j}}, u_{\tilde{c}_{j}}, y_{\tilde{c}_{j}}\right) x_{j}\right)$ 

Subject to

$$\sum_{j=1}^{n} \left( V\left(a_{\tilde{a}_{ij}}^{1}, a_{\tilde{a}_{ij}}^{2}, a_{\tilde{a}_{ij}}^{3}, a_{\tilde{a}_{ij}}^{4}; w_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}}, y_{\tilde{a}_{ij}}\right) x_{j} \right) (\leq, =, \geq) V\left(b_{\tilde{b}_{i}}^{1}, b_{\tilde{b}_{i}}^{2}, b_{\tilde{b}_{i}}^{3}, b_{\tilde{b}_{i}}^{4}; w_{\tilde{b}_{i}}, u_{\tilde{b}_{i}}, y_{\tilde{b}_{i}}\right); i$$
$$= 1, 2, \dots, m,$$

 $x_j \ge 0; \ j = 1, 2, \dots, n.$ 

It is pertinent to mention that Khatter (2020) has used the relation  $V(\tilde{A}_1 \oplus \tilde{A}_2) = V(\tilde{A}_1) + V(\tilde{A}_2)$  to transform the CrLPP ( $P_5$ ) into the CrLPP ( $P_6$ ). While, the following example clearly indicates that  $V(\tilde{A}_1 \oplus \tilde{A}_2) \neq V(\tilde{A}_1) + V(\tilde{A}_2)$  i.e., the CrLPP ( $P_6$ ) is not equivalent to the CrLPP ( $P_5$ ). Hence, it is inappropriate to use Khatter's approach (2020).

Let  $\tilde{A}_1 = (30,40,50,70; 0.7,0.4,0.3)$  and  $\tilde{A}_2 = (40,50,60,70; 0.6,0.5,0.2)$  be two SVTrNNS. Then, using Definition 5, discussed in Section 2,  $\tilde{A}_1 \oplus \tilde{A}_2 = (30 + 40,40 + 50,50 + 60,70 + 70; \min(0.7,0.6), \max(0.4,0.5), \max(0.3,0.2))$ 

$$= (70,90,110,140;0.6,0.5,0.3).$$

Therefore, using the existing expression (Khatter 2020),

$$\begin{split} V(\tilde{A}_{i}) &= \lambda \left( \frac{a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4}}{6} \right) w_{\tilde{A}_{i}}^{2} + (1 - \lambda) \left( \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) u_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) u_{\tilde{A}_{i}}^{2} \right] + \\ \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}}^{2} \right] \right) \\ \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}}^{2} \right] \right) \\ \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}}^{2} \right] \right) \\ \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}}^{2} \right] \right) \\ \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}}^{2} \right] \right) \\ \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}}^{2} \right) \right] \\ \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{3} \right) - \left(a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{3} \right) y_{\tilde{A}_{i}}^{2} \right) \right] \\ \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{$$

 $= 83.83 - 60.96\lambda$ .

 $V(\tilde{A}_2) = V(40,50,60,70; 0.6,0.5,0.2) = 19.8\lambda + (1 - \lambda)(41.25 + 52.8) = 94.05 - 74.25\lambda$ Hence,

(4)

 $V(\tilde{A}_1) + V(\tilde{A}_2) = 177.88 - 135.21\lambda.$ 

It is obvious from (3) and (4) that  $V(\tilde{A}_1 \bigoplus \tilde{A}_2) \neq V(\tilde{A}_1) + V(\tilde{A}_2)$ .

#### 3.3 Inappropriateness of Abdelfattah's approach

Abdelfattah (2021) claimed that on solving the SVTNLPP ( $P_7$ ), the results presented in Table 1, are obtained.

#### SVTNLPP $(P_7)$

Maximize  $((30,40,50; 0.7,0.4,0.3)x_1 \oplus (40,50,60; 0.6,0.5,0.2)x_2)$ 

Subject to

 $(0.5,1,3; 0.6,0.4,0.1)x_1 \oplus (0,2,6; 0.6,0.4,0.1)x_2 \leq (20,40,60; 0.4,0.3,0.5),$ 

 $(1,4,12; 0.4,0.3,0.2)x_1 \oplus (1,3,10; 0.7,0.4,0.3)x_2 \leq (100,120,140; 0.7,0.4,0.3),$ 

 $x_1, x_2 \ge 0.$ 

#### Table 1 Optimal solutions and optimal values (Abdelfattah 2021)

(α,β,γ)	$x_{1(lpha,eta,\gamma)}^{B^*}$	$x^{B^*}_{2(\alpha,\beta,\gamma)}$	$x_{1(\alpha,\beta,\gamma)}^{W^*}$	$x^{W^*}_{2(lpha,eta,\gamma)}$	$Z^{B^*}_{(\alpha,\beta,\gamma)}$	$Z^{W^*}_{(lpha,eta,\gamma)}$
(0,0.5,0.5)	33.64	19.48	12.53	2.24	2523	554.62
(0,1,1)	0	140	6.67	0	8400	200
(0.4,0.5,0.5)	28.43	12.79	16.46	3.68	1874	793.07
(0.4,1,1)	0	106.31	9.36	0	6201	294.29
(0.2,0.8,0.7)	0	77.07	11.42	0.37	4334	398.46

It is pertinent to mention that as in the problem  $(P_7)$ ,  $x_1$  and  $x_2$  are considered as nonnegative real numbers. So, the obtained optimal values of  $x_1$  and  $x_2$  should be same for all values of  $\alpha, \beta, \gamma$ . While, it is obvious from Table 1 that the values of  $x_1$  and  $x_2$  are different for different values of  $\alpha, \beta, \gamma$ . This clearly indicates that  $x_1$  and  $x_2$ , obtained by Abdelfattah's approach (2021), are not non-negative real numbers. Hence, it is inappropriate to use Abdelfattah's approach (2021).

#### 3.4 Inappropriateness of Das et al.'s approach

It is pertinent to mention that in one of the steps of Das et al.'s approach (2021), the scalar multiplication  $\lambda \tilde{A} = (\lambda a_{\tilde{A}}^1, \lambda a_{\tilde{A}}^2, \lambda a_{\tilde{A}}^3, \lambda a_{\tilde{A}}^4; \lambda w_{\tilde{A}}, \lambda u_{\tilde{A}}, \lambda y_{\tilde{A}}), \lambda > 0$ , is used to transform the SVTrNLPP ( $P_1$ ) into the SVTrNLPP ( $P_8$ ).

