Fuzzy analytic hierarchy process with ordered pair of normalized real numbers

Haoyang Cui (563602006@qq.com)
Southwest University of Science and Technology https://orcid.org/0000-0003-2551-0624
Hui Zhang
Southwest University of Science and Technology https://orcid.org/0000-0003-2442-0045
Lei Zhou
Southwest University of Science and Technology
Chunming Yang
Southwest University of Science and Technology
Bo Li
Southwest University of Science and Technology
Xujian Zhao
Southwest University of Science and Technology

Research Article

Keywords: Analytic Hierarchy Process, fuzzy decision-making, Ordered pair of normalized real numbers, machine learning algorithm

Posted Date: July 13th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-1270442/v1

Fuzzy analytic hierarchy process with ordered pair of normalized real numbers

Haoyang Cui[1], Hui Zhang[2*], Lei Zhou[2], Chunming Yang[1], Bo Li[1] and Xujian Zhao[1]

[1]School of computer science and technology, Southwest University of science and technology, Middle Section of Qinglong Avenue, Mianyang, 621010, Sichuan, China.
[2]School of science, Southwest University of science and technology, Middle Section of Qinglong Avenue, Mianyang, 621010, Sichuan, China.

*Corresponding author(s). E-mail(s): zhanghui@swust.edu.cn;
Contributing authors: cuihaoyang@mails.swust.edu.cn;

Abstract

The Analytic hierarchy process (AHP) is a widely used multi-criteria decision theory, and most AHP relies on the judgments of experts to derive priority scales. However, the judgment of experts may be subjective, and different experts may give different judgments for a problem. In order to make the decision results more objective, machine learning algorithms can be used to make the judgment. However, machine learning algorithms are strongly related to the collected data and not being flexible enough. This paper tries to combine expert judgments with algorithmic judgments to improve the bias of experts' judgments while still making decision-making flexibility. The authors combine expert's judgments with the judgments of the machine learning algorithm into Ordered pair of normalized real numbers (OPNs) and then make decisions through OPNs. Experiments on real data sets show that the proposed algorithm can get reasonable decision results. Moreover, when the expert's judgments are wrong or invalid, the judgments given by the machine learning algorithm can correct the expert's judgments to obtain a reasonable decision-making result.

Keywords: Analytic Hierarchy Process, fuzzy decision-making, Ordered pair of normalized real numbers, machine learning algorithm

1 Introduction

The Analytic Hierarchy Process (AHP) was first proposed by T.L.Saaty, it is an effective method to solve complex multi-criteria decision-making problems, and it can help the decision-makers in making the best decisions. The main advantage of the AHP is to realize the combination of quantitative and qualitative analysis, which eliminates the judgment error caused by the excessive influence of qualitative factors in traditional decision-making methods.Until now, many researchers have researched the AHP and applied it in many fields, making the AHP become one of the most widely used multi-criteria decision-making tools. Many outstanding works have been published based on AHP: they include applications of AHP in different fields such as planning, selecting the best alternative, resource allocations, logistics location, resolving conflict, optimization, risk assessment, etc [1, 2].

In 1965, the Fuzzy set theory [3] was first proposed by Zadeh, and it describes the uncertainty of things by the membership. The main idea of the fuzzy set is to use any value on the closed interval [0,1], as the membership of the element to the fuzzy set, instead of the membership of the element to the traditional set can only be 0 or 1. The fuzzy set expands the application scope of mathematics from the deterministic field to the fuzzy field and from the precise phenomenon to the fuzzy phenomenon. At present, fuzzy set theory has been widely used in different domains, such as fuzzy clustering [4], fuzzy programming, fuzzy control, Fuzzy prediction, etc. In 1970, Bellman and Zadeh introduced fuzzy set theory into multi-criteria decision-making for the first time [5]. They proposed the concept and model of fuzzy decision analysis to solve the problem of uncertainty in decision-making. Because most of the decision-making problems in real life are uncertain, fuzzy decision-making is more in line with the actual situation. At present, fuzzy decision-making has become one of the most important applications of fuzzy set theory [6].

Since its establishment, the fuzzy set theory has been continuously developed, and many new fuzzy sets have been proposed one after another. For example, Zadeh introduced the type-2 fuzzy sets in 1975, Smarandache introduced the neutrosophic sets in 1999, Torro and Narukawa introduced the hesitating fuzzy sets in 2010, and Gündoğdu introduced the spherical fuzzy sets in 2019. Among all fuzzy sets, the intuitionistic fuzzy set is one of the most widely used fuzzy sets, which proposed by Atanassov in 1986, as an extension of the traditional fuzzy set [7]. Each fuzzy number in intuitionistic fuzzy sets includes three aspects of information: membership degree, non-membership degree, and hesitation degree. Because the hesitation degree reflects a neutral attitude towards things, so it is more in line with the nature of uncertainty in the real world. Present, researchers have done many kinds of research on the intuitionistic fuzzy set, including using intuitionistic fuzzy set to multi-attribute decision-making. For example, Deng-Feng Li studied the problem of multi-attribute decision-making based on intuitionistic fuzzy sets in 2005, and a corresponding decision-making method was proposed [8].

In the AHP, when comparing the importance of criteria by pairs, we need to choose a certain number from 1 to 9 as the score according to the given scale. However, the decision-makers are limited to their knowledge level and other factors, so it may be difficult or unable to give a certain value. Therefore, researchers combine fuzzy set theory with the AHP to produce the Fuzzy Analytic Hierarchy Process (FAHP). In FAHP, the pairwise comparison results between criteria are represented by a fuzzy number, which will be closer to the actual situation in real life. In order to adapt to more complex situations, according to the intuitionistic fuzzy set theory, Xu proposed the Intuitionistic Fuzzy Analytic Hierarchy Process (IFAHP) in 2014, which extended the AHP and the FAHP into the IFAHP [9]. Moreover, all the preferences are represented by intuitionistic fuzzy numbers in IFAHP. Compared with AHP, IFAHP can handle some more complex problems, especially when the decision-maker has some uncertainty in making decisions, so the IFAHP also has been widely used. Although IFAHP may have more advantages in dealing with complex problems, all the proposed AHP, FAHP and IFAHP decision-making methods depend on the judgment given by experts. However, the judgment given by experts is often subjective, which will be affected by the cognitive level and emotional tendency of experts themselves. And the judgment given by different experts is often not the same, so the decision-making may also be biased.

At present, many decision problems can be attributed to the classification problems in machine learning, so we can also use some machine learning algorithms to make decisions. For example, we can use the decision tree, support vector machine, logistic regression, and other algorithms to make decisions. At present, the decision-making method based on machine learning algorithms has been applied in many fields, such as Huilin Jiang et al. Applied machine learning algorithm to support decision-making in emergency department triage for patients [10]. The decisions made by machine learning algorithms are objective, and the efficiency and accuracy of decision-making often surpass human beings in some cases. However, most of the current machine learning decision algorithms are strongly related to the collected data. In some cases, it is difficult for us to collect enough data, and the quality of data is also difficult to ensure, which will affect the accuracy of machine learning algorithm decision-making. At the same time, it is difficult for machine learning algorithms to introduce background knowledge in related fields explicitly like AHP, and the interpretability of many machine learning algorithms is not very good [11].

To sum up, this paper attempts to combine the subjective opinions given by experts with the objective opinions given by the machine learning algorithm in the traditional FAHP method, so as to make better decisions. The innovations of this paper are as follows:

1. The authors use OPNs, a new mathematical theory, to combine the subjective judgment of experts with the objective judgment given by the machine learning algorithm, and then calculate OPNs. Thus, it avoids the problem that the traditional analytic hierarchy process is too subjective, and avoids

the problem that it is greatly affected by data when only using machine learning algorithms for decision-making.

2. The authors verify that in the method proposed in this paper, when the expert judgment is wrong or invalid, the judgment given by the machine learning algorithm can correct the expert judgment in a certain range, to still get a reasonable decision.

2 Related work

In this section, we briefly review the four fuzzy AHP and three decision tree algorithms in machine learning that are most relevant to the work of this paper.

2.1 fuzzy analytic hierarchy process

Considerable research efforts have been performed in the field of AHP. The fuzzy analytic hierarchy process was first proposed by Buckley et al. in 1985, and this fuzzy analytic hierarchy process is based on the fuzzy set theory proposed by Zadeh [12]. Subsequently, With the continuous development of fuzzy set theory, many FAHP methods have been proposed, such as triangular FAHP, trapezoidal FAHP, Intuitionistic FAHP, Pythagorean FAHP, and Neutrosophic FAHP [13]. Here, we briefly review four widely used FAHP methods proposed in recent years.

2.1.1 Intuitionistic fuzzy analytic hierarchy process

In the complex social environment, people often hesitate to judge things and cannot make effective judgments. In the traditional fuzzy concentration, it cannot reflect the situation of hesitation. So in 1986, Atanassov proposed the Intuitionistic Fuzzy Set (IFS) [14] as an extension for the fuzzy set proposed by Zadeh.

Xu and Liao proposed the intuitionistic fuzzy analytic hierarchy process (IFAHP) in 2014. When using IFAHP, we need to consider the hierarchy of problems and identify the objective, criteria, sub-criteria, and alternates. Then we compare each criterion and sub-criteria in pairs and compare alternatives under each criterion or sub-criteria, score according to the given scale to construct intuitionistic preference relations. After all preference relations are constructed, we need to verify the consistency of all preference relations to avoid self-contradictory situations in preference relations. If the consistency check fails, the decision-maker must reconstruct the intuitionistic preference relations. If the consistency check passes, we should calculate the priority vector of each intuitionistic preference relation and fuse all the weights. Finally, rank the overall weights and make a decision. Moreover, the scale regarding the relative importance degrees is denoted as IFNs and adopts the operation method of IFNs In IFAHP [9].

In various AHP, it is very important to check the consistency of each preference relation. If the check fails, the inconsistent intuitive preference relationship usually returns to the decision-makers, and the decision-makers will re-evaluate it until the intuitive preference relationship is accepted. However, it is very troublesome to make experts re-evaluate, sometimes even challenging to achieve. In IFAHP, Xu and Liao proposed a method to check the preference relation. They also developed a new algorithm that can automatically repair inconsistent intuitionistic fuzzy preference relationships instead of manually repairing them by decision-makers.

2.1.2 Hesitant fuzzy analytic hierarchy process

In many decision-making problems, decision-makers are often hesitant when making decisions. There may be several similar files, and it is not easy to make judgments. To solve this problem, Torro and Narukawa proposed hesitating fuzzy sets in 2010 [15], and the hesitant fuzzy allows the membership of its elements to have several possible values.

In 2015, Öztaysi et al. proposed the hesitant fuzzy analytic hierarchy process. In addition, they use the hesitant fuzzy analytic hierarchy process to solve the problem of multicriteria supplier selection [16].

