Natural Digital image Mixed Noise Removal Using Regularization Perona-Malik Model and Pulse Coupled Neural Networks

Asem Khamg (khmaj2002@gmail.com)
university of zawia https://orcid.org/0000-0002-1360-5346

Research Article

Keywords: Image filter, mixed noise, PCNN, Gaussian noise, wavelet transformation, noise removal, P-M equation

Posted Date: July 12th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-1502808/v1

Natural Digital image Mixed Noise Removal Using Regularization Perona-Malik Model and Pulse Coupled Neural Networks

Asem Khmag*[1]

[1*]Department of Computer Systems Engineering, Faculty of Engineering, University of Zawia, Zawia, Libya
a.khmag@zu.edu.ly

Abstract In the arena of digital image processing, mixed noise is normally shown. Nevertheless, the main limitation of the existing noise removal methods is the exploitation of the statistics of the original contaminated image only. This paper proposes a rapid and high accurate mixed noise removal method by combining Pulse Coupled Neural Networks (PCNN) and regularization of Perona-Malik equation (P-M equation) in order to remove unwanted contamination. In this regard, the locations of impulse noise are positioned by using PCNN, the second-generation wavelet filter is used in order to suppress the mixed noise into small local neighborhood, and then the full noisy image is denoised by exploiting the regularization of P-M equation. The fine details and sharp edges are well preserved in the proposed method. In addition, subjective and objective analyses are showed that the visual quality of the denoised technique outperformed state of the art noise removal methods. The experimental evaluation is conducted on well-known benchmark database images with mixed noise type. Furthermore, the results show that our proposed method can obtain more accurate and more reliable noise removal images than other approaches. Experimental results show that the proposed method provides accurate denoising performance with a low computational cost compared to nonlocal processing like NL-mean method.

Keywords Image filter, mixed noise, PCNN, Gaussian noise, wavelet transformation, noise removal, P-M equation

I. INTRODUCTION

Noise in image signal processing applications is inevitable issue especially when it comes to generation and acquisition processes. Several types of noises can be occurred to the digital images such as additive Gaussian noise, mixed noise and impulse noise. The existing of these types of noise may affect the quality of the whole scene that probably influences the next step in image processing application such as pattern recognition, feature selection and extraction etc. Practically, many researches have been conducted in digital image noise removal in mixed, impulse and Gaussian noise types .Categories such as classical filters are studied in [1]-[4]. However, demerits have been shown in the denoised images that obtained by these filters due to high level noise, distortions and blurred edges. In mixed noise type, there were some studies conducted [5-9]; the image quality has shown improvements compared to the techniques in classical filters up mentioned. On the other hand, some defects are shown up when it comes to the practical applications. Linear filter such as median filter was designed efficiently to deal with impulse noise, but in high noise levels, its efficiency will decrease [10]. Study that is done in [6] has exploited weighted encoding that used nonlocal mean and sparse matrix for mixed noise removal. It shows high performance as they divide the contaminated image to sub-pixels. In study that introduced [8], a fuzzy rule system that exploited averaging weight is proposed. In this method, the mixed and Gaussian noise types are suppressed. Another study that exploited Genetic programing is proposed by [11]. The additive Gaussian noise are located based on the synchronization pulse distribution where the characteristics of the pulse contaminated coupled in clusters to ease remove the noise and filter the corrupted pixels. Several filters are designed based on window size of the noisy images where some are according to fixed size and the other based on varying in window size. Block matching and 3D (BM3D) filtering [12] is well-known as the state-of-the-art Gaussian noise removal method. The main idea of BM3D is to stack similar patches into 3-D fragments by block matching. Hard-Thresholding and Wiener-filtering are used to remove noise in the first and second steps. The denoised image is obtained by aggregating the inverse 3-D transformed patches. Learned simultaneous sparse coding (LSSC) [13], [14] is a nonlocal framework that combines nonlocal means and sparse coding with the help of learned dictionaries to restore the image. As mentioned above, some techniques show low image quality after denoising processes due many reasons such as false positioning, lots of factors in the noise removal method and high noise density in some cases that reduce the quality of the denoised images. Therefore, the proposed study is based on developed a technique of mixed image noise approach with regularization P-M equation according to pulse coupled neural networks (PCNN) in order to suppress the mixed noise and achieve high image quality. The main steps of the proposed framework are implemented as: Firstly, the locations of mixed noise pixels are stated by PCNN. Secondly, the mixed noise pixels are filtered using second-generation wavelets. Finally, the residual of the noise is denoised by the regularization P-M diffusion equation.