#### SVTrNLPP (P<sub>8</sub>)

Maximize/Minimize  $\left(\sum_{j=1}^{n} \left(c_{\tilde{c}_{j}}^{1} x_{j}, c_{\tilde{c}_{j}}^{2} x_{j}, c_{\tilde{c}_{j}}^{3} x_{j}, c_{\tilde{c}_{j}}^{4} x_{j}; w_{\tilde{c}_{j}} x_{j}, u_{\tilde{c}_{j}} x_{j}, y_{\tilde{c}_{j}} x_{j}\right)\right)$ 

$$\sum_{j=1}^{n} \left( a_{\tilde{a}_{ij}}^{1} x_{j}, a_{\tilde{a}_{ij}}^{2} x_{j}, a_{\tilde{a}_{ij}}^{3} x_{j}, a_{\tilde{a}_{ij}}^{4} x_{j}; w_{\tilde{a}_{ij}} x_{j}, u_{\tilde{a}_{ij}} x_{j}, y_{\tilde{a}_{ij}} x_{j} \right) (\leq, \approx,$$
  
$$\geq) \left( b_{\tilde{b}_{i}}^{1}, b_{\tilde{b}_{i}}^{2}, b_{\tilde{b}_{i}}^{3}, b_{\tilde{b}_{i}}^{4}; w_{\tilde{b}_{i}}, u_{\tilde{b}_{i}}, y_{\tilde{b}_{i}} \right); \ i = 1, 2, ..., m,$$

 $x_j \ge 0; \ j = 1, 2, \dots, n.$ 

However, this scalar multiplication is not valid as the following clearly indicates that the number  $(\lambda a_{\tilde{A}}^1, \lambda a_{\tilde{A}}^2, \lambda a_{\tilde{A}}^3, \lambda a_{\tilde{A}}^4; \lambda w_{\tilde{A}}, \lambda u_{\tilde{A}}, \lambda y_{\tilde{A}})$  is not a SVTrNN. Hence, it is inappropriate to use Das et al.'s approach (2021).

According to Definition 4, the number  $(\lambda a_{\tilde{A}}^1, \lambda a_{\tilde{A}}^2, \lambda a_{\tilde{A}}^3, \lambda a_{\tilde{A}}^4; \lambda w_{\tilde{A}}, \lambda u_{\tilde{A}}, \lambda y_{\tilde{A}})$  will be a SVTrNN if

- (i)  $\lambda a_{\tilde{A}}^1 \leq \lambda a_{\tilde{A}}^2 \leq \lambda a_{\tilde{A}}^3 \leq \lambda a_{\tilde{A}}^4$
- (ii)  $0 \le \lambda w_{\tilde{A}} \le 1, 0 \le \lambda u_{\tilde{A}} \le 1, 0 \le \lambda y_{\tilde{A}} \le 1$
- (iii)  $0 \le \lambda w_{\tilde{A}} + \lambda u_{\tilde{A}} + \lambda y_{\tilde{A}} \le 3$

While,

- (i)  $0 \le w_{\tilde{A}} \le 1, 0 \le u_{\tilde{A}} \le 1, 0 \le y_{\tilde{A}} \le 1 \Rightarrow 0 \le \lambda w_{\tilde{A}} \le \lambda, 0 \le \lambda u_{\tilde{A}} \le \lambda, 0 \le \lambda y_{\tilde{A}} \le \lambda$  $\lambda$  i.e., the necessary condition  $0 \le \lambda w_{\tilde{A}} \le 1, 0 \le \lambda u_{\tilde{A}} \le 1, 0 \le \lambda y_{\tilde{A}} \le 1$  is not satisfying.
- (ii)  $0 \le w_{\tilde{A}} + u_{\tilde{A}} + y_{\tilde{A}} \le 3 \Rightarrow 0 \le \lambda w_{\tilde{A}} + \lambda u_{\tilde{A}} + \lambda y_{\tilde{A}} \le 3\lambda$  i.e., the necessary condition  $0 \le \lambda w_{\tilde{A}} + \lambda u_{\tilde{A}} + \lambda y_{\tilde{A}} \le 3$  is not satisfying.

#### 3.5 Inappropriateness of Kar et al.'s approach

It pertinent to mention that in one of the steps of Kar et al.'s approach (2021), it is assumed that if  $\tilde{A}_1 = (a_{\tilde{A}_1}^1, a_{\tilde{A}_1}^2, a_{\tilde{A}_1}^3; a_{\tilde{A}_1}^4, a_{\tilde{A}_1}^5, a_{\tilde{A}_1}^6; a_{\tilde{A}_1}^7, a_{\tilde{A}_1}^8, a_{\tilde{A}_1}^9)$  and  $\tilde{A}_2 = (a_{\tilde{A}_2}^1, a_{\tilde{A}_2}^2, a_{\tilde{A}_2}^3; a_{\tilde{A}_2}^4, a_{\tilde{A}_2}^5; a_{\tilde{A}_2}^7, a_{\tilde{A}_2}^8, a_{\tilde{A}_2}^9)$  are two SVTNNS. Then,  $\frac{\tilde{A}_1}{\tilde{A}_2} = (\frac{a_{\tilde{A}_1}^1}{a_{\tilde{A}_2}^2}, \frac{a_{\tilde{A}_1}^3}{a_{\tilde{A}_2}^2}; a_{\tilde{A}_2}^3; a_{\tilde{A}_2}^6; a_{\tilde{A}_2}^7, a_{\tilde{A}_2}^8, a_{\tilde{A}_2}^9)$  will also be a SVTNN. While, the following clearly indicates that  $(\frac{a_{\tilde{A}_1}^1}{a_{\tilde{A}_2}^3}, \frac{a_{\tilde{A}_1}^2}{a_{\tilde{A}_2}^2}, \frac{a_{\tilde{A}_1}^3}{a_{\tilde{A}_2}^3}; \frac{a_{\tilde{A}_1}^4}{a_{\tilde{A}_2}^3}; \frac{a_{\tilde{A}_1}^3}{a_{\tilde{A}_2}^3}; \frac{a_{\tilde{A}_1}^4}{a_{\tilde{A}_2}^3}; \frac{a_{\tilde{A}_1}^4}{a_{\tilde{A}_2}^3}; \frac{a_{\tilde{A}_1}^3}{a_{\tilde{A}_2}^3}; \frac{a_{\tilde{A}_1}^3}{a_{\tilde{A}_2}^3}; \frac{a_{\tilde{A}_1}^4}{a_{\tilde{A}_2}^3}; \frac{a_$ 

