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Research Article

Keywords: Words: Fractional-order stochastic neural networks, Non-fragile control, Finite-time, Synchronization

Posted Date: May 11th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-1587925/v1

Finite-time non-fragile control for synchronization of fractional-order stochastic neural networks

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Abstract

This study concerned with the synchronization problem in finite-time domain for a class of fractional-order stochastic neural networks via non-fragile controller with discontinuous activation functions. Specifically, the suggested state feedback controller sequentially cope with non-fragile scheme for gain scheduling process. Notably, the main objective of this work is to develop a non-fragile controller to obtained the finite-time synchronization criterion for the resulting synchronized error system over finite bound. For that, some consignment of sufficient conditions is derived systematically by implementing Lyapunovs indirect method and finite-time stability theory which ensures the finite-time stochastic synchronization of the stipulated neural networks. Later on, the controller gain fluctuations that obeying certain white noise sequels derived from Bernoulli distribution are formulated in terms of linear matrix inequalities. Eventually, the illustrated study are substantiated through two numerical examples and the simulation results manifest the advantage and accuracy of the proposed synchronization criteria.

Key Words: Fractional-order stochastic neural networks; Non-fragile control; Finite-time; Synchronization.

I. Introduction

Fractional-order calculus, a natural generalization of integer-order and integration operators has been originally commenced by Riemann and Leibniz [1], and later used in many real process. Scientific researches show that fractional-order approach is the finest description for plentiful natural phenomenon. Moreover, the fractional-order systems has raised promising research attention due to its elegant reflection of dynamical behavior and practical values which confers accurate results [2], [3]. The significant aspect of fractional-order systems are the prominent hereditary properties of diverse materials and superiority of memory [3], [6]. In the present situation, the better understanding of fractional calculus leds to the increasing amount of works on stability and stabilization of fractional-order control systems (see [6]–[8] and the references cited therein).

It is worth noticed that neural networks (NNs) has been extensively inspected owing to its successful applications in parameter estimation, parallel computation, prediction and data analysis, automatic control, artificial intelligence, and so on [4], [5], [9]–[11]. Owing to their viability, neural networks has become a powerful tool to delineate the neurodynamics in human brains with higher quality. Generally, the accuracy can be increased while analyzing the dynamical behavior of neural networks by incorporating the fractional calculus. As mentioned from the above facts and infinite memory property, a huge amount of significant results on the dynamics of fractional-order neural networks (FNNs) systems embracing stability, stabilization and synchronization have been reported, for example

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see [9]–[12]. Namely, Zhang et al. [9] examined the asymptotic stability analysis of Riemann-Liouville derivative fractional based NNs in the presence of distributed and discrete delays. Based on the theory of impulsive and fractional differential equation, Yang et al. [12] presented global synchronization of Mittag-Leffler derivative based fractional-order neural works.

Synchronization is an universal phenomenon which was proposed first by Pecora and Carroll, subsequently, it is enhanced to analyze diverse fields, namely, cryptography, secure communication, image encryption, information sciences and so on [13]–[17]. Consequently, synchronization of two or more systems adjust each other and may leads to common dynamical behavior. However, the study on synchronization of FNNs cannot be ignored and a lot of interesting results have been reported through various control approaches [18], [19], [26]. For instance, the authors proposed globally asymptotically synchronization analysis for FNNs subject to multiple time-delay in [19]. As it is well known that the stochastic noise is usually unavoidable in practice and it is one of the crucial factor which affects the system performance. So far, the synchronization of fractional-order stochastic neural networks (FSNNs) have not been examined fully. This stimulates our research attention towards synchronization of FSNNs.

It should be pointed out that, in reality, the imprecision are frequently inevitable in the controller deployment while designing a practical systems. Obviously, these unmodelled drifts may probably cause performance degradation, round off errors in numerical computation, random perturbation in controller and inherent inaccuracies in simulations [20], [21]. To overwhelm this consequences, it is fundamental to implement a controller that will be able to incorporate some tuning parameters to tolerate the possible gain variations during the synchronization of the underlying system. Desperately, such form of controller are pointed as non-fragile (or resilient) and some important results has been found (see [21]–[25] and references therein). To be more precious, in [25], the synchronization issue with non-fragile control problem for discontinuous NNs subject to randomly occurring controller gain fluctuation and time-varying delay has been investigated, whereas the global asymptotic synchronization has been achieved.

An another important front line in research is finite-time horizon, which is an effective framework for faster synchronization in control systems and it provokes optimality in convergence time. Further, the introduction of finite-time synchronization criterion has revealed the faster convergence rate, better robustness and disturbance rejection performance [27]–[30]. In recent years, a huge enumerates of interesting results based on finite-time synchronization have been reported; for instance, see [31]–[33]. Recently, in [33], the issue of synchronization in finite-time has been concerned for fuzzy cellular NNs under differential inclusions framework. Despite that, the synchronization of FSNNs under non-fragile controller design has not yet been inspected thoroughly over finite-time domain which provokes us to commence the present study.

Accordant from the abovesaid observations, we develop herein the non-fragile controller design for finite-time synchronization of FSNNs with nonlinear activation function over finite interval. Moreover, the crucial benefactions of this study can be encapsulated in the following form:

- Finite-time synchronization of FSNNs under non-fragile controller strategy is investigated.
- The proposed non-fragile state feedback controller assures the synchronization of FSNNs within finite-time domain.
- Sufficient conditions for FSNNs are framed by using Lyapunov technique and the desired non-fragile state feedback gain fluctuations is retrieved in respect of linear matrix inequalities together with neuron nonlinear activation function.
- Finally, two numerical examples along with corresponding simulations delineates the applicability and com-

pactness under the designed control strategy.

II. Problem statement and preliminaries

Consider the FSNNs that can be characterized by the following fractional differential equation:

$$\begin{aligned} &{}^{C}D^{\nu}x(t) = -(A + \Delta A(t))x(t) + B\phi(x(t), \\ &x(0) = 0, \end{aligned} \tag{1}$$

where ${}^{C}D^{\nu}$ denotes the Caputo fractional derivative which takes the commensurate order ν $(0 < \nu \leq 1)$; $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T \in R^n$ represents the n-neuronal state vector; $\phi(x(t) = [\phi_1(x_1(t), \phi_2(x_2(t)), \dots, \phi_n(x_n(t))]^T$ denotes the nonlinear neuron activation function with $\phi(0) = 0$. Moreover, $A = a_i > 0$ is the self-connection diagonal matrix and $B = (b_{lk})_{n\times n}$ is the interconnection weight matrix which represents the synaptic connection for the unit l on the unit k at time t. Matrix $\Delta A(t)$ is unknown time-varying norm bounded uncertain matrix. Let $\Delta A(t) = SG(t)J$, where S and J are known real constant matrices and $G(t)$ is time-varying matrix that satisfies $G^T(t)G(t) \leq I$ for all $t \geq 0$.