II. TRADITIONAL PCNN MODEL

As mentioned earlier, PCNN is considered as third generation of artificial neural network [15]. Practically, the neurons of PCNN have the same structure; this feature makes PCNN widely used in many digital image-processing applications such as image segmentation, enhancement, and image noise removal to name a few [3], [9] and [16]. Three main parts are the pivotal sections of PCNN neuron, receptive part, modulation part and finally pulse generator part, the following formulas describe the structure of PCNN:

$$F_{ij}[n] = e^{-\alpha_F}F_{ij}[n-1] + V_F \sum_{kl} M_{ijkl}Y\,[n-1] + S_{ij} \quad (1)$$
$$L_{ij}[n] = e^{-\alpha_L}L_{ij}[n-1] + V_L \sum_{kl} W_{ijkl}Y\,[n-1] \quad (2)$$
$$U_{ij}[n] = F_{ij}[n](1+\beta L_{ij}[n]) \quad (3)$$
$$\theta_{ij}[n] = e^{-\alpha_\theta}\theta_{ij}[n-1] + V_\theta Y_{ij}[n] \quad (4)$$
$$V_{ij} = \begin{cases} 1, U_{ij}[n] > \theta_{ij}[n] \\ 0, otherwise \end{cases} \quad (5)$$

Where ij denotes the label of each neuron, $S_{ij}, F_{ij}, L_{ij}, U_{ij}, and\ \theta_{ij}$ represent the exterior simulation of each neuron, feedback, linking inputs, core activity and vibrant threshold correspondingly. Linking weight matrices are labeled by M and W, the amplitude constants are denoted by V_F, $V_L$and V_θ, the cyclic iterative time is denoted by n, where α_F, $\alpha_L$and $\alpha_\theta$are the time decay constants, β demonstrates the linking strength, and finally, Y_{ij} represents output of the binary value.

A. Proposed PCNN Model

As can be seen in the previous section, traditional PCNN procedure showed defects in its applications in the field of digital image processing. In order to conquer the demerits such as various artificial set parameters, variation in low quality performance and its tend to repeat the decline of exponential function, the conventional PCNN procedure is improved based on the study in literature [3]. The mathematical formula of the proposed model can be described as follows:

$$F_{ij}[n] = S_{ij}[n] \quad (6)$$
$$L_{ij}[n] = V_L \sum_{kl} W_{ijkl}Y_{ij}[n-1] \quad (7)$$
$$U_{ij}[n] = F_{ij}[n](1+\beta L_{ij}[n]) \quad (8)$$
$$Y_{ij} = \begin{cases} 1, (U_{ij}[n] > T_{ij}[n]) \\ 0, ((U_{ij}[n] \le T_{ij}[n]) \end{cases} \quad (9)$$
$$(\theta_{ij}[n] = \theta_0 e^{-\alpha\theta(n-1)} \quad (10)$$

Where ijrepresent the neuron label, and $S_{ij}, F_{ij}, L_{ij}, U_{ij}, \theta_{ij}$ indicate the external stimulation of each neuron, feedback input, linking input and interior activity, and vibrant threshold correspondingly. In addition, W represents the linking weight matrices, while V_L, θ_0 are amplitude constants, and n is a cyclic iterative time, $\alpha_\theta$is time decay constant, Y_{ij}represents the binary value output. Two-dimensional PCNN network are related to two-dimensional input noisy image when PCNN is taken as an input process. For the first cyclic iteration, the internal activity of neuron is stated to be as equal to external stimulation S_{ij}. In the case where $S_{ij}$is larger than specific threshold, then output neuron is stated as 1 which is the natural activation. Furthermore, the threshold θ_{ij} at this point will be rapidly increased, and as a result, it will have exponential attenuation form with the increasing in time. Consequently, the activated neurons stimulate the neighboring neurons by linking them up with the adjacent neurons. In case the internal activity of adjacent neuron shows higher value than the specific threshold, then it will be fired and activated.

III. P-M MODEL OPTIMIZATION

In the field of digital image processing, partial differential model can be stated as suggested by [17]:

$$\frac{\partial I}{\partial t} = F((I, x, y, t)) \quad (11)$$

Isotropic image smoothing conductivity model is written as follows:

$$\frac{\partial\ I(x,y,t)}{\partial t} = \Delta I(x, y, t) \quad (12)$$

Where I represents the digital image in the change process while F is the format of a given approach; $\Delta I\ (x, y, t)$ is Laplacian image expression, and $I\ (x, y, 0)$ indicated the original condition. Discrete procedure of continuous format of formula equation (12) is represented by:

$$I_{ij}(n) = I_{ij}(n-1) + \frac{\lambda}{m_{ij}} \sum_{pem_{ij}} \nabla I_{ij,p} (13)$$

Where n is the iteration number, while λ represents the distribution coefficient weights, and $m_{ij}$is the neighbor point number. In addition, the features of the digital image should be reserved while the contaminated image is smoothed. The spatial location of each pixel in the tested image should stay independent as considered a linear scale in conduction equations. To be sure of maintaining the features of the noisy image in all directions, Perona and Malik have come up with anisotropy coefficient model $C\ (\mu)$ to smooth the image, and implemented forward nonlinear anisotropic partial differential equations approach [18].

$$\frac{\partial I}{\partial t} = div\ (C(\lfloor \nabla I \rfloor) \nabla I) (14)$$

div represents the divergence operator where diffusion coefficient $C(\nabla I\ ) \in [0,1]$ can be stated as decreasing function that has independent variable known as gradient amplitude.

$$C(0) =\ 1, \lim lim_{\lambda \to \infty}\ C(\mu) = 0 \ (15)$$

The discrete format of Perona-Malik formula is stated as follows:

$$I_{ij}(n) = I_{ij}(n-1) + \frac{\lambda}{m_{ij}} \sum_{pem_{ij}} \mathrm{C}(\nabla I_{ij,p}) \nabla I_{ij,p} (16)$$