Let  $\tilde{A}_1 = (1,2,5;6,7,8;9,10,11)$  and  $\tilde{A}_2 = (2,3,4;8,9,10;11,12,13)$  be two SVTNNS. Then,  $\frac{\tilde{A}_1}{\tilde{A}_2} = \left(\frac{1}{2}, \frac{2}{3}, \frac{5}{4}; \frac{6}{8}, \frac{7}{9}, \frac{8}{10}; \frac{9}{11}, \frac{10}{12}, \frac{11}{13}\right) = (0.5,0.67,1.25;0.75,0.78,0.8;0.81,0.83,0.85)$  is not a SVTNN as the necessary condition  $\frac{a_{\tilde{A}_1}}{a_{\tilde{A}_2}^1} \le \frac{a_{\tilde{A}_1}^2}{a_{\tilde{A}_2}^2} \le \frac{a_{\tilde{A}_1}^3}{a_{\tilde{A}_2}^3} \le \frac{a_{\tilde{A}_1}^5}{a_{\tilde{A}_2}^5} \le \frac{a_{\tilde{A}_1}^6}{a_{\tilde{A}_2}^5} \le \frac{a_{\tilde{A}_1}^7}{a_{\tilde{A}_2}^7} \le \frac{a_{\tilde{A}_1}^8}{a_{\tilde{A}_2}^7} \le \frac{a_{\tilde{A}_1}^8}{a_{\tilde{A}_2}^7}$ 

**Remark 1:** It can be easily verified that the shortcoming, pointed out by Singh et al. (2019) in Abdel-Basset et al.'s approach (2019a), also occurs in the existing approaches (Emam et al. 2020, Lachhwani 2021). Hence, it is inappropriate to use the existing approaches (Emam et al. 2020, Lachhwani 2021).

#### 4. Proposed Mehar approach

In this section, a new approach (named as Mehar approach) is proposed to solve the SVTrNLPP ( $P_1$ ). The proposed Mehar approach can also be used to solve SVTNLPPS.

**Step 1:** Using Definition 7, discussed in Section 2, transform the SVTrNLPP ( $P_1$ ) into its equivalent SVTrNLPP ( $P_9$ ).

#### SVTrNLPP (P9)

Maximize/Minimize  $\left(\sum_{j=1}^{n} \left(c_{\tilde{c}_{j}}^{1} x_{j}, c_{\tilde{c}_{j}}^{2} x_{j}, c_{\tilde{c}_{j}}^{3} x_{j}, c_{\tilde{c}_{j}}^{4} x_{j}; w_{\tilde{c}_{j}}, u_{\tilde{c}_{j}}, y_{\tilde{c}_{j}}\right)\right)$ 

Subject to

$$\sum_{j=1}^{n} \left( a_{\tilde{a}_{ij}}^{1} x_{j}, a_{\tilde{a}_{ij}}^{2} x_{j}, a_{\tilde{a}_{ij}}^{3} x_{j}, a_{\tilde{a}_{ij}}^{4} x_{j}; w_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}}, y_{\tilde{a}_{ij}} \right) (\leq, \approx, \geq) \left( b_{\tilde{b}_{i}}^{1}, b_{\tilde{b}_{i}}^{2}, b_{\tilde{b}_{i}}^{3}, b_{\tilde{b}_{i}}^{4}; w_{\tilde{b}_{i}}, u_{\tilde{b}_{i}}, y_{\tilde{b}_{i}} \right); i$$
$$= 1.2, \dots, m.$$

 $x_j\geq 0;\ j=1,2,\ldots,n.$ 

**Step 2:** Using Definition 5, discussed in Section 2, transform the SVTrNLPP ( $P_9$ ) into its equivalent SVTrNLPP ( $P_{10}$ ).

#### SVTrNLPP (P<sub>10</sub>)

Maximize/

 $\text{Minimize} \left( \sum_{j=1}^{n} c_{\tilde{c}_{j}}^{1} x_{j}, \sum_{j=1}^{n} c_{\tilde{c}_{j}}^{2} x_{j}, \sum_{j=1}^{n} c_{\tilde{c}_{j}}^{3} x_{j}, \sum_{j=1}^{n} c_{\tilde{c}_{j}}^{4} x_{j}; \min_{1 \le j \le n} \left( w_{\tilde{c}_{j}} \right), \max_{1 \le j \le n} \left( u_{\tilde{c}_{j}} \right), \max_{1 \le j \le n} \left( y_{\tilde{c}_{j}} \right) \right)$ 

$$\begin{pmatrix} \sum_{j=1}^{n} a_{\tilde{a}_{ij}}^{1} x_{j}, \sum_{j=1}^{n} a_{\tilde{a}_{ij}}^{2} x_{j}, \sum_{j=1}^{n} a_{\tilde{a}_{ij}}^{3} x_{j}, \sum_{j=1}^{n} a_{\tilde{a}_{ij}}^{4} x_{j}; \min_{\substack{1 \le i \le m \\ 1 \le j \le n}} \left( w_{\tilde{a}_{ij}} \right), \max_{\substack{1 \le i \le m \\ 1 \le j \le n}} \left( u_{\tilde{a}_{ij}} \right), \max_{\substack{1 \le i \le m \\ 1 \le j \le n}} \left( y_{\tilde{a}_{ij}} \right) \end{pmatrix} (\leqslant, n)$$

$$\approx , \geq ) \left( b_{\tilde{b}_{i}}^{1}, b_{\tilde{b}_{i}}^{2}, b_{\tilde{b}_{i}}^{3}, b_{\tilde{b}_{i}}^{4}; w_{\tilde{b}_{i}}, u_{\tilde{b}_{i}}, y_{\tilde{b}_{i}} \right); i = 1, 2, \dots, m,$$

 $x_j \ge 0; \ j = 1, 2, ..., n.$ 

**Step 3:** Using Definition 9, discussed in Section 2, transform the SVTrNLPP ( $P_{10}$ ) into its equivalent CrLPP ( $P_{11}$ ).

#### $CrLPP(P_{11})$

Maximize/

$$\operatorname{Minimize}\left(V\left(\sum_{j=1}^{n} c_{\tilde{c}_{j}}^{1} x_{j}, \sum_{j=1}^{n} c_{\tilde{c}_{j}}^{2} x_{j}, \sum_{j=1}^{n} c_{\tilde{c}_{j}}^{3} x_{j}, \sum_{j=1}^{n} c_{\tilde{c}_{j}}^{4} x_{j}; \min_{1 \le j \le n} \left(w_{\tilde{c}_{j}}\right), \max_{1 \le j \le n} \left(u_{\tilde{c}_{j}}\right), \max_{1 \le j \le n} \left(y_{\tilde{c}_{j}}\right)\right)\right)$$