The noise free system (1) is the drive system and its corresponding response system can be governed by

$$\begin{aligned} &{}^{C}D^{\nu}y(t) = [-(A + \Delta A(t))y(t) + B\phi(y(t)) + Cu(t)] + g(t, y(t))\tfrac{dB^H(t)}{dt}, \\ &y(0) = 0, \end{aligned} \tag{2}$$

where, C depicts the input weight matrix, $u(t) \in R^p$ represents the control input and $g(t, y(t))$ is the noise intensity function.

It is a well-known fact that design of non-fragile controller is established to reduce some amount of parameter variations and fluctuations of the controller without loss of its robustness. Due to clear engineering insights, the non-fragile state feedback control incorporated in this study is represented as

$$u(t) = (K + \Delta K(t))\bar{x}(t), \tag{3}$$

where K is the state feedback controller gain which is to be computed, $\Delta K(t)$ represents the controller gain fluctuations and possess the norm-bounded additive configuration as $\Delta K(t) = U\Phi(t)W$, here U and W are known real constant matrices and unknown norm-bounded function $\Phi(t)$ which is limited as $\Phi^T(t)\Phi(t) \leq I$.

Further, it is to be noted that the synchronization error signal can be denoted as $\bar{x}(t) = x(t) - y(t)$. Thus, the dynamical error system from (1) and (2) with the proposed control strategy (3) can be expressed as follows:

$${}^{C}D^{\nu}\bar{x}(t) =[-(A + \Delta A(t))\bar{x}(t) + B\ell(\bar{x}(t)) + C(K + \Delta K(t))\bar{x}(t)] + g(t, \bar{x}(t))\frac{dB^H(t)}{dt}, \tag{4}$$

where, $\ell(\bar{x}(t)) = \phi(y(t)) - \phi(x(t))$. Further, the stochastic noise is defined over the Wiener process probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where Ω, $\mathcal{F}$ and $\mathcal{P}$ reveals the sample space, algebra of the events and the probability measure which is defined on $[0, T]$ with $1/2 < H < 1$. The function $g(t, \bar{x}(t)) : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is the vector-valued noise intensity function and $B^H(t)$ denotes the fractional Brownian motion of Hurst parameter H. A centered zero mean one-parameter family of Gaussian process $B^H(t)$ on the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with filtration χ_t has covariance $\mathbb{E}\{B^H(t)B^H(s)\} = 1/2(t^{2H} + s^{2H} - |t - s|^{2H})$ for $s \neq t$, where $\mathbb{E}\{.\}$ is the mathematical expectation.

Hereinafter, the following definitions, lemmas and assumption are mandatory for smooth development of finite-time synchronization criterion.

Definition 1: [34] The Riemann-Liouville fractional integral with ν order for the univariate function $x(t)$ is defined as follows:

$$^{C}D^{\nu}x(t) = \frac{1}{\Gamma(k-\nu)}\frac{d^k}{dt^k}\int_0^1 (t-\nu)^{k-\nu-1}x(u)du,$$

where, k is the first integer greater than ν, i.e, $k-1 < \nu < k$, $k \in Z^+$ and $\Gamma(.)$ is the Gamma function and takes $\Gamma(p) = \int_0^\infty t^{p-1}e^{-t}dt$.

Definition 2: [35] The Caputo fractional derivative of order ν for the differentiable function $x(t)$ is expressed as

$$^{C}D^{\nu}x(t) = \frac{1}{\Gamma(k-\nu)}\int_0^1 (t-\nu)^{k-\nu-1}x^{(k)}(u)du,$$

where k is the first-order integer which is bigger than ν. i.e., $k-1 < \nu < k$, $k \in Z^+$ and $x^{(k)}(.)$ is the classical k^{th} derivative.

Definition 3: [35] The Mittag-Leffler function for one parameter $\nu > 0$ is defined as $F_\nu(z) = \sum_{i=0}^{\infty}\frac{z^i}{\Gamma(i\nu+1)}$, where $z \in C$. The Mittag-Leffler function with two parameters $\nu > 0$ and $\beta > 0$ is defined as $F_{\nu,\beta}(z) = \sum_{i=0}^{\infty}\frac{z^i}{\Gamma(i\nu+\beta)}$, where $z \in C$. For $\beta = 1$, one has $F_\nu(z)$=$F_{\nu,1}(z)$ and also $\nu = 1, \beta = 1$ one has further $F_{1,1}(z) = e^z$.

Definition 4: For a given positive definite matrix Q and positive constants c_1, c_2, T^f with $c_1 < c_2$, the fractional-order error system (4) under the controller (3) is said be stochastically synchronized in finite-time, if (c_1, c_2, d_2, T^f, Q) and $\bar{x}^T(0)Q\bar{x}(0) \leq c_1 \Longrightarrow \bar{x}^T(t)Q\bar{x}(t) < c_2, \ \forall t \in [0, T^f]$.

Lemma 1: [36] For any real matrix D, S and F of appropriate dimensions and $F^TF \leq I$. Then for any scalar $\epsilon > 0$ and vectors $x, y \in \mathbb{R}^n$, one has the following inequality:

$$2x^TDF(t)Sy \leq \epsilon^{-1}x^TDD^Tx + \epsilon y^TS^TSy.$$

Assumption 1: For any distinct $x, y \in R, x \neq y$, the neuron activation function $\phi_i(.)(i = 1, \dots, n)$ in (1) is assumed to be continuously bounded, nondecreasing and globally Lipschitz and accomplishes $\chi_i^- \leq \frac{\phi_i(x)-\phi_i(y)}{x-y} \leq \chi_i^+, \ i = 1, \dots, n$, where χ_i^- and χ_i^+ are known real constants which takes either positive, negative or zero. For the purpose of proclamation, we express $\chi_1 = diag\{\chi_1^-\chi_1^+, \dots, \chi_n^-\chi_n^+\}$ and $\chi_2 = diag\{\frac{\chi_1^-+\chi_1^+}{2}, \dots, \frac{\chi_n^-+\chi_n^+}{2}\}$.