The model in (19) keeps the merits for the smoothing features of the tested digital image. On the other hand, the noisy image will be stationary and kept unchanged when the number of iterations reaches to a specific value [19]. As a consequence, study in [20] Y,You and other researchers proposed high order partial differential equations and [21] came up with regularization P-M method [20] in order to deal with the over smoothing results in the denoised images.

$$\frac{\partial I}{\partial t} = div\ (C(\lfloor \nabla I_\delta \rfloor) \nabla I) (17)$$

$$I_\delta\ (x, y, t) = G_\delta\ * I\ (x, y, t) (18)$$

Where Gaussian function is represented by G_δ and the variance is denoted by δ .

IV. PROPOSED DENOISING ALGORITHM

Practically, in the mixed noise digital images the pixels are divided into two main categories: the first part is pixels with impulse noise and the second part is the pixels that contaminated with additive white Gaussian noise. In addition, the proposed denoising technique has three pivotal parts: impulse noise detection, filtering of additive Gaussian noise and impulse noise.

A. Impulse noise detection and filtering processes

In order to detect the impulse noise in the contaminated digital images, we firstly should avoid the missed and false detection or by other word to increase the hit detection rate. The natural of impulse noise is superimposed on the clear image where its gray value has clear difference compared with its surrounding neighborhood pixels, where the high noise pixels denoted by $\mathrm{S_{ij}^{max}}$ and $\mathrm{S_{ij}^{min}}$ denotes to a dark spot or close to low dark pixels. As a result, impulse noise patches are definitely detected based on its characteristics and initiation pulse of PCNN related to its neurons organization that can be analysis as follows: The contaminated digital image S_{ij} is input to PCNN platform and 28×28 template W is used, similarly PCNN makes its pixel gray level for S_{ij}^{max} fire activation using the equations (6)-(10) during the initiate process. Furthermore, in the second round, the following process of PCNN automatically will capture sub-pixels in the range of $[S_{ij}^{max}/(1+\beta L_{ij}), S_{ij}^{max}]$ and then make two activated pixels $Y_{ij}=1$. Secondly, the contaminated image is transformed using the formula $S_{ij}=S^{max}-S_{ij}$, and the transformed image is handled by using PCNN based on the foreword method in order to get output result $Y_{ij}=1$. In addition, once the noise pixels have been detected by PCNN, the pixel patches related to $Y_{ij}=0$ is considered as signal points and at the same time, the pixel points related to $Y_{ij}=1$is the clear pixels as well. The detected noise patches are processed using second-generation wavelet filter (SGW) at this time; as a result, the sharp edges will be protected. In this regard, the impulse noise is filtered in this part of the denoising processes, but at the same time, in high level of noise it will show unsatisfied results due to the high density of noise.

For more pleasant results especially in edges and fine details textures, this study has introduced optimized multi-directional median filtering. In this regard, E_{ij}represents the gray pixel value which acquired after the first step, the size of the template filter is chosen to be $(2K+1)\times(2K+1)$, where K is an integer , $K\geq 1$, and as a result, the filter window is divided to six sub template windows as depicted in figure 1. Furthermore, E_{ij} is managed by using the up-mentioned six median filtering windows accordingly, then the six filtered results $E_{ij}{}^{`\varepsilon}$ , ($`\varepsilon = 1, 2,\ldots 6$) is achieved. At the final stage, the SGW filter with semi-soft thresholding is exploited for the six outputs $E_{ij}{}^{`\varepsilon}$ and E_{ij} , and then the filtered $Q_{ij}$is attained as a consequence of multi-directional median filtered output.

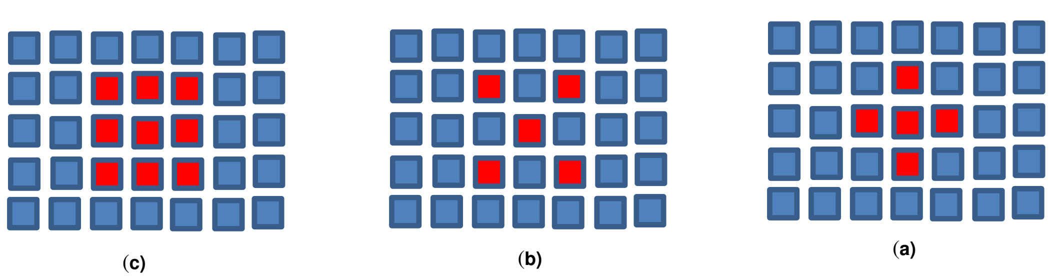

FIGURE 1. Several inter-connection liner filter: (a) four perpendicular pixels inter-connection, (b) four diagonal pixels inter-connection and (c) eight pixels inter-connection (full Neighbor interconnection)

B. Additive white GAUSSIAN Noise Removal process

In order to completely remove the additive white Gaussian noise AWGN from natural digital images, the proposed algorithm is splitting in order to introduce model based on P-M equation, which is stated as follows:

$$I_{ij}(n) = I_{ij}(n-1) + \tau \sum_{pem_{ij}} \frac{C_p(n-1)+C_{ij}(n-1)}{2} \left[I_p(n-1) + I_{ij}(n-1)\right] \quad (19)$$