2.1.3 Pythagorean fuzzy analytic hierarchy process

Although the degree of membership and the degree of non-membership is proposed in IFS, but IFS cannot describe the case that the sum of membership and non-membership is greater than 1, so Yager proposed the Pythagorean fuzzy set (PFS) theory in 2013 [17]. In the PFS, the membership grade μ_α and non-membership ν_α satisfying the condition ${\mu_\alpha}^2 + {\nu_\alpha}^2 \leq 1$. Therefore, it has fewer restrictions than the IFS and can adapt to more situations [18].

According to the PFS theory, Mohd et al. Proposed the Pythagorean fuzzy analytic hierarchy process to multi-criteria decision making in 2017. In this study, the authors proposed the determination of the weight of criteria method in a decision-making problem under the PFS [19].

2.1.4 Spherical fuzzy analytic hierarchy process

In 2019, Gündoğdu et al. introduced the spherical fuzzy sets (SFS) [20]. SFS is one of the latest fuzzy set theories, and it expands other fuzzy sets by setting a membership function on a spherical surface. [21].

The spherical fuzzy analytic hierarchy process is one of the latest fuzzy AHP methods, and it was proposed in 2020. Gündoğdu et al. Proposed the spherical fuzzy analytical hierarchy process [21],and they use this method to locate the wind power farm, which verified the effectiveness of this method.

2.2 decision tree algorithm

In machine learning, many algorithms can be used for decision-making. However, most machine learning algorithms do not have good interpretability due to the complex calculation process, and can not show the processing process of the algorithm. The decision tree algorithm splits the complex decision-making process into a series of simple choices by generating a tree-like model to explain the entire decision-making process intuitively. Therefore, the authors chose the decision tree algorithm as the Calculation method for objective judgment in this paper. This section briefly introduces three currently widely used decision tree algorithms.

2.2.1 ID3 algorithm

The ID3 (Iterative Dichotomiser 3) algorithm is a decision tree algorithm based on the information gain [22]. It is the most classical algorithm in the decision tree algorithms, and many decision tree algorithms are improved based on the ID3 algorithm. The core of the ID3 algorithm is to select attributes on all nodes in the decision tree and use the information gain as the attribute selection standard. The largest category information about the tested example can be obtained when each non-leaf node is tested.

2.2.2 C4.5 algorithm

The C4.5 algorithm was proposed by J.R.Quinlan based on the ID3 algorithm [23]. It uses information gain rate as the criterion to select split attributes, and the algorithm prunes simultaneously during the process of building the tree model, which reduces the size of the tree model. Moreover, the C4. 5 algorithm can handle continuous attributes and datasets with missing values.

2.2.3 CART algorithm

The CART (classification and regression trees) algorithm [24] is a decision tree algorithm used for classification and regression. The CART algorithm selects the attribute with the minimum Gini coefficient value as the splitting attribute. According to the splitting attribute of the node, uses the binary recursive segmentation method to divide each internal node into two child nodes to form a simple binary tree recursively.

By studying the existing fuzzy analytic hierarchy process and decision tree algorithm, the authors find that these methods have the following problems:

1. The current fuzzy AHP methods depend on experts' judgments. However, experts' judgments are subjective, and the judgments given by different experts are often different, which will affect whether the decision-making made by fuzzy AHP is reasonable.
2. The decision made by the decision tree algorithms is objective, but the decision tree algorithms depend on data. In order to have high decision accuracy, it needs a large amount of high-quality data, which is often tricky in reality.

In order to solve these two problems, this paper proposes a new method, which combines the expert's judgments with the decision tree algorithm's judgments to form OPNs, and then calculates the OPNs to obtain the final decision. This method realizes the complementarity of the expert's and algorithm's judgments. It avoids the shortcomings of too subjective judgment of experts, and insufficient accuracy of machine learning algorithms in the case of insufficient data and low quality. Moreover, when the expert opinion is invalid or wrong, the judgments given by the algorithm can correct the judgments given by the expert within a certain range so that the decision-making result is still reasonable.

3 Preliminaries

Ordered pair of normalized real numbers (OPNs) [25] is a new mathematical concept introduced by Zhou as a generalization of the Intuitionistic fuzzy numbers (IFNs). In this section, some basic definitions of OPNs used in this paper are introduced.

Definition 1 $\alpha = (\mu_\alpha, v_\alpha)$ is called an Ordered Pair of Normalized real number (OPN), if $0 < \mu_\alpha$ and $v_\alpha < 1$.

Example 1 if $\alpha = (0.9, 0.8)$, because $0 < 0.9$ and $0.8 < 1$, so $\alpha = (0.9, 0.8)$ is an OPN. But in the define of IFSs, $\mu_\alpha + v_\alpha$ should less than or equal to 1, $0.9 + 0.8 = 1.7$, so α isn't a IFN.

In the previous fuzzy set, if μ_α is larger, then v_α should be smaller. At the same time, there are some requirements for the sum of μ_α and v_α, the sum of squares of μ_α and v_α, etc. But OPNs are different from them. OPNs have fewer restrictions, μ_α can be larger, and v_α can also be larger, as long as $0 < \mu_\alpha, v_\alpha < 1$. So OPNs make it possible to combine expert judgments and the machine learning algorithms judgments.

Definition 2 Let α,β be OPNs, $\alpha = (\mu_\alpha, v_\alpha)$ and $\beta = \left(\mu_\beta, \nu_\beta\right)$. $\alpha = \beta$ if and only if $\mu_\alpha = \mu_\beta$ and $\nu_\alpha = \nu_\beta$.

Definition 3 Let α,β be OPNs. We refer to $s(\alpha) = \mu_\alpha - v_\alpha$ as the score of α, If $s(\alpha) < s(\beta)$, taht means α is smaller than β, denoted by $\beta > \alpha$ or $\alpha < \beta$.

Example 2 if $\alpha = (0.9, 0.8)$ and $\beta = (0.6, 0.1)$, $s(\alpha) = 0.9 - 0.8 = 0.1$, $s(\beta) = 0.6 - 0.1 = 0.5$, because $s(\alpha) < s(\beta)$, so $\alpha < \beta$.

Definition 4 For $c \in R$ and $x, y \in (0,1)$, the basic ψ -operations of ψ -scalar-multiplication, ψ -addition and ψ -multiplication are define as:

$$c \odot x = \psi(c \cdot \psi^{-1}(x)) \tag{1}$$

$$x \oplus y = \psi(\psi^{-1}(x) + \psi^{-1}(y)) \tag{2}$$

$$x \otimes y = \psi(\psi^{-1}(x) \cdot \psi^{-1}(y)) \tag{3}$$

Zhou [25] provide a function ψ, a simple but applied one as (4):

$$\psi(x)=\begin{cases}10^{-n}x+1-\eta_n, & 5-9n\le x<14-9n, n\ge 2;\\ \vdots & \vdots\\ 10^{-4}x+0.0032, & -31\le x<-22;\\ 10^{-3}x+0.023, & -22\le x<-13;\\ 10^{-2}x+0.14, & -13\le x<-4;\\ 10^{-3}x+0.977, & 13\le x<22;\\ 10^{-4}x+0.9968, & 22\le x<31;\\ \vdots & \vdots\\ 10^{-n}x+\eta_n, & 9n-14\le x<9n-5, n\ge 2;\end{cases} \tag{4}$$

$$\psi^{-1}(x)=\begin{cases}10^{n}\left(x-1+\eta_n\right), & 10^{-n}\le x<10^{1-n}, n\ge 2;\\ \vdots & \vdots\\ 10^{3}(x-0.023), & 0.001\le x<0.01;\\ 10^{2}(x-0.14), & 0.01\le x<0.1;\\ 10^{1}(x-0.5), & 0.1\le x<0.9;\\ 10^{2}(x-0.86), & 0.9\le x<0.99;\\ 10^{3}(x-0.977), & 0.99\le x<0.999;\\ \vdots & \vdots\\ 10^{n}\left(x-\eta_n\right), & 1-10^{1-n}\le x<1-10^{-n}, n\ge 2;\end{cases} \tag{5}$$

where

$$\eta_1=0.5, \eta_{n+1}=10^{-(n+1)}(81n-45)+\eta_n, n\ge 1 \tag{6}$$

Definition 5 Let α, β be OPNs, and c be real. We can define the scalar multiplication, addition and multiplication of OPNs as follows:

$$c\alpha=(c\odot\mu_\alpha, c\odot v_\alpha) \tag{7}$$

$$\alpha+\beta=\left(\mu_\alpha\oplus\mu_\beta, v_\alpha\oplus v_\beta\right) \tag{8}$$

$$\alpha\cdot\beta=\left(\mu_\alpha\otimes\left(1-v_\beta\right)\oplus v_\alpha\otimes\left(1-\mu_\beta\right), \mu_\alpha\otimes\left(1-\mu_\beta\right)\oplus v_\alpha\otimes\left(1-v_\beta\right)\right) \tag{9}$$

Example 3 $(0.005, 0.6183)+(0.7563, 0.003)$
$=(0.005\oplus 0.7563, 0.6183\oplus 0.003)$
$=\left(\psi\left(\psi^{-1}(0.005)+\psi^{-1}(0.7563)\right), \psi\left(\psi^{-1}(0.6183)+\ \psi^{-1}(0.003)\right)\right)$
$=\left(\psi\left(10^{3}(0.005-0.023)+10(0.7563-0.5)\right), \psi\left(10(0.6-0.5)+10^{3}(0.003-\ 0.023)\right)\right)$
$=(\psi(-15.437), \psi(-19))=\left(\left(10^{-3}\times(-15.437)+0.023\right), \left(10^{-3}\times(-19)+0.023\right)\right)$
$=(0.007563, 0.004)$

4 The FAHP with OPNs

In this part, the fuzzy analytic hierarchy process with Ordered pair of normalized real numbers(OFAHP) is introduced.

The flowchart of specific steps in OFAHP are shown in Figure 1.

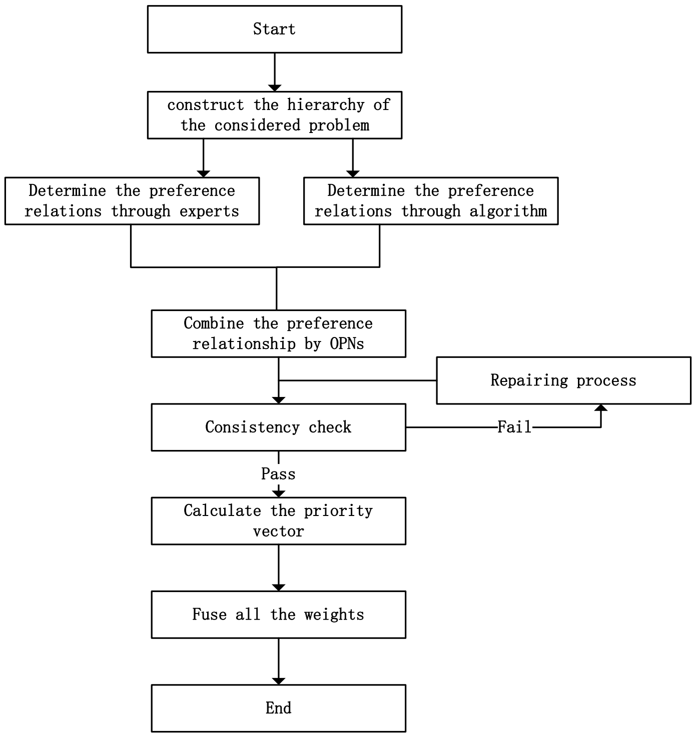

Fig. 1: The steps for OFAHP

Step1: consider the hierarchy of problems, and identify the objective, criteria, sub-criteria, alternates [9]. Then, go to the Step 2.