Assumption 2: Given $\epsilon_1 > 0$, $\epsilon_2 > 0$, the matrix-valued functions $\Delta A(t)$ and $\Delta K(t)$ are assumed to be norm-bounded such that $\|\Delta A(t)\| = \epsilon_1\zeta_a(t)$ and $\|\Delta K(t)\| = \epsilon_2\zeta_k(t)$, where $\zeta_a(t) \in R^n$ and $\zeta_k(t) \in R^n$ with $\|\zeta(t)\| < 1$.

III. Main results

In this section, we derive the finite-time synchronization criterion for FSNNs (1) under the proposed non-fragile controller. For carrying out the above consideration, the upcoming theorem substantiate a quantity of sufficient condition in respect of LMIs to ease the closed-loop error system (4) of FSNNs (1) to be stochastically synchronized over finite-time interval.

Theorem 1: Suppose that Assumption 1 is satisfied, then for a given positive scalars ϵ, ϱ, ν, c_1, c_2, δ and T^f with $c_2 > c_1$, the FSNNs (1) under the controller (3) achieves the stochastic finite-time synchronization criterion with respect to $(c_1, c_2, d_2, T^f, \Psi)$, if there exist a symmetric matrix X, positive diagonal matrix $\hat{\Gamma}_1$ with appropriate dimensions and appropriate dimensioned matrix Y such that the following conditions hold:

$$\begin{bmatrix} -AX - XA^T + \varrho X - \delta X + CY + Y^TC^T - \chi_1\hat{\Gamma}_1 & BX + \chi_2\hat{\Gamma}_1 & -\epsilon_1 S & XJ^T & \epsilon_2 CU & XW^T \\ * & -\hat{\Gamma}_1 & 0 & 0 & 0 & 0 \\ * & * & -\epsilon_1 I & 0 & 0 & 0 \\ * & * & * & -\epsilon_1 I & 0 & 0 \\ * & * & * & * & -\epsilon_2 I & 0 \\ * & * & * & * & * & -\epsilon_2 I \end{bmatrix} < 0, \quad (5)$$

$$\lambda_2 c_1 F_\nu(\delta t^\nu) < \lambda_1 c_2. \quad (6)$$

Furthermore, the anticipated non-fragile controller gain fluctuation is procured by employing the relation $K = YX^{-1}$.

Proof: To obtain the finite-time synchronization criterion for FSNNs (1), let us construct the Lyapunov function for synchronized error system (4) in the following configuration:

$$V(t) = \bar{x}^T(t)P\bar{x}(t). \quad (7)$$

Further, based on Property 1 in [34], the fractional-order Caputo derivative function for $V(t)$ is given as

$$\begin{aligned} {}^C D^\nu V(t) =& [{}^R D^\nu \bar{x}(t)]^T P\bar{x}(t) + \bar{x}^T(t)P[{}^R D^\nu \bar{x}(t)] + P\sum_{k=1}^{\infty} \frac{\Gamma(1+\nu)}{\Gamma(1+k)\Gamma(1-k+\nu)} {}^R D^k \bar{x}(t) {}^R D^{\nu-k}\bar{x}(t) \\ &- \frac{t^{-\nu}P}{\Gamma(1-\nu)}\bar{x}^T(0)\bar{x}(0), \end{aligned}$$

where ${}^R D^\nu$ signifies the Riemann-Liouville fractional-order derivative. For representation convenience, we denote $\Xi_{\bar{x}(t)} = \sum_{k=1}^{\infty} \frac{\Gamma(1+\nu)}{\Gamma(1+k)\Gamma(1-k+\nu)} {}^R D^k \bar{x}(t) {}^R D^{\nu-k}\bar{x}(t)$ such that $\Xi_{\bar{x}(t)} \leq \varrho\|\bar{x}(t)\|^2$, where ϱ is a positive scalar. Since, $\frac{t^{-\nu}P}{\Gamma(1-\nu)}\|\bar{x}(0)\|^2 > 0$ and further replacing the Caputo fractional derivative in place of the Riemann-Liouville fractional derivative, it can be easy to attain that

$$\begin{aligned} {}^C D^\nu V(t) =& [{}^C D^\nu \bar{x}(t)]^T P\bar{x}(t) + \bar{x}^T(t)P[{}^C D^\nu \bar{x}(t)] + P\sum_{k=1}^{\infty} \frac{\Gamma(1+\nu)}{\Gamma(1+k)\Gamma(1-k+\nu)} {}^C D^k \bar{x}(t) {}^C D^{\nu-k}\bar{x}(t) \\ &- \frac{t^{-\nu}P}{\Gamma(1-\nu)}\bar{x}^T(0)\bar{x}(0). \end{aligned}$$

Further, based on Itô's formula with respect to fractional Brownian motion [37] and from the trajectory of synchronized error system (4), the above equation can be rephrased as

$$\begin{aligned} {}^C D^\nu V(t) - \delta V(t) \leq& \bar{x}^T(t)(-P(A+\Delta A(t)) - (A+\Delta A(t))^T P + PC(K+\Delta K(t)) + (K+\Delta K(t))^T C^T P - \delta P \\ &+ \varrho P)\bar{x}(t) + \ell^T(\bar{x}(t),t)B^T P\bar{x}(t) + \bar{x}^T(t)PB\ell(\bar{x}(t),t) + 2\bar{x}^T(t)Pg(t,\bar{x}(t))dB^H(t). \end{aligned} \quad (8)$$

Then, by taking mathematical expectation on both sides of (8), we can obtain that

$$\begin{aligned} \mathbb{E}[{}^C D^\nu V(t)] \leq& \bar{x}^T(t)(-P(A+\Delta A(t)) - (A+\Delta A(t))^T P + PC(K+\Delta K(t)) + (K+\Delta K(t))^T C^T P - \delta P \\ &+ \varrho P)\bar{x}(t) + \ell^T(\bar{x}(t),t)B^T P\bar{x}(t) + \bar{x}^T(t)PB\ell(\bar{x}(t),t). \end{aligned} \quad (9)$$

On the other hand, with the aid of Assumption 1, for the diagonal matrices $\beta_i > 0$ $i = 1, \ldots, n$, one has the following:

$$[\ell(\bar{x}(t)) - \chi_i^- \bar{x}(t)]\beta_i[\chi_i^+ \bar{x}(t) - \ell(\bar{x}(t))] \geq 0. \tag{10}$$

Now, from the equations (9) and (10), we can attain

$$\mathbb{E}[{}^C D^\nu V(t)] \leq \xi^T(t)\Theta\xi(t), \tag{11}$$

where $\xi^T(t) = \begin{bmatrix} \bar{x}^T(t) & l^T(\bar{x}(t)) \end{bmatrix}^T$ and $\Theta = \begin{bmatrix} -PA - A^TP - \delta P + \varrho P + K^TC^TP + PCK - \chi_1\Gamma_1 & PB + \chi_2\Gamma_1 \\ * & -\Gamma_1 \end{bmatrix}$ $- SG(t)J - (SG(t)J)^TP + PCU\Phi(t)W + (U\Phi(t)W)^TC^TP$.