Subject to

$$V\left(\sum_{j=1}^{n} a_{\tilde{a}_{ij}}^{1} x_{j}, \sum_{j=1}^{n} a_{\tilde{a}_{ij}}^{2} x_{j}, \sum_{j=1}^{n} a_{\tilde{a}_{ij}}^{3} x_{j}, \sum_{j=1}^{n} a_{\tilde{a}_{ij}}^{4} x_{j}; \min_{\substack{1 \le i \le m \\ 1 \le j \le n}} \left(w_{\tilde{a}_{ij}}\right), \max_{\substack{1 \le i \le m \\ 1 \le j \le n}} \left(y_{\tilde{a}_{ij}}\right)\right) \right) (\le, n)$$

$$=, \ge) V\left(b_{\tilde{b}_{i}}^{1}, b_{\tilde{b}_{i}}^{2}, b_{\tilde{b}_{i}}^{3}, b_{\tilde{b}_{i}}^{4}; w_{\tilde{b}_{i}}, u_{\tilde{b}_{i}}, y_{\tilde{b}_{i}}\right); i = 1, 2, ..., m,$$

$$x_{j} \ge 0; \ j = 1, 2, ..., n,$$

where,

$$\begin{split} V(\tilde{A}_{i}) &= \lambda \left( \frac{a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4}}{6} \right) w_{\tilde{A}_{i}}^{2} + (1 - \lambda) \left( \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) u_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) u_{\tilde{A}_{i}}^{2} \right] \\ & - \left[ \frac{\left(2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}}^{2} \right] \\ & - \left[ \frac{\left(2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}}^{2} \right) \\ & - \left[ \frac{\left(2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{3} \right) y_{\tilde{A}_{i}}^{2} \right] \\ & - \left[ \frac{\left(2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) y_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A$$

**Step 4:** Find an optimal solution of the CrLPP  $(P_{11})$  for some values of  $\lambda \in [0,1]$ . The obtained optimal solution also represents an optimal solution of the SVTrNLPP  $(P_1)$ .

#### 5. Correct optimal solution of some existing SVNLPPS

In this section, the correct optimal solution of some existing SVNLPPS is obtained by the proposed Mehar approach.

#### 5.1 Correct optimal solution of some existing SVTNLPPS

Hussian et al. (2017) as well as Khatter (2020) have considered the following real-life problem to illustrate their proposed approach.

A Pottery Company, run by a Native American tribal council, desires to find the number of bowls and mugs to be produced each day in order to maximize the profit by considering

- (i) The data presented in Table 2.
- (ii) The data presented in Table 3.
- (iii) The data presented in Table 4.

However, as some mathematical incorrect results are considered in Hussian et al.'s approach (2017) as well as in Khatter's approach (2020). So, the existing optimal solution (Hussian et al. 2017, Khatter 2020) is not correct. In this section, a correct optimal solution of this real-life problem is obtained by the proposed Mehar approach.

Table 2: Resource requirements of two products

Product	Resource requirements			
	Labour (Hr./unit)	Clay (Lb./unit)	Profit(\$/unit)	
Bowl	(0.5,1,3; 0.6,0.4,0.1)	(1,4,12; 0.4,0.3,0.2)	(30,40,50; 0.7,0.4,0.3)	
Mug	(0,2,6; 0.6,0.4,0.1)	(1,3,10; 0.7,0.4,0.3)	(40,50,60; 0.6,0.5,0.2)	
	Total available hr of	Total available pounds of		
	labour =	clay =		
	(20,40,60; 0.4,0.3,0.5)	(100,120,140; 0.7,0.4,0.3)		

# Table 3: Resource requirements of two products

Product	Resource requirements		
	Labour (Hr./unit)	Clay (Lb./unit)	Profit(\$/unit)
Bowl	(3.5,4,4.1; 0.75,0.5,0.25)	(0,1,2; 0.15,0.5,0)	(4,5,6; 0.5,0.8,0.3)
Mug	(2.5,3,3.2; 0.2,0.8,0.4)	(2.8,3,3.2; 0.75,0.5,0.25)	(2.5,3,3.2; 0.6,0.4,0)
	Total available hr of	Total available pounds of	
	labour =	clay =	
	(11,12,13; 0.2,0.6,0.5)	(5.5,6,7.5; 0.8,0.6,0.4)	

# Table 4: Resource requirements of two products

Product	Resource requirements			
	Skilled Labour	Unskilled Labour	Clay (Lb./unit)	Profit(\$/unit)
	(Hr./unit)	(Hr./unit)		
Bowl	15	24	21	(19,25,33; 0.8,0.1,0.4)
Mug	30	6	14	(44,48,54; 0.75,0.25,0)
	Total available hr	Total available hr	Total available	
	of skilled labour	of unskilled labour	pounds of clay	
	= 45000	= 24000	= 28000	

#### **5.1.1 First illustrative example**

If the data, presented in Table 2, is considered. Then, to find an optimal solution of the real-life problem is equivalent to find an optimal solution of the SVTNLPP ( $P_{12}$ ).

# SVTNLPP $(P_{12})$

Maximize  $((30,40,50; 0.7,0.4,0.3)x_1 \oplus (40,50,60; 0.6,0.5,0.2)x_2)$ 

Subject to

 $\begin{aligned} &(0.5,1,3;\,0.6,0.4,0.1)x_1 \oplus (0,2,6;\,0.6,0.4,0.1)x_2 \leqslant (20,40,60;\,0.4,0.3,0.5), \\ &(1,4,12;\,0.4,0.3,0.2)x_1 \oplus (1,3,10;\,0.7,0.4,0.3)x_2 \leqslant (100,120,140;\,0.7,0.4,0.3), \\ &x_1,x_2 \ge 0. \end{aligned}$ 

Using the proposed Mehar approach, an optimal solution of the SVTNLPP  $(P_{12})$  can be obtained as follows:

**Step 1:** Using Step 1 of the proposed Mehar approach, the SVTNLPP  $(P_{12})$  can be transformed into its equivalent SVTNLPP  $(P_{13})$ .

# SVTNLPP (P<sub>13</sub>)

Maximize  $((30x_1, 40x_1, 50x_1; 0.7, 0.4, 0.3) \oplus (40x_2, 50x_2, 60x_2; 0.6, 0.5, 0.2))$ 

Subject to

 $(0.5x_1, 1x_1, 3x_1; 0.6, 0.4, 0.1) \oplus (0x_2, 2x_2, 6x_2; 0.6, 0.4, 0.1) \leq (20, 40, 60; 0.4, 0.3, 0.5),$ 

 $(1x_1, 4x_1, 12x_1; 0.4, 0.3, 0.2) \oplus (1x_2, 3x_2, 10x_2; 0.7, 0.4, 0.3) \leq (100, 120, 140; 0.7, 0.4, 0.3),$ 

$$x_1, x_2 \ge 0.$$

**Step 2:** Using Step 2 of the proposed Mehar approach, the SVTNLPP  $(P_{13})$  can be transformed into its equivalent SVTNLPP  $(P_{14})$ .