Step2: Determine the preference relationship by pairwise comparison between each criterion. At the same time, comparing alternatives under each criterion. The expert first determines the preference relations according to the specified standards and gives the matrix of the corresponding preference relations. In addition, using the algorithm provided in this paper to calculate the preference relations, the corresponding preference relations matrix is also generated. Then, go to the Step 3.

Step3: Combining the preference relations matrix given by the expert and the preference relations matrix calculated by machine learning algorithm using OPNs to form a new preference relations matrix. In the new preference relations matrix, each value is represented by an OPN, and in each OPN, The first number is given by experts, and the machine learning algorithm calculates the second number. Then, go to the Step 4. In Step3, the authors' goal is to fuse the preference relationship given by experts and the preference relationship given by the machine learning algorithm into a new preference relationship by OPNs.

Step4: Checking the consistency of all preference relations. If the consistency check is passed, go to Step 6. If the consistency check is not passed, go to Step 5. The purpose of step 4 is to find out whether there are errors or contradictions in all preference relationships.

Step5: Repairing the inconsistent fuzzy preference relations and then go to Step 4. In most AHPs, inconsistent preference relationships can be repaired by decision-makers, but this is not convenient. Because OPN is a generalization of IFNs, so the automatic repair algorithm proposed by Xu and Liao in IFAHP can also be used. In this paper, we adopt the algorithm to repair inconsistent fuzzy preference relations.

Step6: Calculateing the priority vector $\omega = (\omega_1, \omega_2, ..., \omega_n)$ of each preference relation. Then, go to Step7.

Step7: Fusing all the weights by the operational rules of OPNs, and then choose the best alternative according to overall weights. The purpose of this step is to arrive at the final decision.

Step8: End.

The detailed steps of OFAHP are described below.

4.1 Decomposition of complex multi-criteria decision problems

Similar to AHP, IFAHP, and FAHP, we need to decompose complex multi-criteria decision problems. In order to apply OFAHP, it is necessary to organize comprehensive questions into different levels according to the attributes under consideration.

In this paper, we suppose that $A = \{A_1, A_2, \ldots, A_n\}$ is the finite set of n alternatives, and $C = \{C_1, C_2, \ldots, C_m\}$ is the set of criteria with which the elements of A are compared in the hierarchical structure[9].

4.2 Comparative judgments with ordered pair of normalized real numbers

After we decomposing the complex multi-criteria decision problem into different levels, then we can compare the relative importance of elements in one level relative to the elements in the previous level to establish fuzzy preference relationships.[9]. In Section 4.2, we mainly introduced how to compare the importance of criteria through machine learning algorithms, and get the preference relationship between the alternatives and the criterion.

In the OFAHP, each result of pairwise comparison is represented by an OPN. If all the pairwise comparison values are represented by OPNs and are stored in a fuzzy preference relation, a fuzzy preference relation $\Delta = (b_{ik})_{n\times n}$ can be generated, each $b_{ik} = (\mu_{ik}, v_{ik})$ is an OPN. In the method proposed in this paper, the value of μ_{ik} is determined by expert scores, and the value of v_{ik} is determined by machine learning algorithms according to past data. Both μ_{ik} and v_{ik} take values in the interval (0,1).

Table 1: The 0.1-0.9 scale

The 0.1-0.9 scale	**meaning of the number**
0.1	Extremely not preferred
0.2	Very strongly not preferred
0.3	Strongly not preferred
0.4	Moderately not preferred
0.5	Equally preferred
0.6	Moderately preferred
0.7	Strongly preferred
0.8	Very strongly preferred
0.9	Extremely preferred
Other values between 0 and 1 (not 0 and 1)	Intermediate values used to present compromise

For μ_{ik}, according to the 0.1-0.9 scale proposed [9], it will be determined by decision-makers, the 0.1-0.9 scale is shown in Table 1. In the fuzzy preference relation, μ_{ik} indicates how important A_i is than A_k. For example, $\mu_{ik} = 0.5$ means that A_i and A_k are equally important; $\mu ik > 0.5$ indicates that A_i is more important than A_k, and $\mu ik < 0.5$ indicates that A_k is more important than A_i.

4.2.1 Compare the importance of criteria through the ID3 algorithm

For v_{ik}, in the case of the existence of previous data, the authors use the machine learning algorithms to determine objectively. $C = \{C_1, C_2, ..., C_m\}$ is the set of criteria with which the elements of A are compared in the hierarchical structure, the authors provide an algorithm to compare the importance between C_i and C_m.

To compare the importance of criteria, the authors adopted the ID3 (Iterative Dichotomiser 3) algorithm [26]. ID3 algorithm is one of the widely used machine learning algorithms, this algorithm builds a decision tree from the data which are discrete. For each node in the decision tree, the algorithm will select the attribute with the highest information gain as the best attribute. After constructing the decision tree, we can calculate the importance value of each attribute according to the information gain of each node and the number of samples, this important value is objective [22].

Let the sample is S, its size denoted by $|S|$, and B is one of condition attributes of the set. D is the decision attribute and has k values. S is divided into several categories:$C_1, C_2 ... C_k$ by D, the size of C_i is denoted by $|C_i|$, $i = 1, 2, 3, \ldots, k$. The formula of entropy is shown as (10) [27].

$$Entropy(S) = -\sum_{i=1}^{k} p_i \cdot \log_2 (p_i) \tag{10}$$

Where p_i is the probability that any subset of S belonging to category C_i, and $p_i = |C_i|/|S|$. Supposing that using condition attribute B to divide S , B has j different values, then S can be divided into j subset s$\{S_1, S_2, ..., S_j\}$. The information gain can be represented as (11).

$$IG(B) = Entropy\,(S) - \sum_{i=1}^{j} \frac{|S_i|}{|S|} \cdot Entropy(S_i) \tag{11}$$

After constructing the decision tree, we calculate the importance of each attribute. Suppose we want to calculate the importance of attributes. For each node, first multiply the entropy value of the node by the number of samples of the node, then subtract the entropy value of its child nodes and multiply the number of samples of the child nodes. Finally, we add up all the calculation results. After calculating the importance of each attribute, we normalize all the results. Here, the greater the importance value, the more important the attribute.

Example 4 The authors use iris data set [28] and the ID3 algorithm to construct a decision tree. Each data in iris dataset has four attributes as $X[0]$, $X[1]$, $X[2]$ and $X[3]$, all data are finally divided into three categories. When building decision tree, The authors select the top 20 data of each category to build decision tree. The final decision tree is shown in the figure 2.

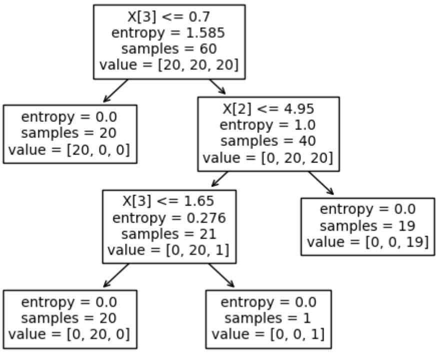

Fig. 2: The decision tree which constructed by ID3 algorithm and 60 data in Iris data set

Through the decision tree, we calculate the importance of each attribute. Because there are no nodes in the decision tree with attributes $X[0]$ and $X[1]$ as judgment conditions, the importance of attributes $X[0]$ and $X[1]$ is 0.

In the decision tree, there is a node whose judgment is based on attribute x[2], so the importance of attribute $X[2]$ is equal to the value of the entropy of the node multiplied by the number of samples, and then the value of the entropy of the two child nodes multiplied by the number of samples of the child nodes is subtracted.

$$importance(X[2]) = (1.0 * 40) - (0.276 * 21) - (0.0 * 19) = 34.204$$

Then we calculate the importance value of attribute $X[3]$. In the decision tree, there are two nodes with $x[3]$ as the judgment condition.

$$importance(X[3]) = [(1.585 * 60) - (0.0 * 20) - (1.0 * 40)] + [(0.276 * 21) - (0.0 * 20) - (0.0 * 1)] = 55.1 + 5.796 = 60.896$$

In conclusion, the importance values of attributes $X[0]$, $X[1]$, $X[2]$, and $X[3]$ are 0, 0, 34.204, and 60.896, respectively. Finally, we normalize all the values, taking 0, 0, 0.3597, 0.6403 as the final value.

Then, we compare the importance of attributes in pairs and generate a matrix. The result of the comparison is that the importance of the two attributes is subtracted, and finally, the method in Definition 6 is used for normalization to obtain the final matrix. In particular, when using expert evaluation, the larger the value of μ_{ik}, the higher the degree of preference. The value of v_{ik} is just the opposite, the smaller the value of v_{ik}, the higher the degree of preference.

Definition 6 Let l be a very small number, $\underline{a} = \min\{m_1, m_2, \ldots, m_n\} - l$, $\bar{a} = \max\{m_1, m_2, \ldots, m_n\} + l$, then map m_n to $1 - (m_n - \underline{a}) / (\bar{a} - \underline{a})$.

Example 5 We take 0, 0, 0.3597, 0.6403 as the final value of attributes $X[0]$, $X[1]$, $X[2]$, and $X[3]$. After we subtract the value of attribute importance in pairs, we get the importance comparison matrix U_1.

$$U_1 = \begin{bmatrix} 0 & 0 & 0.3597 & 0.6403 \\ 0 & 0 & 0.3597 & 0.6403 \\ -0.3597 & -0.3597 & 0 & 0.2806 \\ -0.6403 & -0.6403 & -0.2806 & 0 \end{bmatrix}$$

Then normalize to get the final matrix U_2.

$$U_2 = \begin{bmatrix} 0.5 & 0.5 & 0.2191 & 0.000000781 \\ 0.5 & 0.5 & 0.2191 & 0.000000781 \\ 0.7809 & 0.7809 & 0.5 & 0.2809 \\ 0.9999 & 0.9999 & 0.7191 & 0.5 \end{bmatrix}$$

4.2.2 The algorithm to get the preference relation between the alternatives and the criterion

First of all, we can calculate the average value of each criterion in each alternative from the previous data. Then according to the calculated average value, get the preference relationship between the alternatives and the criterion by algorithm.

Let $A = \{A_1, A_2, A_3 \ldots, A_n\}$ is the finite set of n alternatives, and $C = \{C_1, C_2, C_3, \ldots, C_m\}$ is the set of criteria with which the elements of A are compared in the hierarchical structure.$V_n = \{v_{n1}, v_{n2}, \ldots, v_{nm}\}$ is the average value of m criteria under alternative A_n. $C_{\text{new}} = \{C_{\text{new1}}, C_{\text{new2}}, \ldots, C_{\text{newm}}\}$ is a new set of the values for criteria, we can take the following steps get it‘s preference relation between the alternatives and the criterion, and get the preference relation matrix.