Where $C_{ij}(n)$represents the distribution function at the ij position, it can be achieved by applying the generation of the regularization digital image gradient as it stated in $I_\delta(n) = G_\delta \times I(n)$ into $C(\mu)$ model. The digital image I with AWGN is processed by local rows and columns diffusion, which is changed to dimensional $M \times N$ column vector, then two intermediate results $I^1(n)$ and $I^2(n)$ are achieved respectively.

$$\begin{cases} \left(1 - 2\tau A^x(n)\right) I^1(n+1) = I(n) \\ \left(1 - 2\tau A^x(n)\right) I^2(n+1) = I(n) \end{cases} \quad (20)$$

Where,

$$(n+1) = \frac{1}{2} I^1(n+1) + I^2(n+1) \quad (21)$$

As a result, additive operator splitting formula can be achieved as follows:

$$I(n+1) = \frac{1}{2}\left[\left(1 - 2\tau A^x(n)\right)^{-1} + \left(1 - 2\tau A^y(n)\right)^{-2}\right] I(n) \quad (22)$$

τ is time step, $A^x(n)$ and $A^y(n)$represent $MN \times MN$ –D tri-diagonal matrices. The main procedure for removing AWGN I is as processed as follows: contaminated digital image is processed pixel by pixel from up to down, right to left, the output is then computed using $\left(1 - 2\tau A_i^x(n)\right) I_i^1(n+1) = I_i(n)$ then it applied to the all columns. The solution of the two equations $I^1(n+1)$ and $I^2(n+1)$ are achieved by using catch up method. Then by utilizing equation (15) repeatedly, the processed image $I(n+1)$ is finally got.

V. EXPERIMENTAL RESULTS AND DISCUSSIONS

In order to verify the performance of the proposed noise removal algorithm, qualitative and quantitative assessments have been conducted by utilizing several benchmark digital images that taken from well-known dataset [22]. From practical point of view, in order to achieve the mathematical form of the results, some factors and parameters have been declared as follows: $\alpha_T = 0.5$, T_0 =265, V_L =0.5, β =0.035, τ =3, W is a 5×5 mask matrix, The peak signal-to-noise ratio (PSNR), structural similarity index (SSIM) and Quality index (Q) are used as a quantitative metric for performance evaluation. All the experiments conducted in this paper are run on a 2.2 GHz Intel Core i7 processor with 8GB 1600 MHz DDR3 memory using MATLAB (R2018b). As depicted in figure 2, 3, 4 and 5 benchmark digital images of Couple, Cars, Lena and Jar, (a) is the original image, (b) represents the noisy image with 20% impulse noise density, (c) denoised image using NL-mean [11] (d) denoised image using deep learning-based methods such as DnCNN [23] (e) denoised image using weighted nuclear norm minimization (WNNM) [9] and finally denoised image using proposed method is depicted in (f).

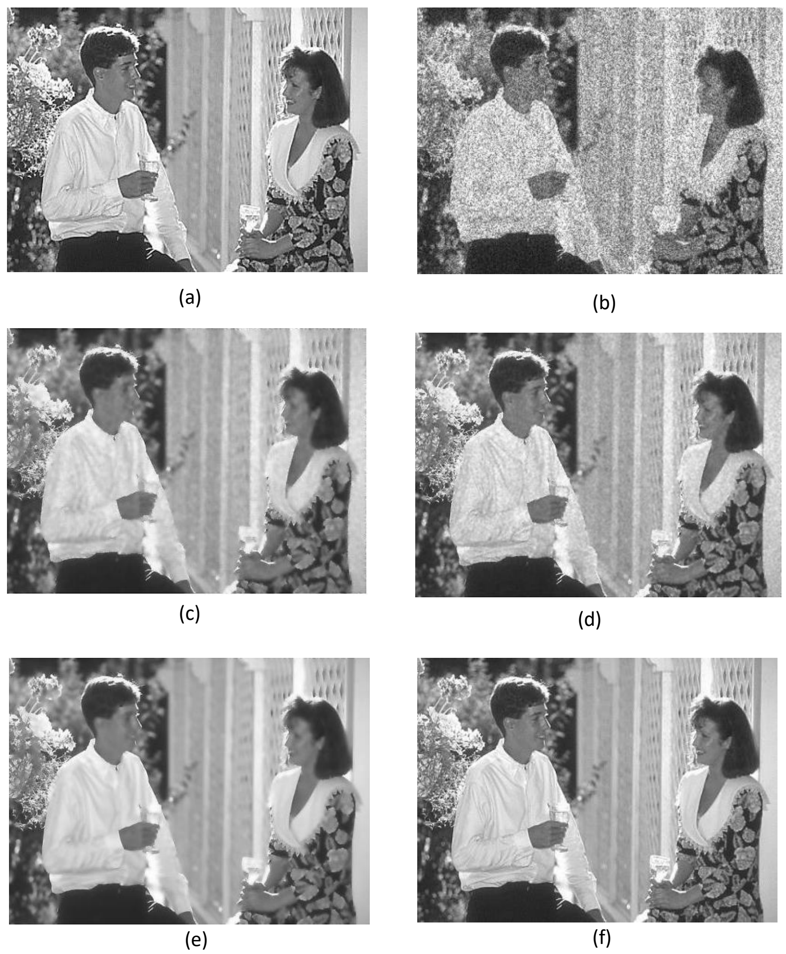

FIGURE 2. Filtering results using different methods for Couple image contaminated by mixed noise image (a) Original image (b) Image with 15% impulse and 0.001variance noise; (c) DnCNN method; (d) WNNM filter, (e) NL-mean filter (f) the proposed method.