# SVTNLPP (P<sub>14</sub>)

Maximize  $(30x_1 + 40x_2, 40x_1 + 50x_2, 50x_1)$ 

$$+ 60x_2; \min(0.7, 0.6), \max(0.4, 0.5), \max(0.3, 0.2))$$

Subject to

$$(0.5x_1 + 0x_2, 1x_1 + 2x_2, 3x_1 + 6x_2; \min(0.6, 0.6), \max(0.4, 0.4), \max(0.1, 0.1)) \\ \leq (20, 40, 60; 0.4, 0.3, 0.5),$$

$$(1x_1 + 1x_2, 4x_1 + 3x_2, 12x_1 + 10x_2; \min(0.4, 0.7), \max(0.3, 0.4), \max(0.2, 0.3))$$
  
$$\leq (100, 120, 140; 0.7, 0.4, 0.3),$$

 $x_1, x_2 \ge 0.$ 

**Step 3:** Using Step 3 of the proposed Mehar approach, the SVTNLPP  $(P_{14})$  can be transformed into its equivalent CrLPP  $(P_{15})$ .

# $CrLPP(P_{15})$

Maximize  $(V(30x_1 + 40x_2, 40x_1 + 50x_2, 50x_1 + 60x_2; 0.6, 0.5, 0.3))$ Subject to  $V(0.5x_1 + 0x_2, 1x_1 + 2x_2, 3x_1 + 6x_2; 0.6, 0.4, 0.1) \le V(20, 40, 60; 0.4, 0.3, 0.5),$  $V(1x_1 + 1x_2, 4x_1 + 3x_2, 12x_1 + 10x_2; 0.4, 0.4, 0.3) \le V(100, 120, 140; 0.7, 0.4, 0.3),$  $x_1, x_2 \ge 0,$ 

where,

$$\begin{split} V(\tilde{A}_{i}) &= \lambda \left( \frac{a_{\tilde{A}_{i}}^{1} + 4a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}}{6} \right) w_{\tilde{A}_{i}}^{2} + (1 - \lambda) \left( \frac{\left[ 2\left(a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) - \left(a_{\tilde{A}_{i}}^{1} - 2a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) u_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 4a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) u_{\tilde{A}_{i}}^{2}} \right] + \\ \frac{\left[ 2\left(a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) - \left(a_{\tilde{A}_{i}}^{1} - 2a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) y_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 4a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) y_{\tilde{A}_{i}}^{2}} \right]}{6} \end{split} \\ \lambda \in [0, 1]. \end{split}$$

**Step 4:** The obtained optimal solution of the CrLPP ( $P_{15}$ ) for some values of  $\lambda \in [0,1]$  are shown in Table 5. It is pertinent to mention that according to Step 4 of the proposed Mehar approach, the obtained optimal solution also represents an optimal solution of the SVTNLPP ( $P_{12}$ ).

λ	Optimal s	solution
	<i>x</i> <sub>1</sub>	<i>x</i> <sub>2</sub>
0	19.55	3.02
0.1	20.60	2.32
0.2	21.89	1.47
0.3	23.49	0.41
0.4	23.87	0
0.5	23.41	0
0.6	22.79	0
0.7	21.92	0
0.8	20.63	0
0.9	18.48	0
1	14.22	0

Table 5 Correct optimal solution for different values of  $\lambda$ 

# 5.1.2 Second illustrative example

If the data, presented in Table 3, is considered. Then, to find an optimal solution of the real-life problem is equivalent to find an optimal solution of the SVTNLPP ( $P_{16}$ ).

# SVTNLPP ( $P_{16}$ )

Maximize  $((4,5,6; 0.5,0.8,0.3)x_1 \oplus (2.5,3,3.2; 0.6,0.4,0)x_2)$ 

Subject to

 $(3.5,4,4.1; 0.75,0.5,0.25)x_1 \oplus (2.5,3,3.2; 0.2,0.8,0.4)x_2 \leq (11,12,13; 0.2,0.6,0.5),$  $(0,1,2; 0.15,0.5,0)x_1 \oplus (2.8,3,3.2; 0.75,0.5,0.25)x_2 \leq (5.5,6,7.5; 0.8,0.6,0.4),$  $x_1, x_2 \ge 0.$ 

Using the proposed Mehar approach, an optimal solution of the SVTNLPP  $(P_{16})$  can be obtained as follows:

**Step 1:** Using Step 1 of the proposed Mehar approach, the SVTNLPP  $(P_{16})$  can be transformed into its equivalent SVTNLPP  $(P_{17})$ .

#### SVTNLPP (P<sub>17</sub>)

Maximize  $((4x_1, 5x_1, 6x_1; 0.5, 0.8, 0.3) \oplus (2.5x_2, 3x_2, 3.2x_2; 0.6, 0.4, 0))$ 

Subject to

 $(3.5x_1, 4x_1, 4.1x_1; 0.75, 0.5, 0.25) \oplus (2.5x_2, 3x_2, 3.2x_2; 0.2, 0.8, 0.4)$ 

 $\leq$  (11,12,13; 0.2,0.6,0.5),

 $(0x_1, 1x_1, 2x_1; 0.15, 0.5, 0) \oplus (2.8x_2, 3x_2, 3.2x_2; 0.75, 0.5, 0.25) \leq (5.5, 6, 7.5; 0.8, 0.6, 0.4),$  $x_1, x_2 \ge 0.$ 

**Step 2:** Using Step 2 of the proposed Mehar approach, the SVTNLPP  $(P_{17})$  can be transformed into its equivalent SVTNLPP  $(P_{18})$ .

# SVTNLPP (P<sub>18</sub>)

Maximize $(4x_1 + 2.5x_2, 5x_1 + 3x_2, 6x_1 + 3.2x_2; \min(0.5, 0.6), \max(0.8, 0.4), \max(0.3, 0))$ Subject to

$$(3.5x_1 + 2.5x_2, 4x_1 + 3x_2, 4.1x_1 + 3.2x_2; \min(0.75, 0.2), \max(0.5, 0.8), \max(0.25, 0.4)) \\ \leq (11, 12, 13; 0.2, 0.6, 0.5),$$

 $(2.8x_2, x_1 + 3x_2, 2x_1 + 3.2x_2; \min(0.15, 0.75), \max(0.5, 0.5), \max(0, 0.25)) \\ \leq (5.5, 6, 7.5; 0.8, 0.6, 0.4),$ 

 $x_1, x_2 \ge 0.$ 

**Step 3:** Using Step 3 of the proposed Mehar approach, the SVTNLPP ( $P_{18}$ ) can be transformed into its equivalent CrLPP ( $P_{19}$ ).