Step1: Under each alternative, the average value of each criterion in the previous data is calculated respectively. $V_n = \{v_{n1}, v_{n2}, \ldots, v_{nm}\}$ is the average value of m criteria under alternative A_n.

Step2: $C_{\text{new}} = \{C_{\text{new1}}, C_{\text{new2}}, \ldots, C_{\text{newm}}\}$ is a new set of the values for criteria. For criteria C_{newm}, the set of its preferences for n alternative is $p_m = \{p_{m1}, p_{m2}, \ldots, p_{mn}\}$, and $p_{mn} = |C_{\text{newm}} - v_{nm}|$.

Step3: Get the preference relation matrix. If there are m criteria, we will establish m matrices, each of which is a $n * n$ matrix. The matrix U_m represents the preference for each alternative on the C_m.And in U_m, $U_m^{(ij)}$ is the result of preference comparison between A_i and A_j on C_m, $U_m^{(ij)} = p_{mj} - p_{mi}$.

Step4: Using the method in Definition 6 to normalize the numbers in each matrix, making every number in the matrix be in the range (0,1).

Example 6 Take iris data set as an example. There are three categories in iris dataset, which are represented by A_1, A_2 and A_3 respectively. Each data has four attributes, which are represented by C_1, C_2, C_3 and C_4. The authors select the first 20 data of each alternative in iris dataset for calculation.

Through calculation, under alternative A_1, the set of average values of the four criteria is

$$V_1 = \{v_{11}, v_{12}, v_{13}, v_{14}\} = \{5.035, 3.480, 1.435, 0.235\}$$

under alternative A_2, the set of average values of the four criteria is

$$V_2 = \{v_{21}, v_{22}, v_{23}, v_{24}\} = \{5.975, 2.760, 4.255, 1.325\}$$

under alternative A_3, the set of average values of the four criteria is

$$V_3 = \{v_{31}, v_{32}, v_{33}, v_{34}\} = \{6.560, 2.920, 5.655, 2.045\}$$

Let $C_{\text{new}} = \{C_{\text{new }1}, C_{\text{new2}}, C_{\text{new }3}, C_{\text{new }4}\} = \{6, 3, 5.1, 1.5\}$, we try to get the preference relation matrix for C_{new}. So

$$p_1 = \{p_{11}, p_{12}, p_{13}\} = \{|6 - 5.035|, |6 - 5.975|, |6 - 6.560|\} = \{0.965, 0.025, 0.560\}$$
$$p_2 = \{p_{21}, p_{22}, p_{23}\} = \{|3 - 3.480|, |3 - 2.760|, |3 - 2.920|\} = \{0.480, 0.240, 0.080\}$$
$$p_3 = \{p_{31}, p_{32}, p_{33}\} = \{|5.1-1.435|, |5.1-4.255|, |5.1-5.655|\} = \{3.665, 0.845, 0.555\}$$
$$p_4 = \{p_{41}, p_{42}, p_{43}\} = \{|1.5-0.235|, |1.5-1.325|, |1.5-2.045|\} = \{1.265, 0.175, 0.545\}$$

Then we build 4 preference relation matrix.

$$U_1 = \begin{bmatrix} p_{11} - p_{11} & p_{11} - p_{12} & p_{11} - p_{13} \\ p_{12} - p_{11} & p_{12} - p_{12} & p_{12} - p_{13} \\ p_{13} - p_{11} & p_{13} - p_{12} & p_{13} - p_{13} \end{bmatrix} = \begin{bmatrix} 0 & 0.940 & 0.405 \\ -0.940 & 0 & -0.535 \\ -0.405 & 0.535 & 0 \end{bmatrix}$$

$$U_2 = \begin{bmatrix} p_{21} - p_{21} & p_{21} - p_{22} & p_{21} - p_{23} \\ p_{22} - p_{21} & p_{22} - p_{22} & p_{22} - p_{23} \\ p_{23} - p_{21} & p_{23} - p_{22} & p_{23} - p_{23} \end{bmatrix} = \begin{bmatrix} 0 & 0.240 & 0.400 \\ -0.240 & 0 & 0.160 \\ -0.400 & -0.160 & 0 \end{bmatrix}$$

$$U_3 = \begin{bmatrix} p_{31} - p_{31} & p_{31} - p_{32} & p_{31} - p_{33} \\ p_{32} - p_{31} & p_{32} - p_{32} & p_{32} - p_{33} \\ p_{33} - p_{31} & p_{33} - p_{32} & p_{33} - p_{33} \end{bmatrix} = \begin{bmatrix} 0 & 2.820 & 3.110 \\ -2.820 & 0 & 0.290 \\ -3.110 & -0.290 & 0 \end{bmatrix}$$

$$U_4 = \begin{bmatrix} p_{41} - p_{41} & p_{41} - p_{42} & p_{41} - p_{43} \\ p_{42} - p_{41} & p_{42} - p_{42} & p_{42} - p_{43} \\ p_{43} - p_{41} & p_{43} - p_{42} & p_{43} - p_{43} \end{bmatrix} = \begin{bmatrix} 0 & 1.090 & 0.720 \\ -1.09 & 0 & -0.370 \\ -0.720 & 0.370 & 0 \end{bmatrix}$$

Finally, normalizing the generated preference relation matrix as Definition 6 . The normalized matrix is described as follows:

$$U_1 = \begin{bmatrix} 0.5 & 0.00000053 & 0.2846 \\ 0.9999 & 0.5 & 0.7846 \\ 0.7154 & 0.2154 & 0.5 \end{bmatrix} \quad U_2 = \begin{bmatrix} 0.5 & 0.2000 & 0.0000012 \\ 0.7999 & 0.5 & 0.3000 \\ 0.9999 & 0.6999 & 0.5 \end{bmatrix}$$

$$U_3=\begin{bmatrix} 0.5 & 0.0466 & 0.00000016 \\ 0.9534 & 0.5 & 0.4534 \\ 0.9999 & 0.5466 & 0.5 \end{bmatrix} \quad U_4=\begin{bmatrix} 0.5 & 0.00000046 & 0.1697 \\ 0.9999 & 0.5 & 0.6697 \\ 0.8303 & 0.3303 & 0.5 \end{bmatrix}$$

4.2.3 Constructing the preference relations with OPNs

In this paper, a preference relation matrix given by experts and the corresponding preference relation matrix calculated by the machine learning algorithm needs to be integrated into a preference relation matrix with OPNs .

If the expert gives a $n * n$ matrix U_1 to show the preference relation of alternatives with respect to a criterion (or the preference relation of criteria with respect to the overall objective), each value in the matrix U_1 is represented by $U_{ik}^{(1)}$. In addition, the corresponding $n*n$ matrix U_2 is calculated through the algorithm described in section 4.2.2 or 4.2.1, each value in the matrix U_2 is represented by $U_{ik}^{(2)}$. The final matrix U is composed of the combination of these two matrices, each value in the matrix U is represented by U_{ik}, U_{ik} is an OPN and $U_{ik} = (\mu_{ik}, v_{ik})$. We let $\mu_{ik} = U_{ik}^{(1)}$,and $v_{ik} = U_{ik}^{(2)}$.

Example 7 If the expert gives a $4 * 4$ matrix U_1 to show the preference relation of alternatives with respect to the criterion C_1, and the algorithm described in section 4.2.2 or 4.2.1 calculates the corresponding matrix U of $4 * 4$.

$$U_1=\begin{bmatrix} 0.5 & 0.7 & 0.9 \\ 0.1 & 0.5 & 0.7 \\ 0.1 & 0.4 & 0.5 \end{bmatrix} \quad U_2=\begin{bmatrix} 0.5 & 0.2000 & 0.0000012 \\ 0.7999 & 0.5 & 0.3000 \\ 0.9999 & 0.6999 & 0.5 \end{bmatrix}$$

So the final matrix U with OPNs is

$$U=\begin{bmatrix} (0.5, 0.5) & (0.7, 0.2000) & (0.9, 0.0000012) \\ (0.1, 0.7999) & (0.5, 0.5) & (0.7, 0.3000) \\ (0.1, 0.9999) & (0.4, 0.6999) & (0.5, 0.5) \end{bmatrix}$$

4.3 Consistency checking

Before deriving the priorities of the alternatives and criteria, we need to check the consistency of the fuzzy preference relationships. Only after the consistency check passes can we proceed to the next step. And if the consistency check fails, we need to repair the inconsistent fuzzy preference relationship.

At present, the most commonly used consistency checking method in AHP is proposed by Saaty [9, 29]. The consistency index (CI) is calculated as (12):

$$CI = \frac{\lambda_{\max} - n}{n - 1} \tag{12}$$

The Consistency ratio (CR) is expressed as (13).

$$CR = \frac{CI}{RI} \tag{13}$$

Among them, n is the dimension of the multiplicative preference relationship, λ_{max} is the maximum eigenvalue of the preference relationship matrix, and RI is a random index dependent on n. It is generally believed that when $CR \leq 0.1$, the multiplicative preference relationship has acceptable consistency, that is, it passes the consistency checking. However, this method has a problem: if the consistency check is not passed, we can only let the experts to give a new preference relation matrix, which is difficult or even unrealistic in many cases.

In 2014, Xu proposed IFAHP and adopted a new consistency checking method in IFAHP [9]. Besides consistency checking,This method can also automatically repair inconsistent fuzzy preference relations into consistent fuzzy preference relations. Because OPN is a new fuzzy mathematical concept as a generalization of the Intuitionistic Fuzzy Numbers, this method is also suitable for the fuzzy analytic hierarchy process with OPNs. Therefore, this paper uses this method to perform a consistency check and repair the inconsistent introductory preference relation.

In the method proposed by xu [9], let $R = (r_{ij})_{n*n}$ be an preference relationan, an algorithm is first required to construct a perfect multiplicative consistent preference relation $\bar{R} = (\bar{r}_{ij})_{n*n}$:

Step1: For $j > i+1$,let $\bar{r}_{ij} = (\bar{\mu}_{ij}, \bar{v}_{ij})$,where

$$\bar{\mu}_{ij} = \frac{\sqrt[j-i-1]{\prod_{m=i+1}^{j-1} \mu_{im} \cdot \mu_{mj}}}{\sqrt[j-i-1]{\prod_{m=i+1}^{j-1} \mu_{im} \cdot \mu_{mj}} + \sqrt[j-i-1]{\prod_{m=i+1}^{j-1} (1-\mu_{im}) \cdot (1-\mu_{mj})}} \quad j > i+1 \tag{14}$$

$$\bar{v}_{ij} = \frac{\sqrt[j-i-1]{\prod_{m=i+1}^{j-1} v_{im} \cdot v_{mj}}}{\sqrt[j-i-1]{\prod_{m=i+1}^{j-1} v_{im} \cdot v_{mj}} + \sqrt[j-i-1]{\prod_{m=i+1}^{j-1} (1-v_{im}) \cdot (1-v_{mj})}} \quad j > i+1 \tag{15}$$

Step2: For $j = i+1$,let $\bar{r}_{ij} = r_{ij}$.