Further, in light of Lemma (1) and Schur complement, Θ can be remodeled as

$$\Theta = \begin{bmatrix} -PA - A^TP + \varrho P - \delta P + K^TC^TP + PCK - \chi_1\Gamma_1 & PB + \chi_2\Gamma_1 & -\epsilon_1 PS & J^T & \epsilon_2 PCU & W^T \\ * & -\Gamma_1 & 0 & 0 & 0 & 0 \\ * & * & -\epsilon_1 I & 0 & 0 & 0 \\ * & * & * & -\epsilon_1 I & 0 & 0 \\ * & * & * & * & -\epsilon_2 I & 0 \\ * & * & * & * & 0 & -\epsilon_2 I \end{bmatrix}. \tag{12}$$

It is noted that the Θ is not strictly in LMIs format, so we need to transform the inequality constraints into LMIs to attain the control gain fluctuation. Considering this fact, first pre- and post-multiplying the right-hand side of (12) with $diag\{P^{-1}, P^{-1}, I, I, I, I\}$ and assume that $P^{-1} = X$. Secondly, denote a new variable as $P^{-1}\Gamma_1 P^{-1} = \hat{\Gamma}_1$ and $Y = KX$. Thus, it is easy to spot from right hand side of (12) that left hand side of (5) is arrived. Thus, if the condition (5) holds, then ${}^C D^\nu V(t) - \delta V(t) < 0$. Suppose that there exists a nonnegative function $L(t)$, the inequality can be rephrased as ${}^C D^\nu V(t) + L(t) - \delta V(t) = 0$. Then, applying Laplace transform, we get

$$\delta V(s) = s^\nu V(s) - V(0)s^{\nu-1} + L(s) \Rightarrow V(s) = (V(0)s^{\nu-1} - L(s))/(s^\nu - \delta). \tag{13}$$

Further, taking inverse Laplace transform to (13), we procure $V(t) = V(0)F_\nu(\delta t^\nu) - \int_0^1 j(u)(t-u)^{\nu-1}F_{\nu,\nu}(\delta(t-u)^\nu)du$. Since $(t-u)^{\nu-1}$ and $F_{\nu,\nu}(\delta(t-u)^\nu)$ are non negative functions, then we get

$$V(t) \leq V(0)F_\nu(\delta t^\nu) \tag{14}$$

Let $\tilde{P} = \Psi^{-1/2}P\Psi^{-1/2}, \lambda_1 = \lambda_{min}(\tilde{P})$ and $\lambda_2 = \lambda_{max}(\tilde{P})$, we can obtain

$$V(t) = x^T(t)\Psi^{1/2}\tilde{P}\Psi^{1/2}x(t) \geq \lambda_{min}(\tilde{P})x^T(t)\Psi x(t) = \lambda_1 x^T(t)\Psi x(t). \tag{15}$$

Moreover, $V(0)F_\nu(\delta t^\nu) = x^T(0)Px(0) = x^T(0)\Psi^{1/2}\tilde{P}\Psi^{1/2}x(0) \leq \lambda_{max}(\tilde{P})x^T(0)\Psi x(0)$. If $x^T(0)\Psi x(0) \leq c_1$, then the above inequality can be easily written as

$$V(0)F_\nu(\delta t^\nu) \leq \lambda_2 c_1 F_\nu(\delta t^\nu). \tag{16}$$

Further, from the inequalities (14), (15) and (16), we can attain $\lambda_1 x^T(t)\Psi x(t) < V(t) < V(0)F_\nu(\delta t^\nu) < \lambda_2 c_1 F_\nu(\delta t^\nu)$. Hence, $x^T(t)\Psi x(t) < \lambda_2 c_1 F_\nu(\delta t^\nu)/\lambda_1$. Therefore, if the condition (6) holds, then it is clear that

$x^T(t)\Psi x(t) < c_2$, for all $t \in [0, T^f]$. Thus, from Definition 4, it can be easily deduced FSNNs (1) is stochastically synchronized within finite-time interval through the controller (3). ■

In the forthcoming theorem, we derive the finite-time synchronization of FSNNs (1) by assuming the system parameter and control gain variation with norm bounded uncertainty which is defined in Assumption (2).

Theorem 2: With the assistance of Assumption 1 and 2, the FSNNs (1) under the controller (3) achieves the stochastic finite-time synchronization criterion with respect to $(c_1, c_2, d_2, T^f, \Psi)$, if there exist a matrix X, diagonal matrix $\hat{\Gamma}_1$, any appropriate matrix Y and some positive scalars $\epsilon_1,\ \epsilon_2,\ \varrho,\ \nu,\ c_1,\ c_2,\ \delta$ and T^f with $c_2 > c_1$, such that the following conditions hold:

$$\begin{bmatrix} -X^TA^T - AX + \rho X - \delta X + CZ + Z^TC^T - G_1\hat{\Gamma}_1 & BX + G_2\hat{\Gamma}_1 & -I & I & X & X \\ * & -\hat{\Gamma}_1 & 0 & 0 & 0 & 0 \\ * & * & -\epsilon_1 & 0 & 0 & 0 \\ * & * & * & -\epsilon_2 & 0 & 0 \\ * & * & * & * & -\frac{1}{\epsilon_1 {\zeta_a}^2} & 0 \\ * & * & * & * & * & -\frac{1}{\epsilon_2 {\zeta_k}^2} \end{bmatrix} < 0. \tag{17}$$

$$\lambda_2 c_1 F_\nu(\delta t^\nu) < \lambda_1 c_2. \tag{18}$$

Moreover, the controller gain fluctuation is acquired by the relation $K = YX^{-1}$.