#### CrLPP (*P*<sub>19</sub>)

Maximize  $(V(4x_1 + 2.5x_2, 5x_1 + 3x_2, 6x_1 + 3.2x_2; 0.5, 0.8, 0.3))$ 

Subject to

 $V(3.5x_1 + 2.5x_2, 4x_1 + 3x_2, 4.1x_1 + 3.2x_2; 0.2, 0.8, 0.4) \le V(11, 12, 13; 0.2, 0.6, 0.5),$  $V(2.8x_2, x_1 + 3x_2, 2x_1 + 3.2x_2; 0.15, 0.5, 0.25) \le V(5.5, 6, 7.5; 0.8, 0.6, 0.4),$   $x_1, x_2 \ge 0,$ 

where,

$$\begin{split} V(\tilde{A}_{i}) &= \lambda \left( \frac{a_{\tilde{A}_{i}}^{1} + 4a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}}{6} \right) w_{\tilde{A}_{i}}^{2} + (1 - \\ \lambda) \left( \frac{\left[ 2\left(a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) - \left(a_{\tilde{A}_{i}}^{1} - 2a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) u_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 4a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) u_{\tilde{A}_{i}}^{2}} \right] + \\ \frac{\left[ 2\left(a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) - \left(a_{\tilde{A}_{i}}^{1} - 2a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) y_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 4a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) y_{\tilde{A}_{i}}^{2}} \right]}{6} \end{split} \\ \lambda \in [0,1]. \end{split}$$

**Step 4:** The obtained optimal solution of the CrLPP ( $P_{19}$ ) for some values of  $\lambda \in [0,1]$  are shown in Table 6. It is pertinent to mention that according to Step 4 of the proposed Mehar approach, the obtained optimal solution also represents an optimal solution of the SVTNLPP ( $P_{16}$ ).

λ	Optimal solution		
	<i>x</i> <sub>1</sub>	<i>x</i> <sub>2</sub>	
0	3.574	0	
0.1	3.572	0	
0.2	3.570	0	
0.3	3.567	0	
0.4	3.563	0	
0.5	3.557	0	
0.6	3.549	0	
0.7	3.536	0	
0.8	3.512	0	
0.9	3.452	0	
1	3.051	0	

	Table 6 Correct o	optimal	solution fo	or different	values of A	λ
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#### 5.1.3 Third illustrative example

If the data, presented in Table 4, is considered. Then, to find an optimal solution of the real-life problem is equivalent to find an optimal solution of the SVTNLPP ( $P_{20}$ ).

# SVTNLPP ( $P_{20}$ )

Maximize( $(19,25,33; 0.8,0.1,0.4)x_1 \oplus (44,48,54; 0.75,0.25,0)x_2$ )

Subject to

 $15x_1 + 30x_2 \le 45000,$ 

 $24x_1 + 6x_2 \le 24000,$ 

 $21x_1 + 14x_2 \le 28000,$ 

 $x_1, x_2 \ge 0.$ 

Using the proposed Mehar approach, an optimal solution of the SVTNLPP  $(P_{20})$  can be obtained as follows:

**Step 1:** Using Step 1 of the proposed Mehar approach, the SVTNLPP  $(P_{20})$  can be transformed into its equivalent SVTNLPP  $(P_{21})$ .

# SVTNLPP (P<sub>21</sub>)

 $Maximize((19x_1, 25x_1, 33x_1; 0.8, 0.1, 0.4) \oplus (44x_2, 48x_2, 54x_2; 0.75, 0.25, 0))$ 

Subject to

Constraints of the problem  $(P_{20})$ .

**Step 2:** Using Step 2 of the proposed Mehar approach, the SVTNLPP  $(P_{21})$  can be transformed into its equivalent SVTNLPP  $(P_{22})$ .

# SVTNLPP (P<sub>22</sub>)

Maximize  $(19x_1 + 44x_2, 25x_1 + 48x_2, 33x_1)$ 

$$+ 54x_2; \min(0.8, 0.75), \max(0.1, 0.25), \max(0.4, 0))$$

Subject to

Constraints of the problem  $(P_{20})$ .

**Step 3:** Using Step 3 of the proposed Mehar approach, the SVTNLPP ( $P_{22}$ ) can be transformed into its equivalent CrLPP ( $P_{23}$ ).

# $CrLPP(P_{23})$

Maximize 
$$(V(19x_1 + 44x_2, 25x_1 + 48x_2, 33x_1 + 54x_2; 0.75, 0.25, 0.4))$$

Subject to

Constraints of the problem  $(P_{20})$ 

where,

$$\begin{split} V\big(\tilde{A}_{i}\big) &= \lambda \left(\frac{a_{\tilde{A}_{i}}^{1} + 4a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}}{6}\right) w_{\tilde{A}_{i}}^{2} + (1 - \\ \lambda) \left(\frac{\left[2\left(a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) - \left(a_{\tilde{A}_{i}}^{1} - 2a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) u_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 4a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) u_{\tilde{A}_{i}}^{2}}\right] + \\ \frac{\left[2\left(a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) - \left(a_{\tilde{A}_{i}}^{1} - 2a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) y_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 4a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3}\right) y_{\tilde{A}_{i}}^{2}}\right]}{6} \end{split}, \lambda \in [0, 1]. \end{split}$$

**Step 4:** The obtained optimal solution of the CrLPP ( $P_{23}$ ) for some values of  $\lambda \in [0,1]$  are shown in Table 7. It is pertinent to mention that according to Step 4 of the proposed Mehar approach, the obtained optimal solution also represents an optimal solution of the SVTNLPP ( $P_{20}$ ).

λ	Optimal solution		
	<i>x</i> <sub>1</sub>	<i>x</i> <sub>2</sub>	
0	500	1250	
0.1	500	1250	
0.2	500	1250	
0.3	500	1250	
0.4	500	1250	
0.5	500	1250	
0.6	500	1250	
0.7	500	1250	
0.8	500	1250	
0.9	500	1250	
1	500	1250	

Table 7 Correct optimal solution for different values of  $\lambda$ 

#### 5.2 Correct optimal solution of an existing SVTrNLPP

Das et al. (2021) have considered the following real-life problem to illustrate their proposed approach.

An electric cable maker desires to find the number of cable 1 and cable 2 to be produced each day in order to maximize the profit by considering the data presented in Table 8.

 Table 8: Resource requirements of two cables

	Metal (meter)	Plastic (meter)	Profit
Cable 1	(2,4,6,8; 0.6,0.1,0.3)	(4,7,10,13; 0.7,0.4,0.2)	(1,3,4,7; 0.8,0.2,0.4)
Cable 2	(3,5,9,12; 0.7,0.2,0.1)	(3,6,9,14; 0.8,0.5,0.3)	(4,6,8,10; 0.9,0.3,0.5)
	Total available meters of	Total available meters of	
	metal =	plastic =	
	(10,15,20,25; 0.6,0,0.5)	(10,20,25,30; 0.9,0.45,0.3)	

However, as some mathematical incorrect results are considered in Das et al.'s approach (2021). So, the existing optimal solution (Das et al. 2021) is not correct. In this section, a correct optimal solution of this real-life problem is obtained by the proposed Mehar approach.