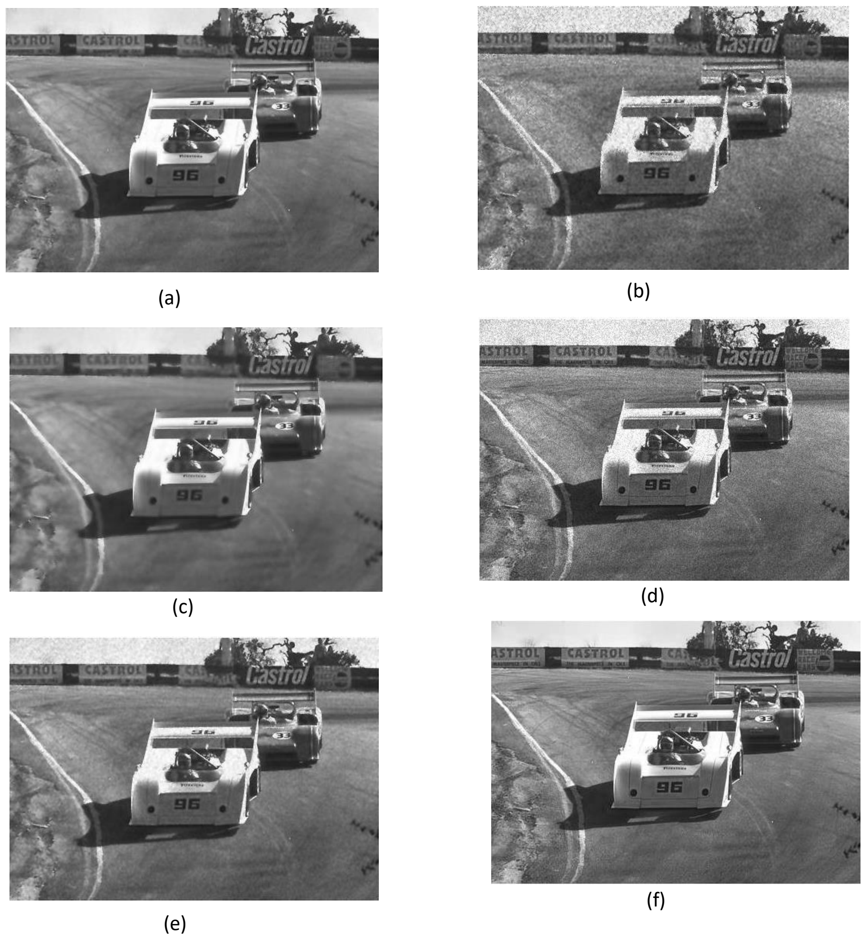

FIGURE 3. Filtering results using different methods for Cars image contaminated by mixed noise image (a) Original image (b) Image with 15% impulse and 0.001variance noise; (c) DnCNN method; (d) WNNM filter; (e) NL-mean filter (f) the proposed method.