Proof: To derive the anticipated result, we assume that Assumption 2 holds for $\Delta A(t)$ and $\Delta K(t)$. Then, by following the similar lines of Theorem 1, (11) can be rewritten as

$$E[{}^CD^\nu V(t)] \le \xi^T\Theta_1\xi, \tag{19}$$

where $\xi^T = \begin{bmatrix} \bar{x}^T(t) & l^T(\bar{x}(t)) \end{bmatrix}^T$, $\Theta_1 = \begin{bmatrix} \theta_{11} & PB + \chi_2\Gamma_1 \\ * & -\Gamma_1 \end{bmatrix} + \epsilon_1^{-1}\bar{x}^T(t)\bar{x}(t) + \epsilon_2^{-1}\bar{x}^T(t)\bar{x}(t)$ and $\theta_{11} = -PA - A^TP - \delta P + \varrho P + K^TC^TP + PCK - \chi_1\Gamma_1 + \epsilon_1{\zeta_a}^2 + \epsilon_2{\zeta_k}^2$. Next, using Lemma (1) and Schur complement, Θ_1 can be rewritten as

$$\Theta_1 \le \begin{bmatrix} -PA - A^TP + \varrho P - \delta P + K^TC^TP + PCK - \chi_1\Gamma_1 + \epsilon_1{\zeta_a}^2 + \epsilon_2{\zeta_k}^2 & PB + \chi_2\Gamma_1 & -I & I \\ * & -\Gamma_1 & 0 & 0 \\ * & * & -\epsilon_1 & 0 \\ * & * & * & -\epsilon_2 \end{bmatrix}. \tag{20}$$

Further, pre- and post-multiply the right-hand side of (20) with $diag\{P^{-1}, P^{-1}, I, I\}$ and letting $X = P^{-1}$, $P^{-1}\Gamma_1P^{-1} = \hat{\Gamma}_1$ and $Y = KX$, the right hand side of (20) is arrived which is similar to the left hand side of (17). Thus, if the conditions (17), (18) holds, then from Definition 4, it can be concluded that FSNNs (1) is stochastically synchronized over finite-time interval. ■

For the case neither parameter uncertainty nor control variations ($\Delta A(t) = 0$ and $\Delta K(t) = 0$), the error dynamics of the FSNNs has the following framework:

$${}^CD^\nu e(t) = -A\bar{x}(t) + B\ell(\bar{x}(t)) + CK\bar{x}(t) + g(t, \bar{x}(t))\frac{dB^H(t)}{dt}. \tag{21}$$

Corollary 1: Given some positive constants $\varrho,\ \nu,\ c_1,\ c_2,\ \delta$ and T^f with $c_2 > c_1$, the FSNNs (21) achieves the finite-time synchronization criterion, if there exists X, $\hat{\Gamma}_1$ and Y such that the following conditions hold:

$$\begin{bmatrix} -AX - XA^T + \varrho X - \delta X + CY + Y^TC^T - \chi_1\hat{\Gamma}_1 & B + \chi_2\hat{\Gamma}_1 \\ * & -\hat{\Gamma}_1 \end{bmatrix} < 0, \tag{22}$$

$$\lambda_2 c_1 F_\nu(\delta t^\nu) < \lambda_1 c_2. \tag{23}$$

The non-fragile state feedback controller gain fluctuation can be computed as $K = YX^{-1}$.

Proof: In order to procure the required consequence, we consider the uncertain terms as $\Delta A(t) = 0$ and $\Delta K(t) = 0$, the conditions (22) and (23) can be secured easily from Theorem 2. The proof is similar to that of Theorem 2 and hence it is neglected. ■

IV. Simulation results

This section confers two numerical examples with its simulations for FSNNs (1) to validate the applicability of the developed theoretical approach through finite-time synchronization criterion under the Theorem 1.

Example 1: Consider the FSNNs (1) with two neuron function and the dynamics of parameter coefficients are retrieved as follows:

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 8 & 8 \\ -8 & -8 \end{bmatrix}, \quad C = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix}, \quad S = \begin{bmatrix} 0.5 \\ 1.5 \end{bmatrix}, \quad J = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, \quad U = 5, \quad W = \begin{bmatrix} 4 \\ 3 \end{bmatrix},$$
$$G(t) = \Phi(t) = \sin(t).$$

The activation function revealed for the FSNNs (1) is $\phi(x(t)) = [0.2\tanh(x(t)); 0]$ and the anticipated values from Assumption (1) are $\chi_1 = 0.10$ and $\chi_2 = 0.50$. Moreover, the fractional-order here is taken $\nu = 0.93$ and the rest of the parameters involved in the inequality (5) and (6) are portrayed as $\varrho = 0.2,\ c_1 = 1.4,\ \delta = 0.1$ and $T^f = 10$. Now, based on these substantiate values and solving the LMIs developed in Theorem 1, the feasibility can be succeeded with the aid of MATLAB software. Further, the feedback gain fluctuation is computed as $K = [-87.8836 \quad -65.1513]$. Moreover, the initial conditions of the considered network system are randomly taken as $x^T(0) = [2 \quad 1]$ and $y^T(0) = [12 \quad 10]$. Further, the associated simulation results are rendered in Figs. 1-6. For compressed note, Fig. 1 delineates the response of state trajectory curve, ie., Fig. 1 portrays that the response system exactly traces the drive system. Further, the phase portrait of the drive and its response system is delineated in Fig. 2. Moreover, the corresponding synchronized error state is demonstrated in Fig. 3. Fig. 4 shows the time history $x^T(t)\Psi x(t)$ of the considered system (1), wherein, it is noted that the state trajectories of all states indeed satisfy the condition $x^T(t)\Psi x(t) < 5.6414$ for all $t \in [0, 10]$. From which, it is evident that the synchronization criterion for FSNNS (1) is attained over finite-time domain. Further, Figs. 5 exhibit the controller responses and the fractional Brownian motion curve is represented in Fig. 5. Finally, TABLE I displays the optimal bound value of c_2 for various values of c_1. From TABLE I, it is noticed that the optimum bound value of c_2 increasing when the optimum bound value of c_1 is increasing.