If the data, presented in Table 8, is considered. Then, to find an optimal solution of the real-life problem is equivalent to find an optimal solution of the SVTrNLPP ( $P_{24}$ ).

# SVTrNLPP (P<sub>24</sub>)

Maximize  $((1,3,4,7; 0.8,0.2,0.4)x_1 \oplus (4,6,8,10; 0.9,0.3,0.5)x_2)$ 

Subject to

 $(2,4,6,8;0.6,0.1,0.3)x_1 \oplus (3,5,9,12;0.7,0.2,0.1)x_2 \leq (10,15,20,25;0.6,0,0.5),$ 

 $(4,7,10,13; 0.7,0.4,0.2)x_1 \oplus (3,6,9,14; 0.8,0.5,0.3)x_2 \leq (10,20,25,30; 0.9,0.45,0.3),$ 

 $x_1, x_2 \ge 0.$ 

Using the proposed Mehar approach, an optimal solution of the SVTrNLPP  $(P_{24})$  can be obtained as follows:

**Step 1:** Using Step 1 of the proposed Mehar approach, the SVTrNLPP ( $P_{24}$ ) can be transformed into its equivalent SVTrNLPP ( $P_{25}$ ).

# SVTrNLPP (P<sub>25</sub>)

Maximize  $((x_1, 3x_1, 4x_1, 7x_1; 0.8, 0.2, 0.4) \oplus (4x_2, 6x_2, 8x_2, 10x_2; 0.9, 0.3, 0.5))$ 

Subject to

 $\begin{array}{l} (2x_1, 4x_1, 6x_1, 8x_1; 0.6, 0.1, 0.3) \oplus (3x_2, 5x_2, 9x_2, 12x_2; 0.7, 0.2, 0.1) \\ \leqslant (10, 15, 20, 25; 0.6, 0, 0.5), \end{array}$ 

 $\begin{array}{l} (4x_1, 7x_1, 10x_1, 13x_1; 0.7, 0.4, 0.2) \oplus (3x_2, 6x_2, 9x_2, 14x_2; 0.8, 0.5, 0.3) \\ \leqslant (10, 20, 25, 30; 0.9, 0.45, 0.3), \end{array}$ 

 $x_1, x_2 \ge 0.$ 

**Step 2:** Using Step 2 of the proposed Mehar approach, the SVTrNLPP ( $P_{25}$ ) can be transformed into its equivalent SVTrNLPP ( $P_{26}$ ).

# SVTrNLPP (P<sub>26</sub>)

Maximize $(x_1 + 4x_2, 3x_1 + 6x_2, 4x_1 + 8x_2, 7x_1)$ 

$$+ 10x_2; \min(0.8, 0.9), \max(0.2, 0.3), \max(0.4, 0.5))$$

Subject to

$$(2x_1 + 3x_2, 4x_1 + 5x_2, 6x_1 + 9x_2, 8x_1 + 12x_2; \min(0.6, 0.7), \max(0.1, 0.2), \max(0.3, 0.1)) \\ \leq (10, 15, 20, 25; 0.6, 0, 0.5),$$

$$(4x_1 + 3x_2, 7x_1 + 6x_2, 10x_1 + 9x_2, 13x_1 + 14x_2; \min(0.7, 0.8), \max(0.4, 0.5), \max(0.2, 0.3))$$
$$\leq (10, 20, 25, 30; 0.9, 0.45, 0.3),$$

 $x_1, x_2 \ge 0.$ 

**Step 3:** Using Step 3 of the proposed Mehar approach, the SVTrNLPP ( $P_{26}$ ) can be transformed into its equivalent CrLPP ( $P_{27}$ ).

# $\mathrm{CrLPP}\left(\boldsymbol{P}_{\mathbf{27}}\right)$

Maximize  $(V(x_1 + 4x_2, 3x_1 + 6x_2, 4x_1 + 8x_2, 7x_1 + 10x_2; 0.8, 0.3, 0.5))$ 

Subject to

$$V(2x_1 + 3x_2, 4x_1 + 5x_2, 6x_1 + 9x_2, 8x_1 + 12x_2; 0.6, 0.2, 0.3) \le V(10, 15, 20, 25; 0.6, 0, 0.5),$$
  
$$V(4x_1 + 3x_2, 7x_1 + 6x_2, 10x_1 + 9x_2, 13x_1 + 14x_2; 0.7, 0.5, 0.3)$$

 $\leq V(10,20,25,30;0.9,0.45,0.3),$ 

 $x_1, x_2 \ge 0$ , where,

$$\begin{split} V(\tilde{A}_{i}) &= \lambda \left( \frac{a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4}}{6} \right) w_{\tilde{A}_{i}}^{2} + (1 - \lambda) \left( \frac{\left[ \left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) u_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) u_{\tilde{A}_{i}}^{2} \right] \\ & - \left[ \frac{\left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) v_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) u_{\tilde{A}_{i}}^{2} \right] \\ & - \left[ \frac{\left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) v_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) u_{\tilde{A}_{i}}^{2} \right) \\ & - \left[ \frac{\left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{4} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) v_{\tilde{A}_{i}} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{3} \right) u_{\tilde{A}_{i}}^{2} \right] \\ & - \left[ \frac{\left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{3} \right) - \left(a_{\tilde{A}_{i}}^{1} - a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) u_{\tilde{A}_{i}}^{2} - \left(a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{3} \right) u_{\tilde{A}_{i}}^{2} \right] \right] \\ & - \left[ \frac{\left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{3} \right) - \left(a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{3} \right) u_{\tilde{A}_{i}}^{2} + 2a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{3} \right) u_{\tilde{A}_{i}}^{2} \right] \right] \\ & - \left[ \frac{\left( 2a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{3} + 2a_{\tilde{A}_{i}}^{3} \right) - \left(a_{\tilde{A}_{i}}^{2} - a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{3} \right) u_{\tilde{A}_{i}}^{3} \right] \right] \\ & - \left[ \frac{\left( 2a_{\tilde{A}_{i}}^{1} + 2a_{\tilde{$$

**Step 4:** The obtained optimal solution of the CrLPP ( $P_{27}$ ) for some values of  $\lambda \in [0,1]$  are shown in Table 9. It is pertinent to mention that according to Step 4 of the proposed Mehar approach, the obtained optimal solution also represents an optimal solution of the SVTrNLPP ( $P_{24}$ ).