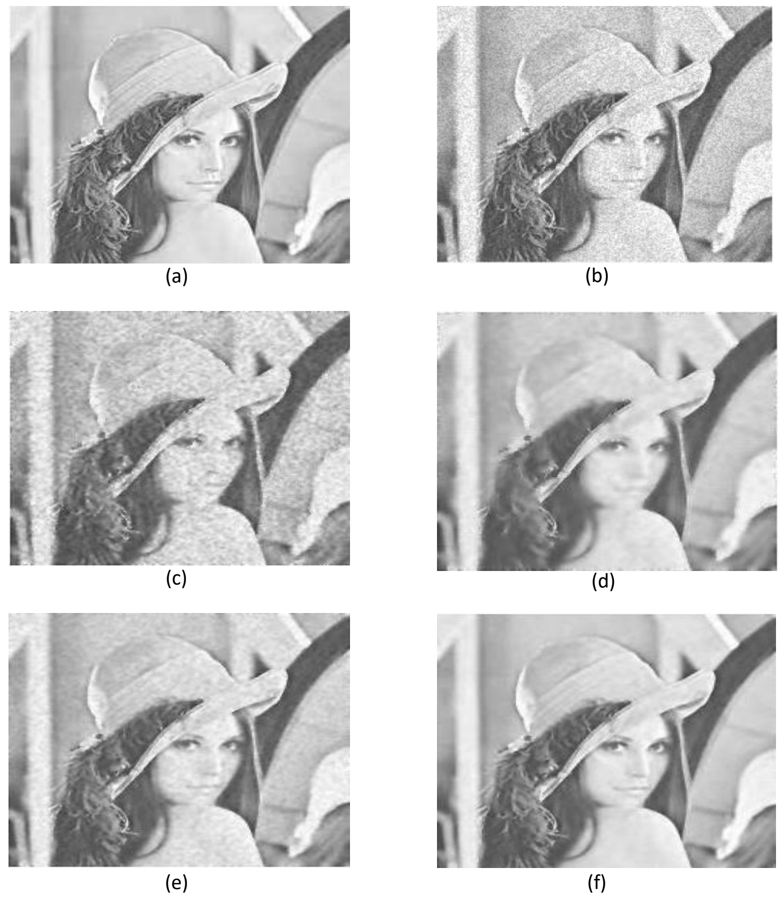

FIGURE 4. Filtering results using different methods for Lena image contaminated by mixed noise image (a) Original image (b) Image with 15% impulse and 0.001variance noise; (c) DnCNN method; (d) WNNM filter; (e) NL-mean filter (f) the proposed method.

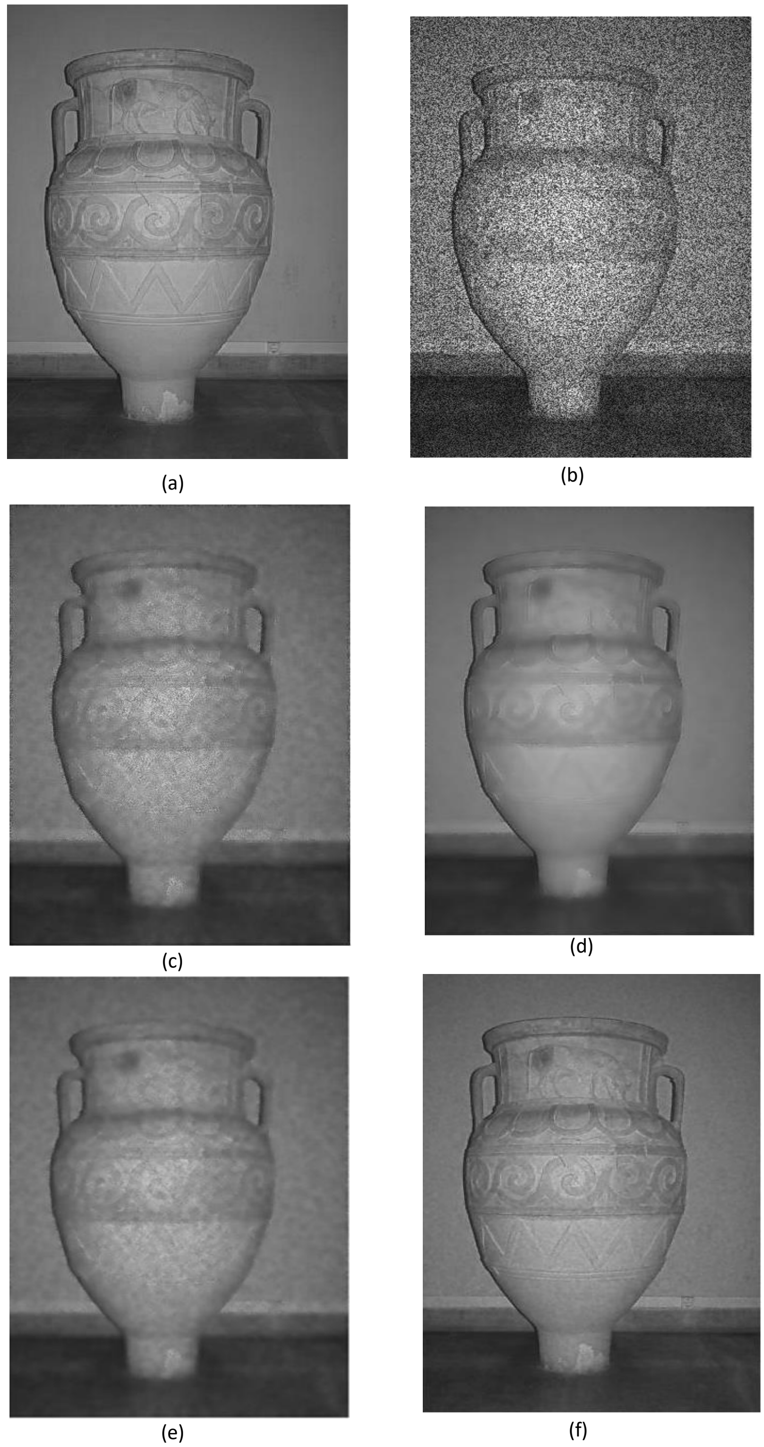

FIGURE 5. Filtering results using different methods for Jar image contaminated by mixed noise image (a) Original image (b) Image with 15% impulse and 0.001variance noise; (c) DnCNN method; (d) WNNM filter, (e) NL-mean filter (f) the proposed method.