Step3: For $j < i$, let $\bar{r}_{ji} = (\bar{v}_{ji},\ \bar{\mu}_{ji})$.

if R is an acceptable multiplicative consistent preference relation, then $d(R, \bar{R}) < \tau$, where

$$d(\bar{R}, R) = \frac{1}{2 \cdot (n-1) \cdot (n-2)} \sum_{j=1}^{n} \sum_{k=1}^{n} (|\bar{\mu}_{jk} - \mu_{jk}| + |\bar{v}_{jk} - v_{jk}| + |\bar{\pi}_{jk} - \pi_{jk}|) \tag{16}$$

and τ is the consistency threshold.

If the consistency check fails, Xu provides an algorithm to repair the inconsistent preference relation, the steps of the algorithm are described as follows [9]:

Step1: Let t be the number of iterations and $t = 1$, construct the complete multiplicative consistent preference relationship $barR$, from $R^{(t)}$, through the algorithm of constructing the complete multiplication consistent preference relationship.

Step2: Calculating the distance $d(R^{(t)}, \bar{R})$ between $R^{(t)}$ and $\bar{R}$. If $d(R^{(t)}, \bar{R}) < \tau$, then output $R^{(t)}$; If not, go to the Step 3.

Step3: Constructing the fused preference relation $\widetilde{R}^{(t)} = (\widetilde{r}_{ij}^{(t)})_{n*n}(\widetilde{r}_{ij}^{(t)} = (\widetilde{\mu}_{ij}^{(t)}, \widetilde{v}_{ij}^{(t)})$, where

$$\widetilde{\mu}_{ij}^{(t)} = \frac{\left(\mu_{ij}^{(t)}\right)^{1-\lambda} (\bar{\mu}_{ij})^{\lambda}}{\left(\mu_{ij}^{(t)}\right)^{1-\lambda} (\bar{\mu}_{ij})^{\lambda} + \left(1-\mu_{ij}^{(t)}\right)^{1-\lambda} (1-\bar{\mu}_{ij})^{\lambda}} \quad i,j = 1,2,\ldots,n \tag{17}$$

$$\widetilde{\mu}_{ij}^{(t)} = \frac{\left(v_{ij}^{(t)}\right)^{1-\lambda} (\bar{v}_{ij})^{\lambda}}{\left(v_{ij}^{(t)}\right)^{1-\lambda} (\bar{v}_{ij})^{\lambda} + \left(1-v_{ij}^{(t)}\right)^{1-\lambda} (1-\bar{v}_{ij})^{\lambda}} \quad i,j = 1,2,\ldots,n \tag{18}$$

And λ is a controlling parameter determined by the decision maker: The smaller the value of λ, the closer $\widetilde{R}^{(t)}$ is to $R^{(t)}$. Let $R^{(t+1)} = \widetilde{R}^{(t)}, i.e.,$ $\mu_{ij}^{(t+1)} = \widetilde{\mu}_{ij}^{(t)}$ and $v_{ij}^{(t+1)} = \widetilde{v}_{ij}^{(t)}$. Let$t = t+1$, and then go to Step2.

Through this algorithm, we can automatically improve the consistency level of the fuzzy preference relationships without losing a large amount of original information to pass the consistency check.

4.4 Calculate the priority vector

After performing consistent checking and get the consistent preference relation, we can calculate the priority vector $\omega = (\omega_1, \omega_2, \omega_3, ..., \omega_n)$ of each preference relation. Because OPNs are generalization of the IFNs, so we adopt the calculation method of priority vector in IFAHP, The priority vector is calculated as (19) [9]:

$$\omega_i = \left(\frac{\sum_{j=1}^{n} \mu_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n} (1-v_{ij})}, 1 - \frac{\sum_{j=1}^{n} (1-v_{ij})}{\sum_{i=1}^{n} \sum_{j=1}^{n} (1-\mu_{ij})} \right) \tag{19}$$

After calculating all priority vectors, we fuse all the weights from the lowest level to the highest level by the operational rules of OPNs, ranking the overall weights, and then choose the best alternative. OPNs calculation method is described in Definition 4, and method of ranking the overall weights is described by Definition 3. Finally, choose the best alternative.

5 Experiment

Because the method proposed in this paper needs to use some past data, the authors use the Iris data set [28] to test the method and get reasonable results.

The Iris data set is a very widely used dataset in machine learning. In this part, the authors use three examples, all of these examples are from the iris data set. The first two examples are different data in the iris data set. And in these two examples, both experts and the machine learning algorithms give reasonable judgments. Through these two examples, the authors have verified that the method proposed in this paper can make a reasonable decision. The third example is modified on the second example, the authors changes the reasonable judgments given by experts to wrong judgments. The third example is used to verify that with the proposed method , the judgment given by the machine learning algorithm can correct the wrong judgment given by the expert, so that the final decision made is still correct.

In the beginning, we need to construct the hierarchy of problems. The iris data set contains three classes: Iris Setosa, Iris Versicolour, and Iris Virginica, each class contains 50 instances, and each class is linearly separable from the other two classes. For each instance, it has four attributes: sepal length (unit: cm), sepal width (unit: cm), petal length (unit: cm), and petal width (unit: cm). Therefore, the hierarchical structure of the problem we constructed is shown in Figure 3.

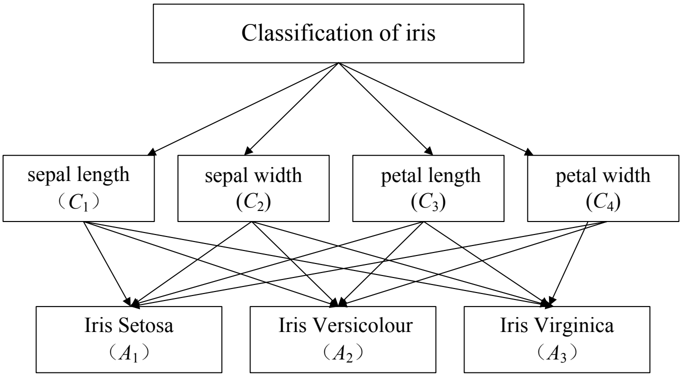

Fig. 3: Hierarchical structure of iris classification problem

There are a total of four criteria considered for this question: sepal length(C_1), sepal width(C_2), petal length(C_3) , and petal width(C_4). And three alternatives: Iris Setosa(A_1), Iris Versicolour(A_2), Iris Versicolour(A_3). The hierarchy consists of three levels. The overall objective is placed at Level 1, criteria at Level 2 and alternatives at Level 3.

There are a total of 150 instances in the Iris data set. After removing the three instances we want to test, the remaining 147 instances are used to pass the algorithm and compare the criteria' importance to the overall goal. We use the algorithm proposed in section 4.2.1 to calculate, the decision tree generated by the ID3 algorithm for these 147 instances is shown in Figure 4.

According to the method mentioned above and Figure 4, we can calculate the importance scores of the four criteria as:

$$\text{importance}(C_1)=0, \text{ importance}(C_2)=0.0118$$

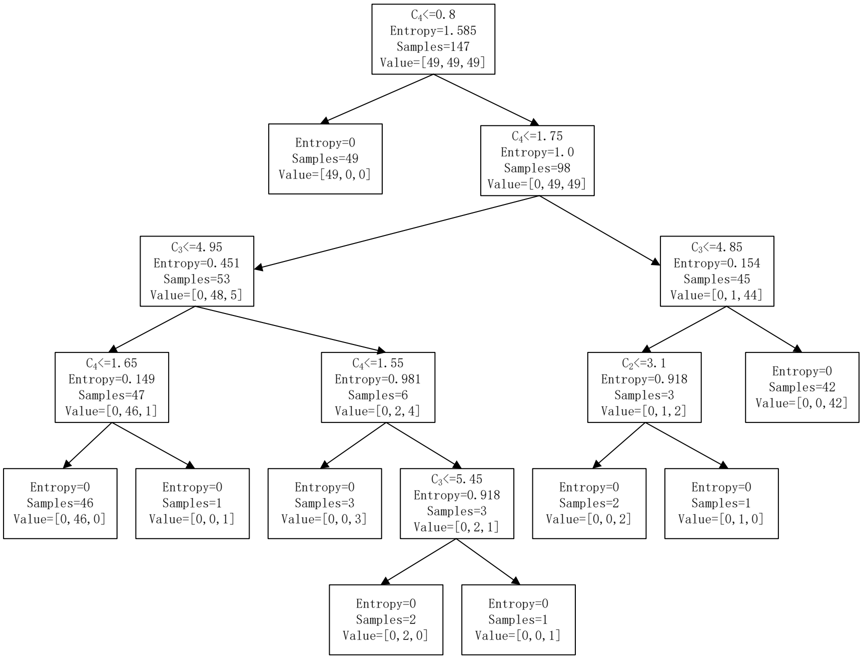

Fig. 4: The decision tree which constructed by ID3 algorithm and 147 data in iris data set

$$\text{importance}(C_3)=0.0786,\ \text{importance}(C_4)=0.9096$$

The given preference relationship is shown in Table 2.

Table 2: preference relation of criteria with respect to the overall objective calculated by the algorithm

U_2	C_1	C_2	C_3	C_4
C_1	0.5	0.4935001	0.4567783	0.0000005
C_2	0.5064999	0.5	0.4632783	0.0065005
C_3	0.5432217	0.5367217	0.5	0.0432222
C_4	0.9999995	0.9934995	0.9567778	0.5

5.1 Examples with experts give reasonable judgments

We compare the importance of the criteria to the overall goal.According to the 0.1-0.9 scale in Table 1, the preference relation of criteria with respect to the overall objective which was given by expert is shown in Table 3.

We have synthesized the preference relations given in Table 3 and Table 2, and obtained the final preference relation of criteria with respect to the overall objective which was expressed in OPNs. The final preference relation of criteria with respect to the overall objective is shown in Table 4.

Table 3: Preference relation of criteria with respect to the overall objective given by expert

U_1	C_1	C_2	C_3	C_4
C_1	0.5	0.5	0.7	0.9
C_2	0.5	0.5	0.6	0.9
C_3	0.3	0.4	0.5	0.7
C_4	0.1	0.1	0.3	0.5

Table 4: The preference relation of criteria with respect to the overall objective

U	C_1	C_2	C_3	C_4
C_1	(0.5,0.5)	(0.5,0.4935001)	(0.7,0.4567783)	(0.9,0.0000005)
C_2	(0.5,0.5064999)	(0.5,0.5)	(0.6,0.4632783)	(0.9,0.0065005)
C_3	(0.3,0.5432217)	(0.4,0.5367217)	(0.5,0.5)	(0.7,0.0432222)
C_4	(0.1,0.9999995)	(0.1,0.9934995)	(0.3,0.9567778)	(0.5,0.5)

We check the consistency of the preference relations in Table 4, and repair the inconsistent preference relations. The results after repair are shown in Table 5.