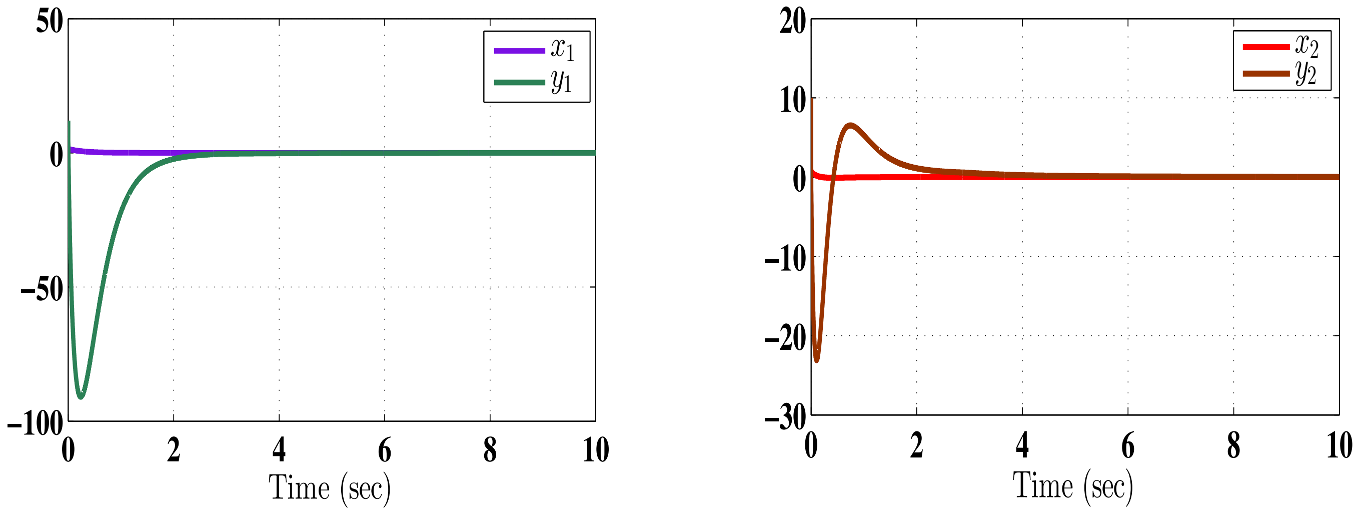

Fig. 1: Evaluation of state trajectories

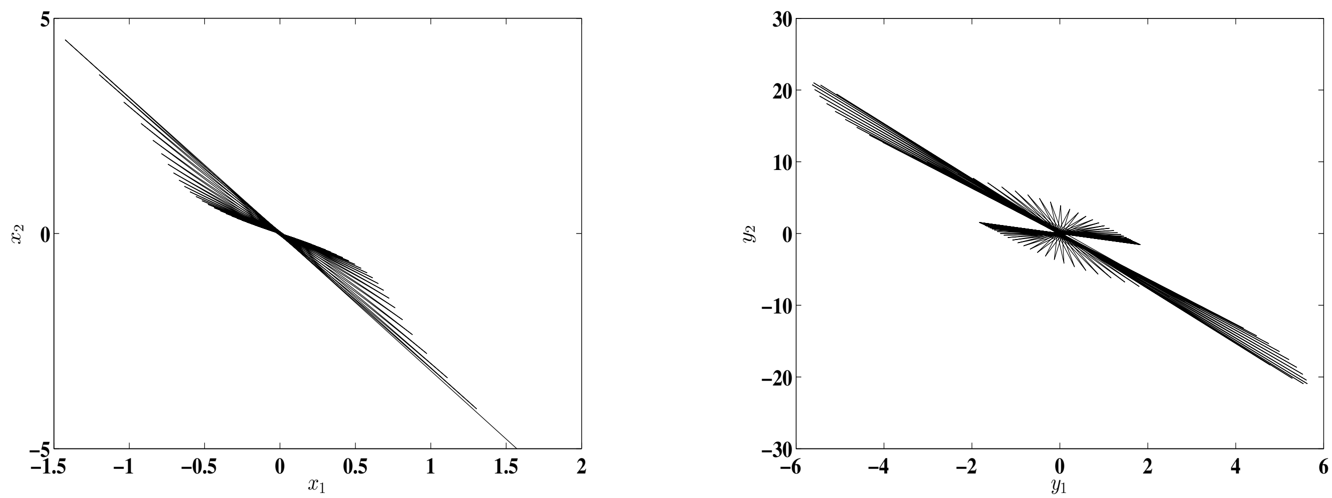

Fig. 2: Phase portrait of the state trajectories

TABLE I: Optimum bound value of c_2 for various values of c_1

c_1	1.4	1.5	1.6	1.7	1.8	1.9
c_2	5.6414	9.1277	10.1320	15.3480	19.8304	23.2460

Example 2: Consider the three-neuron mode FSNNs (1) with the corresponding coefficient matrices as follows:

$$A = \begin{bmatrix} 2.3 & 0 & 0 \\ 0 & 2.8 & 0 \\ 0 & 0 & 2.4 \end{bmatrix}, \quad B = \begin{bmatrix} 0.9 & 1.5 & 0.1 \\ 1.2 & 1 & 0.2 \\ 0.2 & 0.3 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2.8 \\ 1.5 \\ 0.8 \end{bmatrix}, \quad \zeta_a = 0.3, \quad \zeta_k = 0.5.$$

At this point, the nonlinear neuron activation functions is spotted as $\phi(x(t)) = [0.5\tanh(x(t)); 0; 0]$, from which we can hold the values $\chi_1 = 0.10$ and $\chi_2 = 0.50$ satisfying Assumption 1. Further, the other parameters engaged in the simulation process are stipulated as $\nu = 0.93,\ \varrho = 0.9,\ c_1 = 1.4,\ \delta = 0.1$ and $T^f = 10$. Moreover, by resorting the LMIs formulated in Theorem 2 with the prior mentioned parameter values, the feasibility can be endorsed and