As can be seen from several benchmark images, different denoising techniques have certain inhibitory influence for impulse in the contaminated images, but each the algorithm of denoising effect is not identical. The denoised images by using NL-means showed poor results in the flat images such as Jar, and when it comes to the fine edges, it shows good quality results. On the other hand, digital images that resulted from DnCNN shows poor results in fine details as it depicted in Lena hat and hair. The results that achieved from WNNM filter have Over-smoothing in the edges, as it can be clear noticeable in Cars benchmark image. Furthermore, the benchmark images that resulted from the proposed method showed high quality performances as can be seen in edges and fine details and textures of several investigated images. The proposed technique holds outstanding performance when it comes to the visual effect and objective assessment. In addition as Tables 1, 2, 3 and 4 depicted objective analyses with PSNR, SSIM and Q-index with impulse noise density 15%. As can be seen, still the proposed method showed its superior performances where it reflects better capability for removing impulse noise, more steady robustness and wider range of adaptability.

Table. 1: Experimental results of Couple benchmark image represented by PSNR, SSIM and Q-INDEX using several noise removal methods for mixed image (15% impulsive noise)

Method	Assessments		
	PSNR	SSIM	Q-index
Noisy image	14.23	0.5633	18.12
NL-mean	22.18	0.7662	22.22
WNNM	22.85	0.8021	25.09
DnCNN	23.19	0.8555	27.12
Proposed Method	24.66	0.8787	27.89

Table. 2: Experimental results of Cars benchmark image represented by PSNR, SSIM and Q-INDEX using several noise removal methods for mixed image (15% impulsive noise)

Method	Assessments		
	PSNR	SSIM	Q-index
Noisy image	15.26	0.6233	17.09
NL-mean	24.31	0.7732	31.25
WNNM	24.05	0.7709	26.20
DnCNN	25.29	0.8109	28.15
Proposed Method	26.17	0.8443	29.06

Table. 3: Experimental results of Lena benchmark image represented by PSNR, SSIM and Q-INDEX using several noise removal methods for mixed image (15% impulsive noise)

Method	Assessments		
	PSNR	SSIM	Q-index
Noisy image	13.12	0.6767	17.77
NL-mean	17.24	0.7772	20.09
WNNM	17.87	0.8013	22.12
DnCNN	19.99	0.8323	23.37
Proposed Method	21.22	0.8377	23.76

Table. 4: Experimental results of Jar benchmark image represented by PSNR, SSIM and Q-INDEX using several noise removal methods for mixed image (15% impulsive noise)

Method	Assessments		
	PSNR	SSIM	Q-index
Noisy image	15.55	0.6303	17.09
NL-mean	16.18	0.6462	18.99
WNNM	17.09	0.7039	19.97
DnCNN	19.89	0.7855	20.62
Proposed Method	22.03	0.7939	23.09

In order to increase the reliability of the proposed method, The execution time of the proposed method is much faster than NL-mean and other learning methods due to the internal structure of the proposed method which is the locality, i.e., it processes on only the target patch. On the other hand, the nonlocal methods like DnCNN and WNNM need huge computation costs since they have to search for similar patches as the target patch throughout the whole image. In addition, our method does not need self-exemplars and a huge number of parameters as DnCNN

Table.5: Comparison of computational time [s] in the test images with 15% impulsive noise for several noise removal techniques

Denoising Method	Benchmark Image			
	Couple	Cars	Lena	Jar
NL-mean	6.4	3.4	1.92	1.41
WNNM	1.62	0.5	0.5	0.4
DnCNN	8.3	7.4	3.27	4.4
Proposed Method	1.8	1.31	0.8	0.7

VI. CONCLUSIONS

In this study, a novel technique of mixed image denoising based on PCNN and regularization P-M equation is introduced. In the first stage, the sub-pixel locations of impulse noise are positioned by utilizing PCNN and then the impulse noise is suppressed throughout multi-direction information of the median filter. Furthermore, the image Gaussian noise is successfully removed by using additive operator splitting method based on the PM equation in the second stage of the proposed method. Experimental results showed that the proposed technique can effectively filter the mixed image noise as well as preserve the denoised digital image fine details and its tiny textures, and has better processing time and subjective vision effect and objective quality than state of the art denoising methods. In the future, we will extend to RGB color image denoising and apply to images with unknown noise levels using noise level estimation methods. Furthermore, it is important to extend to real-world image denoising as well.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Author Contributions Asem Khmag conceived the presented idea and performed the computations. In addition, he investigates smart queue on edge and supervised the findings of this work. Moreover, the author discussed the results and contributed to the final manuscript.

Funding The author received no financial support for the research, authorship, and/or publication of this article.