Table 5: The repaired preference relation of criteria with respect to the overall objective

$\overline{U}$	C_1	C_2	C_3	C_4
C_1	(0.5,0.5)	(0.5,0.4935001)	(0.62662689,0.45680912)	(0.88182123,0.00112069)
C_2	(0.49515217,0.50165227)	(0.5,0.5)	(0.6,0.4632783)	(0.81650303,0.02417208)
C_3	(0.41471401,0.58575809)	(0.446997,0.58414323)	(0.5,0.5)	(0.7,0.0432222)
C_4	(0.02500672,0.99417174)	(0.04842056,0.90138613)	(0.07410227,0.80521252)	(0.5,0.5)

According to Table 5, we can use (19) to calculate the priority vector:

$$\omega_1 = (0.307952, 0.665952),\ \omega_2 = (0.296069,\ 0.670889)$$
$$\omega_3 = (0.253108,\ 0.700253),\ \omega_4 = (0.079495,\ 0.895243)$$

From the remaining 147 instances of iris data set, the average values of three alternates are calculated as

$$Average(A_1) = [5.00612244898, 3.420408163267, 1.465306122449, 0.244897959184];$$
$$Average(A_2) = [5.94081632653, 2.769387755102, 4.263265306122, 1.326530612245];$$
$$Average(A_3) = [6.60204081633, 2.973469387755, 5.561224489796, 2.030612244898].$$

5.1.1 Experiment case 1

We select the last instance of the first class from the iris data set for an experiment, and judge the category of the example through the method proposed in this paper, and compare them with the actual situation to see whether the algorithm proposed in this paper can give a reasonable conclusion. The value of this instance is $I_1 = [5,\ 3.3,\ 1.4,\ 0.2]$.

R_1	A_1	A_2	A_3
A_1	0.5	0.1	0.1
A_2	0.9	0.5	0.3
A_3	0.9	0.7	0.5

Table 6: The preference relation of alternatives with respect to the criterion C_1 given by experts

R_2	A_1	A_2	A_3
A_1	0.5	0.2	0.4
A_2	0.8	0.5	0.7
A_3	0.6	0.3	0.5

Table 7: The preference relation of alternatives with respect to the criterion C_2 given by experts

R_3	A_1	A_2	A_3
A_1	0.5	0.1	0.1
A_2	0.9	0.5	0.3
A_3	0.9	0.7	0.5

Table 8: The preference relation of alternatives with respect to the criterion C_3 given by experts

R_4	A_1	A_2	A_3
A_1	0.5	0.2	0.1
A_2	0.8	0.5	0.4
A_3	0.9	0.6	0.5

Table 9: The preference relation of alternatives with respect to the criterion C_4 given by experts

Table 10: The preference relation of alternatives with respect to the criterion C_1 calculated by algorithm

R_1	A_1	A_2	A_3
A_1	0.5	0.7928387	0.9999997
A_2	0.2071613	0.5	0.707161
A_3	0.00000031	0.292839	0.5

Firstly, four preference relations of alternatives with respect to the criteria are given by experts, the four preference relations are shown in Table 6, Table 7, Table 8 and Table 9.

By comparing the value of I_1 with $Average(A_1)$, $Average(A_2)$ and $Average(A_3)$, we can get four preference relations of alternatives with respect to the criteria through the algorithm in section 4.2.2. The four preference relations are shown in Table 10, Table 11, Table 12 and Table 13.

We combine Table 6 and Table 10, Table 7 and Table 11, Table 8 and Table 12, Table 9 and Table 13 to form four preference relations of alternatives with respect to the criteria expressed by OPNs, as shown in Table 14, 15, 16 and 17.

We check the consistency of the four preference relations and fix the inconsistent preference relations. Finally, we get the preference relations shown in Tables 18, 19, 20 and 21.

Let $\omega_{ci} = [\omega_{1i}, \omega_{2i}, ..., \omega_{ji}]$ are weights of the alternatives over the criteria C_i . According to Table 18, 19, 20, and 21,we can calculate that
$\omega_{c1} = [\omega_{11}, \omega_{21}, \omega_{31}] = [(0.142768, 0.837404), (0.353982, 0.618301), (0.459583, 0.498633)]$;
$\omega_{c2} = [\omega_{12}, \omega_{22}, \omega_{32}] = [(0.258751, 0.900595), (0.530707, 0.562021), (0.423963, 0.713268)]$;
$\omega_{c3} = [\omega_{13}, \omega_{23}, \omega_{33}] = [(0.143234, 0.849369), (0.362278, 0.606369), (0.453793, 0.501840)]$;
$\omega_{c4} = [\omega_{14}, \omega_{24}, \omega_{34}] = [(0.190357, 0.843941), (0.386970, 0.653775), (0.475120, 0.552118)]$.

Table 11: The preference relation of alternatives with respect to the criterion C_2 calculated by algorithm

R_2	A_1	A_2	A_3
A_1	0.5	0.9999988	0.7512432
A_2	0.00000121	0.5	0.2512444
A_3	0.24875683	0.7487556	0.5

Table 12: The preference relation of alternatives with respect to the criterion C_3 calculated by algorithm

R_3	A_1	A_2	A_3
A_1	0.5	0.8415545	0.9999999
A_2	0.15844552	0.5	0.6584454
A_3	0.00000012	0.3415546	0.5

Table 13: The preference relation of alternatives with respect to the criterion C_4 calculated by algorithm

R_4	A_1	A_2	A_3
A_1	0.5	0.802857	0.9999997
A_2	0.19714302	0.5	0.6971427
A_3	0.00000027	0.3028573	0.5

Table 14: The preference relation of alternatives with respect to the criterion C_1

R_1	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.1,0.79283869)	(0.1,0.99999968)
A_2	(0.9,0.2071613)	(0.5,0.5)	(0.3,0.70716099)
A_3	(0.9,0.00000031)	(0.7,0.292839)	(0.5,0.5)

Table 15: The preference relation of alternatives with respect to the criterion C_2

R_2	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.2,0.99999878)	(0.4,0.75124316)
A_2	(0.8,0.00000121)	(0.5,0.5)	(0.7,0.25124438)
A_3	(0.6,0.24875683)	(0.3,0.74875561)	(0.5,0.5)

Table 16: The preference relation of alternatives with respect to the criterion C_3

R_3	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.1,0.84155447)	(0.1,0.99999987)
A_2	(0.9,0.15844552)	(0.5,0.5)	(0.3,0.6584454)
A_3	(0.9,0.00000012)	(0.7,0.34155459)	(0.5,0.5)

Table 17: The preference relation of alternatives with respect to the criterion C_4

R_4	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.2,0.80285697)	(0.1,0.99999972)
A_2	(0.8,0.19714302)	(0.5,0.5)	(0.4,0.69714274)
A_3	(0.9,0.00000027)	(0.6,0.30285725)	(0.5,0.5)

Table 18: The preference relation of alternatives with respect to the criterion C_1 (after repair)

R_1	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.1,0.79283869)	(0.05464577,0.99415464)
A_2	(0.82313625,0.11903456)	(0.5,0.5)	(0.3,0.70716099)
A_3	(0.90182665,0.00308994)	(0.70553163,0.29835298)	(0.5,0.5)

Table 19: The preference relation of alternatives with respect to the criterion C_2 (after repair)

R_2	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.2,0.99999878)	(0.37262816,0.999983)
A_2	(0.99999363,0.04567091)	(0.5,0.5)	(0.7,0.25124438)
A_3	(0.99998131,0.350808)	(0.25751346,0.70689509)	(0.5,0.5)

Table 20: The preference relation of alternatives with respect to the criterion C_3 (after repair)

R_3	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.1,0.84155447)	(0.0557691,0.99687514)
A_2	(0.85862229,0.11272833)	(0.5,0.5)	(0.3,0.6584454)
A_3	(0.90833116,0.00179437)	(0.66927423,0.3102892)	(0.5,0.5)

Finally, we calculate the score W_i of alternative A_i.

$W_1 = \bigoplus_{j=1}^{4} (\omega_j \otimes \omega_{1j})$
$= (0.307952, 0.665952) \otimes (0.142768, 0.837404) \oplus (0.296069,\ 0.670889) \otimes (0.258751, 0.900595)$
$\oplus\ (0.253108,\ 0.700253) \otimes (0.143234, 0.849369) \oplus (0.079495,\ 0.895243) \otimes (0.190357, 0.843941)$
$= (0.9999999966,\ 0.0000000045)$

$$W_2 = \bigoplus_{j=1}^{4} (\omega_j \otimes \omega_{2j}) = (0.9992590903, 0.0008351801)$$

$$W_3 = \bigoplus_{j=1}^{4} (\omega_j \otimes \omega_{3j}) = (0.9740059483, 0.0330835253)$$

As described in Definition 3, because $s(\alpha) = \mu_\alpha - v_\alpha$, so

$$s(W_1) = 0.9999999921,$$
$$s(W_2) = 0.9984239102,$$
$$s(W_3) = 0.9409224230.$$

Because $s(W_1) > s(W_2) > s(W_3)$, so we can judge that the instance belonging to alternative A_1 (Iris Setosa), This is consistent with the actual situation, so we can think that the decision is accurate.

5.1.2 Experiment case 2

We select the last instance of the second class from iris data set for experiment, and judge the category of the instance through the method proposed in this

Table 21: The preference relation of alternatives with respect to the criterion C_4 (after repair)

R_4	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.2,0.80285697)	(0.13751022,0.97452033)
A_2	(0.80254587,0.19968589)	(0.5,0.5)	(0.4,0.69714274)
A_3	(0.90321921,0.03730346)	(0.68715852,0.38880805)	(0.5,0.5)

R_1	A_1	A_2	A_3
A_1	0.5	0.7	0.2
A_2	0.3	0.5	0.1
A_3	0.8	0.9	0.5

Table 22: The preference relation of alternatives with respect to the criterion C_1 given by experts

R_2	A_1	A_2	A_3
A_1	0.5	0.8	0.8
A_2	0.2	0.5	0.5
A_3	0.2	0.5	0.5

Table 23: The preference relation of alternatives with respect to the criterion C_2 given by experts

paper, and compare with the actual situation to see whether the algorithm proposed in this paper can give a reasonable conclusion. The value of this example is $I_1 = [5.7,\ 2.8,\ 4.1,\ 1.3]$.

Firstly, four preference relations of alternatives with respect to the criteria are given by experts, the four preference relations are shown in Table 22, Table 23, Table 24 and Table 25.

R_3	A_1	A_2	A_3
A_1	0.5	0.9	0.8
A_2	0.1	0.5	0.2
A_3	0.2	0.8	0.5

Table 24: The preference relation of alternatives with respect to the criterion C_3 given by experts

R_4	A_1	A_2	A_3
A_1	0.5	0.95	0.75
A_2	0.05	0.5	0.2
A_3	0.25	0.8	0.5

Table 25: The preference relation of alternatives with respect to the criterion C_4 given by experts

By comparing the value of I_1 with $Average(A_1)$, $Average(A_2)$ and $Average(A_3)$, and through the algorithm in section 4.2.2, we can get four preference relations of alternatives with respect to the criteria. The four preference relations are shown in Table 26, Table 27, Table 28 and Table 29.