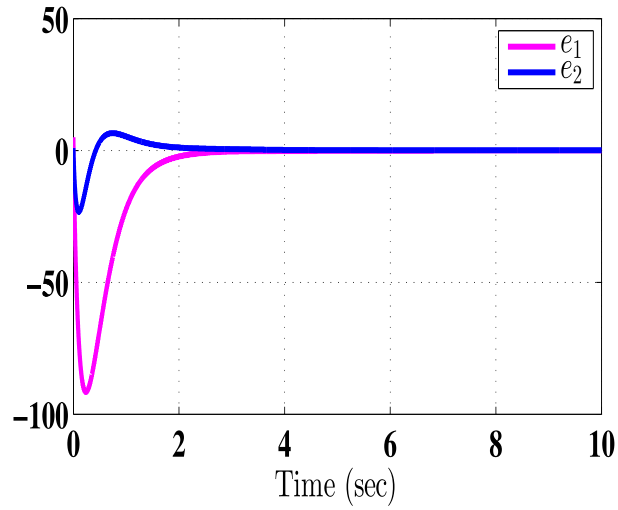

Fig. 3: Evaluation of error response

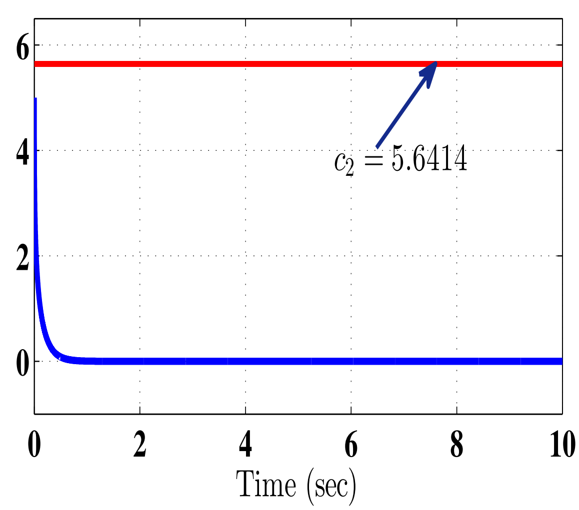

Fig. 4: Time history of $x^T(t)\Psi x(t)$

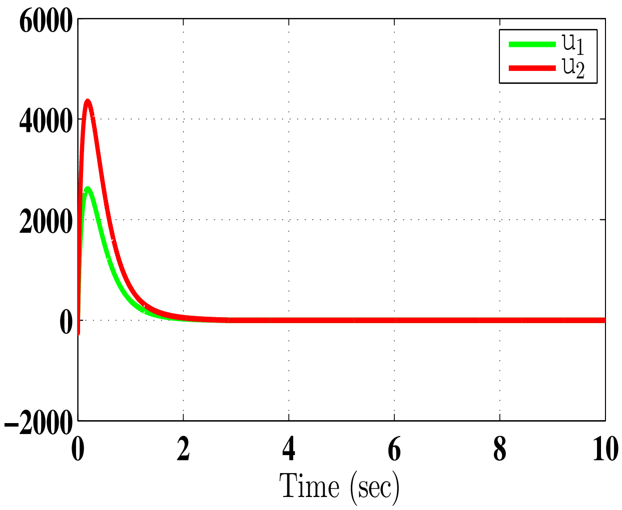

Fig. 5: Evaluation of control response

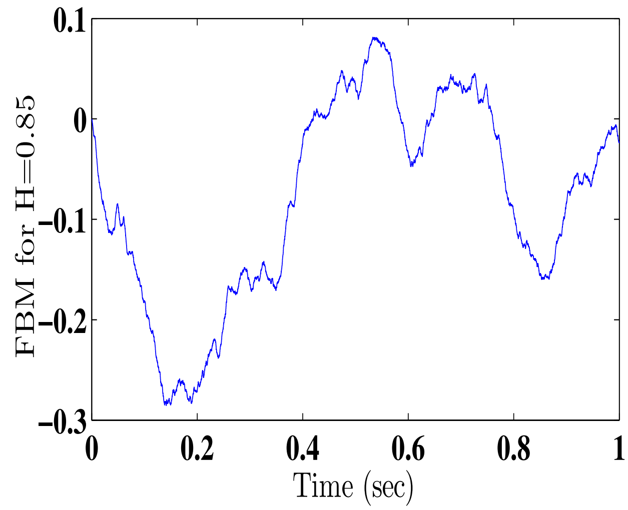

Fig. 6: Sample path of fractional Brownian motion

the corresponding state feedback gain fluctuation is quantified as $K = [-10.2966 \quad -11.2866 \quad -13.4298]$. Further, by substituting the obtained feedback gain fluctuation, the simulation results are drawn in Figs. 7-11, whereas the initial conditions are randomly chosen as $x^T(0) = [1 \quad 2 \quad 1]$ and $y^T(0) = [2 \quad 3 \quad 4]$. To mention in short, 7 and 8 illustrate the trajectories of drive and its response state which trace exactly each other within a specified finite-time period. Furthermore, the synchronized error state are depicted in Fig. 9. Fig. 10 spots the response of the controller. Further, the finite-time synchronization criterion is displayed in Fig. 11, whereas the time evolution of $x^T(t)\Psi x(t)$ does not exceeds the optimum bound value $c_2 = 7.8881$ and contained within the finite interval. Additionally, the optimal bound value of c_2 for various values of c_1 are established and listed in TABLE II. Thus, TABLE II reveals that the value of c_1 increases, the bound value of c_2 will also increases subsequently.