References

[1] Amitab Khwairakpam1, Debdatta Kandar,Babusena Paul, "Noise reduction in synthetic aperture radar images using fuzzy logic and genetic algorithm, " *Microsystem Technologies*,vol.25,pp.1743-1752,June,2019.
[2] H. Hosseini,F. Hessar, and F. Marvasti,"Real-Time Impulse Noise Suppression from Images Using an Efficient Weighted-Average Filtering, " *Signal Processing Letters,* vol.22, pp.1050-1054, Aug. 2015.
[3] Q. Liu, L.P. Xu, Y.D. Ma, Y. Wang,"Bilateral Filtering Algorithm for Image Processing Based on Pulse Coupled Neural Networks， " *International Journal of Computers and Application,* vol.34, pp. 217-222,Oct. 2012.
[4] S. Shaik Majeeth, C. Nelson Kennedy Babu, "Gaussian Noise Removal in an Image using Fast Guided Filter and its Method Noise Thresholding in Medical Healthcare Application，"*Journal of Medical Systems*,vol. 43,pp. 280-283 ,July,2019.
[5] A Palacios-Enriquez, V. Ponomaryov, R. Reyes-Reyes1,S.Sadovnychiy, "Sparse Technique for Images Corrupted by Mixed Gaussian-Impulsive Noise， "*Circuits Syst Signal Process* ,vol.37,pp.5389-5416,Apr. 2018.
[6] Chun Li, Yuepeng Li, Zhicheng Zhao, Longlong Yu, Ze Luo, "A mixed noise removal algorithm based on multi-fidelity modeling with nonsmooth and nonconvex regularization,"*Multimedia Tools and Applications* vol.78,pp.23117-23140, Apr.2019.
[7] J.D. Mendiola, and I.R. Terol, "Filtering of mixed Gaussian and impulsive noise using morphological contrast detectors," *Image Processing, IET,* vol. 8, pp.13 -14, Mar.
2014.
[8] J.G. Camarena, V. Gregori, S. Morillas, and A. Sapena,"A Simple Fuzzy Method to Remove Mixed Gaussian-Impulsive Noise From Color Images," *IEEE Transactions on Fuzzy Systems,* vol.21,pp. 971-978,May . 2012.
[9] S. Gu, L. Zhang, W. Zuo, and X. Feng, "Weighted nuclear norm minimization with application to image denoising," in 2014 IEEE Conference on Computer Vision and Pattern Recognition, 2014.
[10] A Khmag, AR Ramli, SJ bin Hashim, SAR Al Haddad, Additive noise reduction in natural images using second-generation wavelet transform hidden Markov models. IEEJ Transactions on Electrical and Electronic Engineering 11 (3), 339-347 (2017).
[11] Khmag, A., Al Haddad, S. A. R., Suhimi, Y., Kamarudin, N. Denoising of natural images through robust wavelet thresholding and genetic programming. The Visual Computer, 33(9), 1141-1154, 2017.
[12] K. Dabov, A. Foi, V. Katkovnik, and K. Egiazarian, "Image denoising by sparse 3-d transform-domain collaborative filtering," IEEE Transactions on Image Processing, vol. 16, no. 8, pp. 2080–2095, 2007.
[13] J. Mairal, M. Elad, and G. Sapiro, "Sparse representation for color image restoration," IEEE Trans. Image Process., vol. 17, no. 1, pp. 53–69, Jan 2008.
[14] J. Mairal, F. Bach, J. Ponce, G. Sapiro, and A. Zisserman, "Non-local sparse models for image restoration," in Proc. IEEE 12th Int. Conf. Comput. Vis., 2009, pp. 2272–2279.
[15] Khmag, A., Al Haddad, S. A. R., Ramlee, R. A., Kamarudin, N., & Malallah, F. L. Natural image noise removal using nonlocal means and hidden Markov models in transform domain. The Visual Computer, 34(12), 1661-1675, 2018.
[16] Chong Shen, Ding Wang, Shuming Tang, Huiliang Cao, Jun Liu, "Hybrid image noise reduction algorithm based on genetic ant colony and PCNN," *Vision Computer,* vol. 33, pp.1373-1384, Oct, 2017.
[17] Chin Sheng Chen, Chi-Min Weng, Chien-Chuan Tseng, "An efficient detection algorithm based on anisotropic diffusion for low-contrast defect," *The International Journal of Advanced Manufacturing Technology,* vol.94, pp.4427-4449, Oct.2018.
[18] Xiao juan Zhang, Wanzhou Ye, "An adaptive second-order partial differential equation based on TV equation and p-Laplacian equation for image denoising,"*Multimedia Tools and Applications,*vol.78, pp.18095-18112, Jan.2019.

[19] Haider Ali, Shah Faisal, Ke Chen, Lavdie Rada, "Image selective segmentation model for multi-regions within the object of interest with application to medical disease," *The Visual Computer,* vol. 20, pp.1845-1, Apr.2020.
[20] Y.L. You, and M. Kaveh, "Four-Order Partial Differential Equations for Noise Removal," *IEEE Trans.On Image Processing*, vol.9, pp. 1723-1729, Oct .2000.
[21] F. Gatte, P.L. Lions, J. M. Morel, and T. Coll, "Image selective smoothing and edge detection by nonlinear diffusion," *SIAM Journal on Numerical Analysis,* vol.29, pp.182-193, Jan.1999.
[22] M.Gharbi, G.Chaurasia, S.Paris, and F.Durand, "Deep joint demosaicking and denoising ," ACM Trans. Graph., vol. 35, no.6, 2016, Art. No. 19.
[23] K. Zhang, W. Zuo, Y. Chen, D. Meng, and L. Zhang, "Beyond a gaussian denoiser: Residual learning of deep cnn for image denoising," IEEE Trans. Img. Proc., vol. 26, no. 7, pp. 3142–3155, July 2017.