We combine Table 22 and Table 26, Table 23 and Table 27, Table 24 and Table 28, Table 25 and Table 29 to form four preference relations of alternatives with respect to the criteria expressed by OPNs, as shown in Table 30, 31, 32 and 33.

We check the consistency of the four preference relations and fix the inconsistent preference relations. Finally, we get the preference relations shown in Tables 34, 35, 36 and 37.

Let $\omega_{ci} = [\omega_{1i}, \omega_{2i}, ..., \omega_{ji}]$ are weights of the alternatives over the criteria C_i. According to Table 34,35,36,37, we can calculate that
$\omega_{c1} = [\omega_{11}, \omega_{21}, \omega_{31}] = [(0.322214, 0.713823), (0.180672, 0.838520), (0.573571, 0.518683)]$;

Table 26: The preference relation of alternatives with respect to the criterion C_1 calculated by algorithm

R_1	A_1	A_2	A_3
A_1	0.5	0.15740792	0.65740716
A_2	0.84259207	0.5	0.99999924
A_3	0.34259283	0.00000075	0.5

Table 27: The preference relation of alternatives with respect to the criterion C_2 calculated by algorithm

R_2	A_1	A_2	A_3
A_1	0.5	0.00000084	0.1211079
A_2	0.99999915	0.5	0.62110706
A_3	0.87889209	0.37889293	0.5

Table 28: The preference relation of alternatives with respect to the criterion C_3 calculated by algorithm

R_3	A_1	A_2	A_3
A_1	0.5	0.0000002	0.26259299
A_2	0.99999979	0.5	0.76259279
A_3	0.737407	0.2374072	0.5

Table 29: The preference relation of alternatives with respect to the criterion C_4 calculated by algorithm

R_4	A_1	A_2	A_3
A_1	0.5	0.00000048	0.34226205
A_2	0.99999951	0.5	0.84226157
A_3	0.65773794	0.15773842	0.5

Table 30: The preference relation of alternatives with respect to the criterion C_1

R_1	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.7,0.15740792)	(0.2,0.65740716)
A_2	(0.3,0.84259207)	(0.5,0.5)	(0.1,0.99999924)
A_3	(0.8,0.34259283)	(0.9,0.00000075)	(0.5,0.5)

Table 31: The preference relation of alternatives with respect to the criterion C_2

R_2	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.8,0.00000084)	(0.8,0.1211079)
A_2	(0.2,0.99999915)	(0.5,0.5)	(0.5,0.62110706)
A_3	(0.2,0.87889209)	(0.5,0.37889293)	(0.5,0.5)

Table 32: The preference relation of alternatives with respect to the criterion C_3

R_3	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.9,0.0000002)	(0.8,0.26259299)
A_2	(0.1,0.99999979)	(0.5,0.5)	(0.2,0.76259279)
A_3	(0.2,0.737407)	(0.8,0.2374072)	(0.5,0.5)

Table 33: The preference relation of alternatives with respect to the criterion C_4

R_4	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.95,0.00000048)	(0.75,0.34226205)
A_2	(0.05,0.99999951)	(0.5,0.5)	(0.2,0.84226157)
A_3	(0.25,0.65773794)	(0.8,0.15773842)	(0.5,0.5)

Table 34: The preference relation of alternatives with respect to the criterion C_1 (after repair)

R_1	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.7,0.15740793)	(0.20437509,0.99991946)
A_2	(0.18746302,0.74237707)	(0.5,0.5)	(0.1,0.99999924)
A_3	(0.99993318,0.23641019)	0.99998444,0.00537034)	(0.5,0.5)

Table 35: The preference relation of alternatives with respect to the criterion C_2 (after repair)

R_2	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.8,0.00000085)	(0.8,0.00000563)
A_2	(0.00000392,0.948703686)	(0.5,0.5)	(0.5,0.621107061)
A_3	(0.000006050.81133619)	0.60686673,0.48498221)	(0.5,0.5)

$\omega_{c2} = [\omega_{12,}\omega_{22,}\omega_{32}] = [(0.453185, 0.405736), (0.215803, 0.778888), (0.238866, 0.713878)]$;
$\omega_{c3} = [\omega_{13,}\omega_{23,}\omega_{33}] = [(0.444075, 0.392555), (0.145424, 0.820657), (0.265434, 0.617107)]$;
$\omega_{c4} = [\omega_{14,}\omega_{24,}\omega_{34}] = [(0.474717, 0.414192), (0.148229, 0.845773), (0.280261, 0.632868)]$.

Finally, we calculate the score W_i of alternative A_i.

$$W_1 = \bigoplus_{j=1}^{4} (\omega_j \otimes \omega_{1j}) = (0.5209125426,\ 0.1326041202),$$

$$W_2 = \bigoplus_{j=1}^{4} (\omega_j \otimes \omega_{2j}) = (0.9999999952, 0.0000000046),$$

$$W_3 = \bigoplus_{j=1}^{4} (\omega_j \otimes \omega_{3j}) = (0.9999127478, 0.0000643935).$$

As described in Definition 3, because $s(\alpha) = \mu_\alpha - v_\alpha$, so

$$s(W_1) = 0.3883084224,$$
$$s(W_2) = 0.9999999906,$$
$$s(W_3) = 0.9998483543.$$

Because $s(W_2) > s(W_3) > s(W_1)$, so we can judge that the instance belonging to alternative A_2 (Iris Versicolour), This is consistent with the actual situation, so we can think that the decision is accurate.

5.2 Examples with experts give invalid or unreasonable judgments

Similar to the examples when experts give reasonable judgments in section 5.1 ,we compare the importance of the criteria to the overall goal. Preference

Table 36: The preference relation of alternatives with respect to the criterion C_3 (after repair)

R_3	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.9,0.0000002)	0.73765931,0.00010866)
A_2	(0.00003386,0.9993366)	(0.5,0.5)	(0.2,0.76259279)
A_3	(0.00009475,0.71028823)	(0.77763324,0.21394382)	(0.5,0.5)

Table 37: The preference relation of alternatives with respect to the criterion C_4 (after repair)

R_4	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.95,0.00000049)	(0.79221875,0.00089088)
A_2	(0.00012436,0.99979433)	(0.5,0.5)	(0.2,0.84226157)
A_3	(0.00072007,0.75497543)	(0.8230271,0.17880654)	(0.5,0.5)

Table 38: Preference relation of criteria with respect to the overall objective given by experts (invalid).

U_1	C_1	C_2	C_3	C_4
C_1	0.5	0.5	0.5	0.5
C_2	0.5	0.5	0.5	0.5
C_3	0.5	0.5	0.5	0.5
C_4	0.5	0.5	0.5	0.5

relation of criteria with respect to the overall objective which was given by experts is shown in Table 38.

It can be seen from Table 38 that the standard preference relationship related to the overall goal given by the experts is invalid (all values are 0.5), which cannot give decisions in the previous FAHP. However, in the method proposed in this paper, we can use the results given by machine learning algorithms to modify the results given by experts. The preference relation of criteria with respect to the overall objective calculated by the machine learning algorithm can be seen in Table 2.

We have synthesized the preference relations given in Table 1 and Table 2, and obtained the final preference relation of criteria with respect to the overall objective which was expressed in OPNs. The final preference relation of criteria with respect to the overall objective is shown in Table 39.

We check the consistency of the preference relations in Table 39, and repair the inconsistent preference relations. The results after repair are shown in Table 40.

According to Table 40, we can use (10) to calculate the priority vector:

$$\omega_1 = (0.222379,\ 0.611806),\ \omega_2 = (0.221727,\ 0.617862),$$
$$\omega_3 = (0.214366,\ 0.626333),\ \omega_4 = (0.070227,\ 0.771689).$$

From the remaining 147 instances of iris data set, the average values of three alternates are calculated as

$$Average(A_1) = [5.00612244898, 3.420408163267, 1.465306122449, 0.244897959184];$$

Table 39: The preference relation of criteria with respect to the overall objective

U	C_1	C_2	C_3	C_4
C_1	(0.5,0.5)	(0.5,0.4935001)	(0.5,0.4567783)	(0.5,0.0000005)
C_2	(0.5,0.5064999)	(0.5,0.5)	(0.5,0.4632783)	(0.5,0.0065005)
C_3	(0.5,0.5432217)	(0.5,0.5367217)	(0.5,0.5)	(0.5,0.0432222)
C_4	(0.5,0.9999995)	(0.5,0.9934995)	(0.5,0.9567778)	(0.5,0.5)

Table 40: The repaired preference relation of criteria with respect to the overall objective

$\overline{U}$	C_1	C_2	C_3	C_4
C_1	(0.5,0.5)	(0.5,0.4935001)	(0.5,0.45681556)	(0.5 ,0.00558671)
C_2	(0.49414013,0.50064013)	(0.5,0.5)	(0.5,0.4632783)	(0.5, 0.03167166)
C_3	(0.46105374,0.50426689)	(0.46688338,0.50362271)	(0.5,0.5)	(0.5, 0.0432222)
C_4	(0.02291032,0.80670965)	(0.05093762,0.62134858)	(0.05774648,0.57566555)	(0.5,0.5)

R_1	A_1	A_2	A_3
A_1	0.5	0.5	0.5
A_2	0.5	0.5	0.5
A_3	0.5	0.5	0.5

Table 41: The preference relation of alternatives with respect to the criterion C_1 given by experts (invalid)

R_2	A_1	A_2	A_3
A_1	0.5	0.5	0.5
A_2	0.5	0.5	0.5
A_3	0.5	0.5	0.5

Table 42: The preference relation of alternatives with respect to the criterion C_2 given by experts (invalid)

$$Average(A_2) = [5.94081632653, 2.769387755102, 4.263265306122, 1.326530612245];$$
$$Average(A_3) = [6.60204081633, 2.973469387755, 5.561224489796, 2.030612244898].$$

We also select the last instance of the second class from iris data set for experiment, the value of this instance is $I_1 = [5.7,\ 2.8,\ 4.1,\ 1.3]$ (consistent with instance in section 5.2.2).

Firstly, four preference relations of alternatives with respect to the criteria are given by experts, the four preference relations are shown in Table 41, Table 42, Table 43 and Table 44.

From Table 41 to Table 44, we can see all values are 0.5, the preference relation of alternatives with respect to the criteria given by expert are invalid.

The preference relation of alternatives with respect to the criteria calculated by algorithm are shown in Table 26,27,28,29. We combine Table 41 and Table 26, Table 42 and Table 27, Table 43 and Table 28, Table 44 and Table 29 to form four preference relations of alternatives with respect to the criteria expressed by OPNs, as shown in Table 45,46,47 and 48.

We check the consistency of the four preference relations and fix the inconsistent preference relations. Finally, we get the preference relations shown in Tables 49,50,51 and 52.