λ	Optimal solution		
	<i>x</i> <sub>1</sub>	<i>x</i> <sub>2</sub>	
0	0	2.243	
0.1	0	2.248	
0.2	0	2.252	
0.3	0	2.258	
0.4	0	2.266	
0.5	0	2.275	
0.6	0	2.287	
0.7	0	2.304	
0.8	0	2.329	
0.9	0	2.368	
1	0	2.442	

Table 9 Correct optimal solution for different values of  $\lambda$ 

#### 6. Conclusions and future work

It is shown that some mathematical incorrect results are considered in all existing approaches for solving mathematical programming problems under neutrosophic environment. Hence, it is inappropriate to use any existing approach to solve mathematical programming problems under neutrosophic environment. Also, a new approach (named as Mehar approach) is proposed to solve SVNLPPS. Furthermore, correct optimal solutions of some existing real-life problems under neutrosophic environment (Hussian et al. 2017, Khatter 2020, Das et al. 2021) are obtained by the proposed Mehar approach.

The following work may be considered as a future work.

- (i) The proposed Mehar approach may be extended for solving SVTNLFPPS (Hussian et al. 2018, Abdel-Basset et al. 2019b, Das et al. 2020, Das and Edalatpanah 2022) and SVTrNLFPPS (ElHadidi et al. 2021b).
- (ii) It can be easily verified that the relation  $S(\tilde{A}_1 \otimes \tilde{A}_2) = S(\tilde{A}_1) \times S(\tilde{A}_2)$  is considered in Khalifa and Kumar's approach (2020) for solving single-valued trapezoidal fully neutrosophic linear programming problems (linear programming problems in which all the parameters including decision variables are represented by SVTrNNS),

where,

- (a)  $\tilde{A}_i = \left(a_{\tilde{A}_i}^1, a_{\tilde{A}_i}^2, a_{\tilde{A}_i}^3, a_{\tilde{A}_i}^4; w_{\tilde{A}_i}, y_{\tilde{A}_i}\right); i = 1, 2$  is a non-negative SVTrNN i.e.,  $a_{\tilde{A}_i}^1 \ge 0.$
- (b)  $\tilde{A}_1 \otimes \tilde{A}_2 = (a_{\tilde{A}_1}^1 a_{\tilde{A}_2}^1, a_{\tilde{A}_1}^2 a_{\tilde{A}_2}^2, a_{\tilde{A}_1}^3 a_{\tilde{A}_2}^3, a_{\tilde{A}_1}^4 a_{\tilde{A}_2}^4; \min(w_{\tilde{A}_1}, w_{\tilde{A}_2}), \max(u_{\tilde{A}_1}, u_{\tilde{A}_2}), \max(y_{\tilde{A}_1}, y_{\tilde{A}_2})).$

(c) 
$$S(\tilde{A}_i) = \frac{1}{16} \left( a_{\tilde{A}_i}^1 + a_{\tilde{A}_i}^2 + a_{\tilde{A}_i}^3 + a_{\tilde{A}_i}^4 \right) \left( w_{\tilde{A}_i} + \left( 1 - u_{\tilde{A}_i} \right) + \left( 1 - y_{\tilde{A}_i} \right) \right); i = 1, 2.$$

While, the following example clearly indicates that  $S(\tilde{A}_1 \otimes \tilde{A}_2) \neq S(\tilde{A}_1) \times S(\tilde{A}_2)$ .

Let  $\tilde{A}_1 = (1,3,4,5; 0.1,0.8,0.1)$  and  $\tilde{A}_2 = (3,4,6,7; 0.1,0.8,0.9)$  be two SVTrNNS. Then,

$$\tilde{A}_1 \otimes \tilde{A}_2 = (3,12,24,35; \min(0.1,0.1), \max(0.8,0.8), \max(0.1,0.9))$$
  
= (3,12,24,35; 0.1,0.8,0.9).

Therefore, using the existing expression (Khalifa and Kumar 2020),

$$S(\tilde{A}_{i}) = \frac{1}{16} \left( a_{\tilde{A}_{i}}^{1} + a_{\tilde{A}_{i}}^{2} + a_{\tilde{A}_{i}}^{3} + a_{\tilde{A}_{i}}^{4} \right) \left( w_{\tilde{A}_{i}} + \left( 1 - u_{\tilde{A}_{i}} \right) + \left( 1 - y_{\tilde{A}_{i}} \right) \right),$$

$$S(\tilde{A}_{1} \otimes \tilde{A}_{2}) = S(3,12,24,35; 0.1,0.8,0.9)$$

$$= \frac{1}{16} (3 + 12 + 24 + 35) \left( 0.1 + (1 - 0.8) + (1 - 0.9) \right) = 1.85$$

$$S(\tilde{A}_{1}) = S(1,3,4,5; 0.1,0.8,0.1)$$

$$= \frac{1}{16} (1 + 3 + 4 + 5) \left( 0.1 + (1 - 0.8) + (1 - 0.1) \right) = 0.975$$
(5)

$$S(\tilde{A}_2) = S(3,4,6,7; 0.1,0.8,0.9)$$
  
=  $\frac{1}{16}(3 + 4 + 6 + 7)(0.1 + (1 - 0.8) + (1 - 0.9)) = 0.5$   
Hence,

$$S(\tilde{A}_1) \times S(\tilde{A}_2) = 0.975 \times 0.5 = 0.4875$$
 (6)

It is obvious from (5) and (6) that  $S(\tilde{A}_1 \otimes \tilde{A}_2) \neq S(\tilde{A}_1) \times S(\tilde{A}_2)$ .

Hence, it is inappropriate to use Khalifa and Kumar's approach (2020). In future, the proposed Mehar approach may be extended for solving single-valued trapezoidal fully neutrosophic linear programming problems (Khalifa and Kumar 2020).

(iii) It is pertinent to mention that the shortcoming pointed out in Khalifa and Kumar's approach (2020) also occurs in the existing approaches (Bera and Mahapatra 2020a, Bera and Mahapatra 2020b) for solving SVTrNLPPS with single-valued trapezoidal neutrosophic decision variables. Hence, it is inappropriate to use the existing approaches (Bera and Mahapatra 2020a, Bera and Mahapatra 2020b). In future, the proposed Mehar approach may be extended for solving SVTrNLPPS with single-valued trapezoidal neutrosophic decision variables. Gera and Mahapatra 2020b). In future, the proposed Mehar approach may be extended for solving SVTrNLPPS with single-valued trapezoidal neutrosophic decision variables (Bera and Mahapatra 2020b).

#### Compliance with ethical standards

Conflict of interest Author declares that there is no conflicts of interest.

**Human and animal rights** This article does not contain any studies with human participants or animals performed by any of the authors.

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