TABLE II: Optimum bound value of c_2 for various values of c_1

c_1	1.4	1.6	1.8	2.0	2.2	2.4
c_2	7.8881	8.9734	10.6570	11.8996	13.1348	14.3854

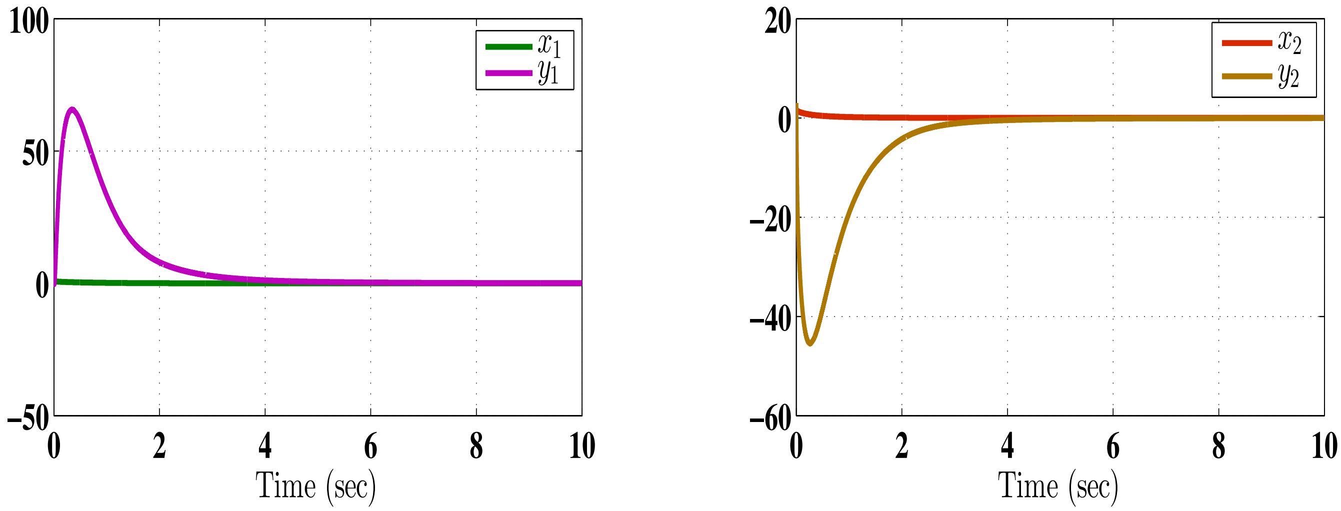

Fig. 7: Evaluation of state trajectories

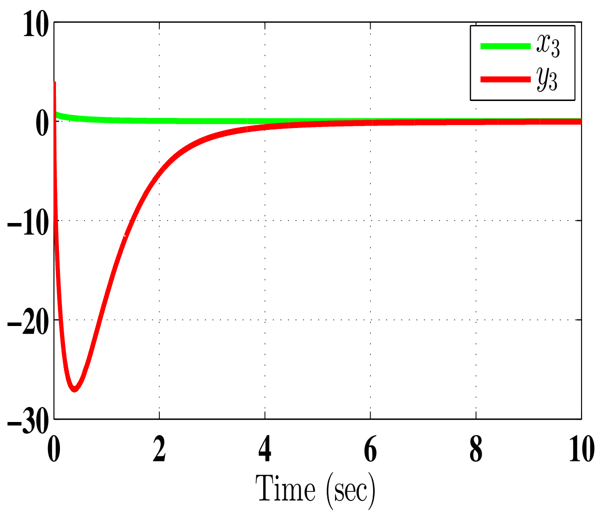

Fig. 8: Evaluation of state trajectories

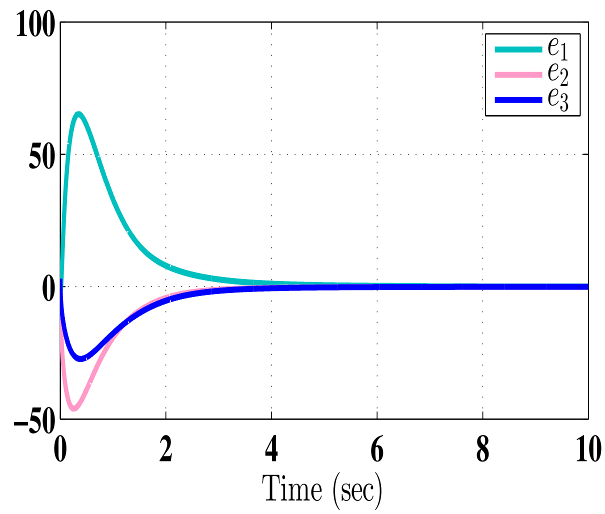

Fig. 9: Evaluation of error response

Thus, it is conferred from the above simulation results, the finite-time synchronization criterion under the proposed non-fragile control strategy for the addressed FSNNs (1) is achieved with respect to $(c_1, c_2, d_2, T^f, \Psi)$ at specified finite-time interval.

V. Conclusion

In this paper, we have examined the synchronization problem in finite-time domain for a class of fractional-order stochastic neural networks with discontinuous activation functions via a non-fragile controller. Precisely, the phenomenon of randomness in the controller is characterized by stochastic variables that satisfies the Bernoulli distributed sequences with white noise. Further, by utilizing indirect Lyapunov method along with linear matrix inequalities, a novel set of adequate conditions is procured and it assures the finite-time stochastic synchronization criterion over specified finite domain. Moreover, the desired form of non-fragile state feedback controller gain fluctuation has been exhibited by using aforementioned sufficient conditions. At last, two numerical examples with its simulations have been proffered to validate the superiority and potency of the synchronization criterion proposed

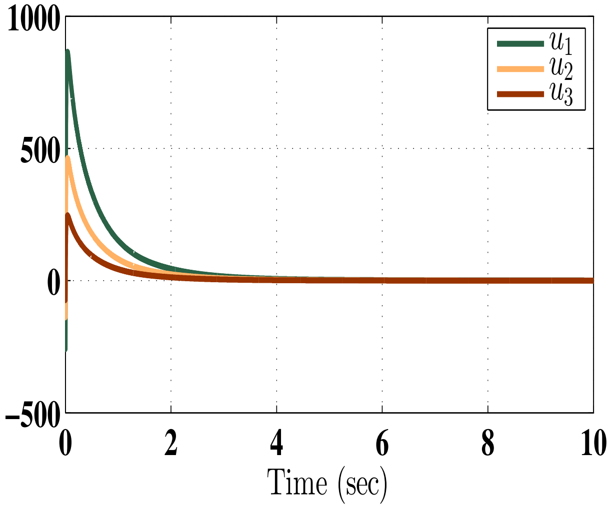

Fig. 10: Evaluation of control input

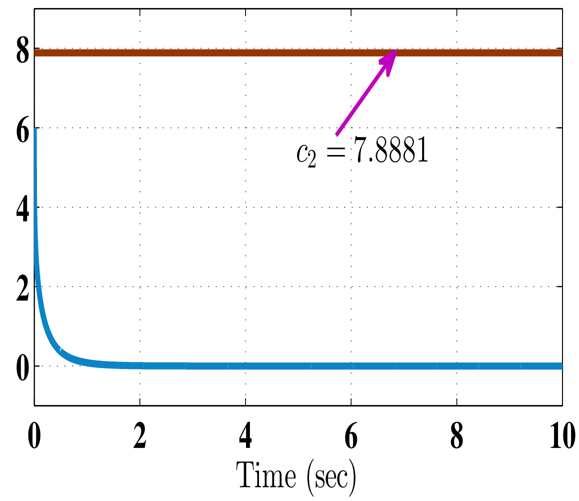

Fig. 11: Time history of $x^T(t)\Psi x(t)$

in this study.

Compliance with ethical standards

Conflict of Interest: The authors declare that they have no conflict of interest.

Human and animal rights: The article does not contain any studies with human participants or animals performed by any of the authors.

Informed Consent: There is no individual participant included in the study.

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