Let $\omega_{ci} = [\omega_{1i}, \omega_{2i}, ..., \omega_{ji}]$ are weights of the alternatives over the criteria C_i . According to Table 49,50,51,52, we can calculate that

R_1	A_1	A_2	A_3
A_1	0.5	0.5	0.5
A_2	0.5	0.5	0.5
A_3	0.5	0.5	0.5

Table 43: The preference relation of alternatives with respect to the criterion C_3 given by experts (invalid)

R_2	A_1	A_2	A_3
A_1	0.5	0.5	0.5
A_2	0.5	0.5	0.5
A_3	0.5	0.5	0.5

Table 44: The preference relation of alternatives with respect to the criterion C_4 given by experts (invalid)

Table 45: The preference relation of alternatives with respect to the criterion C_1

R_1	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.5,0.15740792)	(0.5,0.65740716)
A_2	(0.5,0.84259207)	(0.5,0.5)	(0.5,0.99999924)
A_3	(0.5,0.34259283)	(0.5,0.00000075)	(0.5,0.5)

Table 46: The preference relation of alternatives with respect to the criterion C_2

R_2	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.5,0.00000084)	(0.5,0.1211079)
A_2	(0.5,0.99999915)	(0.5,0.5)	(0.5,0.62110706)
A_3	(0.5,0.87889209)	(0.5,0.37889293)	(0.5,0.5)

Table 47: The preference relation of alternatives with respect to the criterion C_3

R_3	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.5,0.0000002)	(0.5,0.26259299)
A_2	(0.5,0.99999979)	(0.5,0.5)	(0.5,0.76259279)
A_3	(0.5,0.737407)	(0.5,0.2374072)	(0.5,0.5)

Table 48: The preference relation of alternatives with respect to the criterion C_4

R_4	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.5,0.00000048)	(0.5,0.34226205)
A_2	(0.5,0.99999951)	(0.5,0.5)	(0.5,0.84226157)
A_3	(0.5,0.65773794)	(0.5,0.15773842)	(0.5,0.5)

Table 49: The preference relation of alternatives with respect to the criterion C_1 (after repair)

R_1	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.5,0.15740792)	(0.5,0.99999292)
A_2	(0.16817413 0.5197445)	(0.5,0.5)	(0.5,0.99999924)
A_3	(0.9999927 0.49232591)	(0.99999852 0.33977916)	(0.5,0.5)

Table 50: The preference relation of alternatives with respect to the criterion C_2 (after repair)

R_2	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.5,0.00000084)	(0.5,0.00000236836964)
A_2	(0.00000162350337,0.658897089)	(0.5,0.5)	(0.5,0.62110706)
A_3	(0.00000260011426,0.523321519)	(0.615613312,0.494180312)	(0.5,0.5)

Table 51: The preference relation of alternatives with respect to the criterion C_3 (after repair)

R_3	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.5,0.0000002)	(0.5,0.00000112541441)
A_2	(0.000000384597989,0.657418936)	(0.5,0.5)	(0.5,0.76259279)
A_3	(0.00000117576809,0.510940839)	(0.753520814,0.487635458)	(0.5,0.5)

Table 52: The preference relation of alternatives with respect to the criterion C_4 (after repair)

R_4	A_1	A_2	A_3
A_1	(0.5,0.5)	(0.5,0.00000048)	(0.5,0.00000430271371)
A_2	(0.000000889407021,0.649284402)	(0.5,0.5)	(0.5,0.84226157)
A_3	(0.00000442352536,0.506922351)	(0.832596325,0.482254542)	(0.5,0.5)

$$W_1 = \bigoplus_{j=1}^{4} (\omega_j \otimes \omega_{1j}) = (0.2907763691, 0.0008584455)$$
$$W_2 = \bigoplus_{j=1}^{4} (\omega_j \otimes \omega_{2j}) = (0.9999994677, 0.0000000953)$$
$$W_3 = \bigoplus_{j=1}^{4} (\omega_j \otimes \omega_{3j}) = (0.9997186612, 0.0000578631)$$

As described in Definition 3, because $s(\alpha) = \mu_\alpha - v_\alpha$, so

$$s(W_1) = 0.2899179236,$$
$$s(W_2) = 0.9999993724,$$
$$s(W_3) = 0.9996607981.$$

Because $s(W_2) > s(W_3) > s(W_1)$, so we can judge that the instance belonging to alternative A_2 (Iris Versicolour), This is consistent with the actual situation, so we can think that the decision is accurate. When experts gives wrong or invalid judgment, the expert's judgment is corrected by the machine learning algorithm, so that the correct conclusion is still drawn in the end.

6 Conclusion

This paper uses OPNs, a new fuzzy number theory, to improve the original fuzzy analytic hierarchy process, and proposed the FAHP with OPNs for the first time. The authors use the machine learning algorithm to improve the disadvantages that the original fuzzy analytic hierarchy process depends too much on the subjective opinions of experts, and avoid the decision-making errors caused by the deviation of expert opinions to a certain extent. We show that the algorithm proposed in this paper can get reasonable results through two examples. Finally, we use an example to show that when the expert can not give a correct (or effective) judgment, the method proposed in this paper allows the judgment given by the machine learning algorithm to correct the invalid judgment given by the expert. So that In the end, this method can still draw reasonable conclusions.

In this paper, the weight of subjective judgment given by experts and objective judgment given by the machine learning algorithm is the same. The future work could consider giving different weights to the subjective judgment given by experts and the objective judgment given by the machine learning algorithms. It could adapt to more complex situations in real society better.

Acknowledgements

This work was supported by the key R&D project of the science and technology department of Sichuan Province, China [grant number: 2021YFG0031]. And the provincial scientific research institutes' achievement transformation project of the science and technology department of Sichuan Province, China [grant number:22YSZH0021].

References

[1] Vaidya, O.S., Kumar, S.: Analytic hierarchy process: An overview of applications. European Journal of operational research **169**(1), 1–29 (2006)

[2] Ho, W., Ma, X.: The state-of-the-art integrations and applications of the analytic hierarchy process. European Journal of Operational Research **267**(2), 399–414 (2018)

[3] Kahraman, C., Öztayşi, B., Çevik Onar, S.: A comprehensive literature review of 50 years of fuzzy set theory. International Journal of Computational Intelligence Systems **9**(sup1), 3–24 (2016)

[4] Ruspini, E.H., Bezdek, J.C., Keller, J.M.: Fuzzy clustering: A historical perspective. IEEE Computational Intelligence Magazine **14**(1), 45–55 (2019)

[5] Bellman, R.E., Zadeh, L.A.: Decision-making in a fuzzy environment. Management science **17**(4), 141 (1970)

[6] Blanco-Mesa, F., Merigó, J.M., Gil-Lafuente, A.M.: Fuzzy decision making: A bibliometric-based review. Journal of Intelligent & Fuzzy Systems **32**(3), 2033–2050 (2017)

[7] Atanassov, K.T.: More on intuitionistic fuzzy sets. Fuzzy sets and systems **33**(1), 37–45 (1989)

[8] Li, D.-F.: Multiattribute decision making models and methods using intuitionistic fuzzy sets. Journal of computer and System Sciences **70**(1), 73–85 (2005)

[9] Xu, Z., Liao, H.: Intuitionistic fuzzy analytic hierarchy process. IEEE transactions on fuzzy systems **22**(4), 749–761 (2013)

[10] Jiang, H., Mao, H., Lu, H., Lin, P., Chen, X.: Machine learning-based models to support decision-making in emergency department triage for patients with suspected cardiovascular disease. International Journal of Medical Informatics **145**, 104326 (2021)

[11] Laat, D., Paul, B.: Algorithmic decision-making based on machine learning from big data: Can transparency restore accountability? Philosophy & Technology (2017)

[12] Buckley, J.J.: Fuzzy hierarchical analysis. Fuzzy sets and systems **17**(3), 233–247 (1985)

[13] Yucesan, M., Gul, M.: Failure modes and effects analysis based on neutrosophic analytic hierarchy process: method and application. Soft Computing, 1–18 (2021)

[14] Wu, G., Xu, J.: Optimized approach of feature selection based on information gain. In: 2015 International Conference on Computer Science and Mechanical Automation (CSMA), pp. 157–161 (2015). IEEE

[15] Xia, M., Xu, Z.: Some studies on properties of hesitant fuzzy sets. International Journal of Machine Learning and Cybernetics **8**(2), 1–7 (2015)

[16] Öztaysi, B., Onar, S.Ç., Boltürk, E., Kahraman, C.: Hesitant fuzzy analytic hierarchy process. In: 2015 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), pp. 1–7 (2015). IEEE

[17] Peng, X., Selvachandran, G.: Pythagorean fuzzy set: state of the art and future directions. Artificial Intelligence Review (1), 1–55 (2017)

[18] Peng, X., Selvachandran, G.: Pythagorean fuzzy set: state of the art and future directions. Artificial Intelligence Review **52**(3), 1873–1927 (2019)

[19] Mohd, W.R.W., Abdullah, L.: Pythagorean fuzzy analytic hierarchy process to multi-criteria decision making. In: AIP Conference Proceedings, vol. 1905, p. 040020 (2017). AIP Publishing LLC

[20] Kutlu Gündoğdu, F., Kahraman, C.: Spherical fuzzy sets and spherical fuzzy topsis method. Journal of intelligent & fuzzy systems **36**(1), 337–352

(2019)

[21] Gündoğdu, F.K., Kahraman, C.: A novel spherical fuzzy analytic hierarchy process and its renewable energy application. Soft Computing **24**(6), 4607–4621 (2020)

[22] Yasser, YasamiSaadat, Pour, Mozaffari: A novel unsupervised classification approach for network anomaly detection by k-means clustering and id3 decision tree learning methods. Journal of Supercomputing (2010)

[23] Damanik, I.S., Windarto, A.P., Wanto, A., Andani, S.R., Saputra, W., *et al.*: Decision tree optimization in c4. 5 algorithm using genetic algorithm. In: Journal of Physics: Conference Series, vol. 1255, p. 012012 (2019). IOP Publishing

[24] Rutkowski, L., Jaworski, M., Pietruczuk, L., Duda, P.: The cart decision tree for mining data streams. Information Sciences **266**, 1–15 (2014)

[25] Zhou, L.: Ordered pair of normalized real numbers. Information Sciences **538**, 290–313 (2020)

[26] Bhardwaj, R., Vatta, S.: Implementation of id3 algorithm. International Journal of Advanced Research in Computer Science and Software Engineering **3**(6) (2013)

[27] Guan, C., Zeng, X.: An improved id3 based on weighted modified information gain. In: 2011 Seventh International Conference on Computational Intelligence and Security, pp. 1283–1285 (2011). IEEE

[28] Dua, D., Graff, C.: UCI Machine Learning Repository (2017). http://archive.ics.uci.edu/ml

[29] Saaty, T.L.: A scaling method for priorities in hierarchical structures. Journal of mathematical psychology **15**(3), 234–281 (1977)

Statements and Declarations

Funding

This work was supported by the key R&D project of the science and technology department of Sichuan Province, China [grant number: 2021YFG0031]. And the provincial scientific research institutes' achievement transformation project of the science and technology department of Sichuan Province, China [grant number:22YSZH0021].

Competing Interests

The authors declare that they have no conflict of interest.

Data availability

All data generated or analysed during this study are included in this article.